# Lesson 13.1 • Defining the Circular Functions

**Name** _____ **Period** _____ **Date** _____

**1.** Find the exact value of each expression.

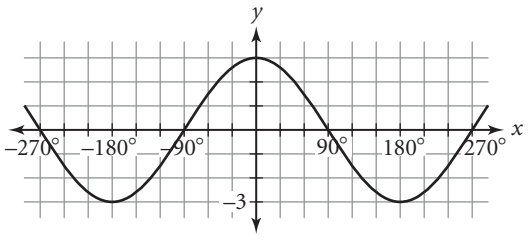
**a.** $\cos 45°$ **b.** $\sin(-30°)$ **c.** $\cos 240°$ **d.** $\sin 360°$

**2.** Use your calculator to find each value, approximated to four decimal places. Then draw a diagram in a unit circle to represent the value. Name each reference angle.

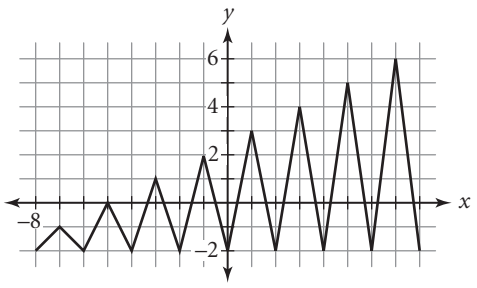
**a.** $\sin 37°$ **b.** $\cos 115°$ **c.** $\sin(-21°)$

**3.** Determine whether each function whose graph is shown below is periodic or not periodic. For each periodic function, identify the period.

**a.**




**b.**




**4.** Identify an angle $\theta$ that is coterminal with the given angle. Use domain $0° \leq \theta \leq 360°$.

**a.** $-42°$ **b.** $415°$ **c.** $913°$ **d.** $-294°$

**5.** Let $\theta$ represent the angle between the $x$-axis and the ray with endpoint $(0, 0)$ passing through $(-3, 3)$. Find $\sin\theta$ and $\cos\theta$.

# Lesson 13.2 • Radian Measure

Name _____ Period _____ Date _____

**1.** Convert between radians and degrees. Give exact answers.

**a.** $\frac{5\pi}{4}$ **b.** $15°$ **c.** $330°$ **d.** $-\frac{2\pi}{3}$

**e.** $-140°$ **f.** $780°$ **g.** $-\frac{11\pi}{6}$ **h.** $\frac{17\pi}{15}$

**2.** Find the length of the intercepted arc for each central angle.

**a.** $r = 8$ and $\theta = \frac{5\pi}{4}$ **b.** $r = 5.4$ and $\theta = 2.5$ **c.** $d = 3$ and $\theta = \frac{\pi}{12}$

**3.** Solve for $\theta$.

**a.** $\sin\theta = \frac{\sqrt{3}}{2}$ and $90° \leq \theta \leq 180°$ **b.** $\sin\theta = -1$ and $0° \leq \theta \leq 360°$

**c.** $\cos\theta = \frac{1}{2}$ and $\pi \leq \theta \leq 2\pi$ **d.** $\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ and $0 \leq \theta \leq \frac{\pi}{2}$

**4.** The minute hand on a watch is 85 mm long. Round your answers in 4a and b to the nearest tenth, and in 4c to the nearest thousandth.

**a.** What is the distance the tip of the minute hand travels, in mm?

**b.** At what speed is the tip moving, in mm/min?

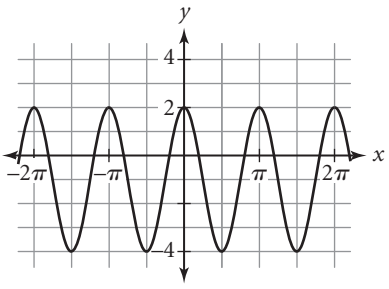
**c.** What is the angular speed of the tip, in radians/min?

