# Mathematics

## Key Concept Chart

## Form 3

## Classification of Numbers

The diagram below summarises how numbers are classified.

- Real numbers ($\mathbb{R}$)
  - Rational numbers ($\mathbb{Q}$)
    - Integers ($\mathbb{Z}$)
      - Negative integers
      - Natural numbers ($\mathbb{N}_0$)
        - Zero
        - Whole numbers ($\mathbb{N}$)
    - Fractions
  - Irrational numbers ($\mathbb{Q}'$)

## Patterns and Sequences

- A sequence or pattern of numbers is how a set or group of numbers is generated from a single number.
- A sequence may be linear or geometrical. A linear sequence can be defined by the formula: $T_n = (n - 1)\,d + a$, where $T_n$ is the n<sup>th</sup> term, $d$ is the common difference between the terms and $a$ the first number.
- Example: the 10<sup>th</sup> number in the sequence: 3, 9, 15, 21 is $T_n = (10 - 1) \times 6 + 3 = 57$.

## Money

| Concept | Formulae | |
|---|---|---|
| Interest | Simple: $A = P(1 + in)$ where $A$ is the accumulated amount, $P$ the initial amount, $i$ the rate and $n$ the period | Compound: $A = P(1 + i)^n$ where $A$ is the accumulated amount, $P$ the initial amount, $i$ the interest and $n$ the period |
| Depreciation | Straight line $A = P(1 - ni)$ | Reducing balance $A = P(1 - i)^n$ |
| Appreciation | Straight line $A = P(1 + ni)$ | Increasing balance $A = P(1 + i)^n$ |

## Indices

Indices are used to write repeated multiplication or division in a shorter way.

### Rules of indices

$a^m = a \times a \times a \times a \times \ldots (m \text{ times})$

$a^{-m} = \dfrac{1}{a \times a \times a \times a \times (m \text{ times})} = \dfrac{1}{a^m}$

$a^m \times a^n = a^{m+n}$ $\quad a^m \div a^n = a^{m-n}$

$(ab)^n = a^n b^n$ $\quad \left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

### Standard form

Standard form is where numbers are written in the form $a \times 10^n$; $a$ is a decimal number such that $1 \leq a < 10$ and $n$ is an integer.

Standard form is used to represent very large or very small numbers.

## Matrices

A **matrix** (plural **matrices**) is an array of numbers written in rows and columns in parentheses.

$$A = \begin{bmatrix} a & b & c & d \\ l & m & n & o \\ w & x & y & z \end{bmatrix} \qquad B = \begin{bmatrix} a & b & c \\ d & l & m \\ n & o & w \\ x & y & z \end{bmatrix}$$

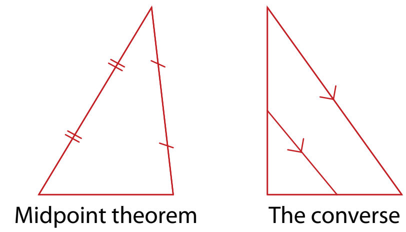
- The order of a matrix is the number of rows by the number of columns, in that order.
  - Matrix A above has the order 3 by 4
  - Matrix B above has the order 4 by 3
- Entries in a matrix are the individual numbers.
- Two matrices are equal only if corresponding entries are the same.
- Matrix addition is possible only when two matrices have the same order.
- Matrix multiplication is possible if the number of columns in the first matrix is the same as