# Lesson 13.3 • Graphing Trigonometric Functions

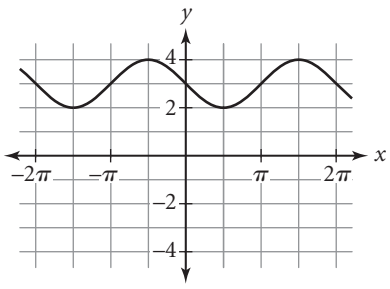
**Name** _____ **Period** _____ **Date** _____

**1.** Write an equation for each sinusoid as a transformation of the graph of either $y = \sin x$ or $y = \cos x$. More than one answer is possible. Describe the amplitude, period, phase shift, and vertical shift of each graph.

**a.**

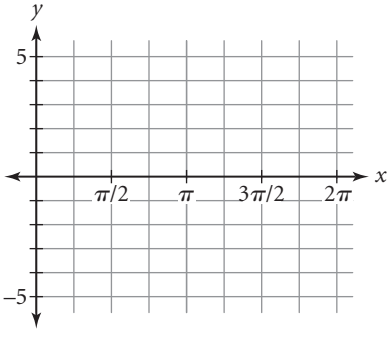



**b.**

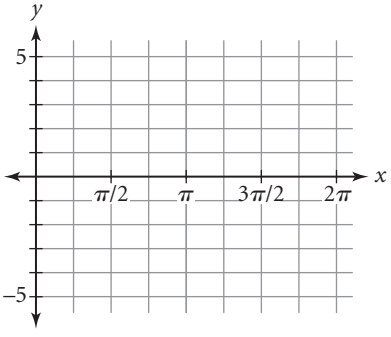



**2.** Graph each function for the interval $0 \leq \theta \leq 2\pi$.

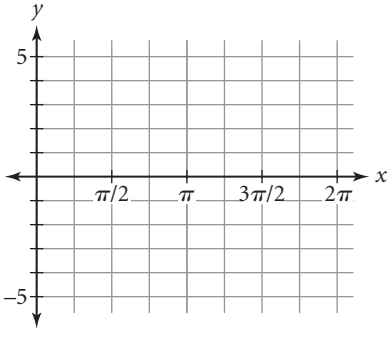
**a.** $y = 2\sin x + 1$




**b.** $y = -\tan x$




**c.** $y = 3\cos\left(x + \frac{\pi}{4}\right)$




**3.** Write an equation for each sinusoid with the given characteristics.

**a.** A cosine curve with amplitude 2.5, period $2\pi$, and phase shift $\frac{\pi}{4}$

**b.** A sine function with minimum value 2, maximum value 8, and one cycle starting at $x = 0$ and ending at $x = \frac{3\pi}{2}$

# Lesson 13.4 • Inverses of Trigonometric Functions

Name _____ Period _____ Date _____

**1.** Find the principal value of each expression to the nearest tenth of a degree and then to the nearest hundredth of a radian.

**a.** $\sin^{-1} 0.5976$

**b.** $\cos^{-1}(-0.0315)$

**c.** $\cos^{-1}(0.8665)$

**d.** $\sin^{-1}(-0.6789)$

**2.** Find all four values of $x$ between $-2\pi$ and $2\pi$ that satisfy each equation.

**a.** $\sin x = \sin\frac{2\pi}{3}$

**b.** $\cos x = \cos\frac{5\pi}{12}$

**c.** $\sin x = \sin 1.25$

**d.** $\cos x = \cos 0.73$

**e.** $\cos x = \cos\frac{3\pi}{5}$

**f.** $\sin x = \sin\left(-\frac{5\pi}{6}\right)$

**3.** Find values of $x$ approximate to three decimal places that satisfy the conditions given.

**a.** Find the first two positive solutions of $\sin x = 0.9827$.

**b.** Find the first two positive solutions of $\cos x = 0.7205$.

**4.** Find the measure of the smallest angle of a triangle in which two of the sides have lengths 13 cm and 28 cm, and in which the angle opposite the 28 cm side measures 110°.

# Lesson 13.5 • Modeling with Trigonometric Equations

Name _____ Period _____ Date _____

**1.** Find all solutions for $0 \leq x < 2\pi$. Give exact values in radians.

**a.** $\sin x = 1$

**b.** $\cos x = 0$

**c.** $\cos 3x = 0.5$

**d.** $2\sin\left(\frac{1}{2}x\right) = 1$

**e.** $\sin\frac{x}{2} = \frac{\sqrt{2}}{2}$

**f.** $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$

**2.** Find all solutions for $0 \leq x < 2\pi$, rounded to the nearest hundredth.

**a.** $3\cos(x + 0.4) = 2.6$

**b.** $5 + 0.5\sin 2x = 4.7$

**3.** Consider the graph of the function

$$h = 8.5 + 5\sin\left[\frac{2\pi(t-4)}{7}\right]$$

**a.** What is the vertical translation?

**b.** What is the average value?

**c.** What is the vertical scale factor?

**d.** What is the minimum value?

**e.** What is the maximum value?

**f.** What is the amplitude?

**g.** What is the horizontal scale factor?

**h.** What is the period?

**i.** What is the horizontal translation?

**j.** What is the phase shift?

**4.** The number of hours of daylight on any day of the year in Philadelphia, Pennsylvania, is modeled using the equation

$$y = 12 + 2.4\sin\left[\frac{2\pi(x-80)}{365}\right]$$

where $x$ represents the day number (with January 1 as day 1). This equation assumes a 365-day year (not a leap year).

**a.** Find the number of hours of daylight in Philadelphia on day 172, the longest day of the year (the summer solstice).

**b.** Find the day numbers of the two days when the number of hours of daylight is closest to 13.

# Lesson 13.6 • Fundamental Trigonometric Identities

Name _____ Period _____ Date _____

**1.** Evaluate. Give exact values.

**a.** $\tan\frac{\pi}{3}$ **b.** $\cot\frac{5\pi}{6}$ **c.** $\sec\frac{\pi}{4}$

**d.** $\csc\frac{4\pi}{3}$ **e.** $\cot\pi$ **f.** $\csc\frac{7\pi}{6}$

**2.** Find another function that has the same graph as each function below. (More than one answer is possible.)

**a.** $y = \tan(x + \pi)$ **b.** $y = \sin(x - 2\pi)$ **c.** $y = -\csc(x - 2\pi)$

**3.** Use trigonometric identities to rewrite each expression in a simplified form containing only sines and cosines, or as a single number.

**a.** $\tan\theta + \sec\theta$ **b.** $\left(\sec^2\theta - \tan^2\theta\right)\cos^2\theta$

**c.** $\cot\theta\sin^2\theta - \tan\theta\cos^2\theta$ **d.** $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta)$

**4.** Determine whether each equation is an identity or not an identity.

**a.** $\sin(A + \pi) = \cos A$ **b.** $\tan\left(A - \frac{\pi}{2}\right) = -\cot A$

**c.** $\csc^2 A = \cot A(\tan A + \cot A)$ **d.** $\sec A \cot A = \csc A$

# Lesson 13.7 • Combining Trigonometric Functions

Name _____ Period _____ Date _____

**1.** Use a graph or substitute values of $A$ and $B$ to decide whether each equation is an identity or not an identity.

**a.** $\cos 2A = 1 - 2\sin^2 A$

**b.** $\sin(2\pi - A) = \sin A$

**c.** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$

**d.** $\cos(A - B) = \cos A \cos B - \sin A \sin B$

**2.** Use identities from this lesson to derive an identity for $\sin 3A$ in terms of $\sin A$ and $\cos A$. Show the steps you used to derive the identity.

**3.** Rewrite each expression with a single sine or cosine.

**a.** $\sin 3.2 \cos 2.5 - \cos 3.2 \sin 2.5$

**b.** $2 \sin 4.8 \cos 4.8$

**c.** $\cos^2 0.8 - \sin^2 0.8$

**d.** $\cos 0.6 \cos 2.1 + \sin 0.6 \sin 2.1$

**4.** Use a sum or difference identity to find the exact value of each expression.

**a.** $\sin\left(-\dfrac{\pi}{12}\right)$

**b.** $\sin 105°$

**c.** $\cos 285°$

**5.** Find the exact values of $\sin 2x$, $\cos 2x$, and $\tan 2x$ for each set of conditions.

**a.** $\sin x = \dfrac{3}{5}$, $0 \le x \le \dfrac{\pi}{2}$

**b.** $\cos x = -\dfrac{5}{13}$, $\dfrac{\pi}{2} \le x \le \pi$

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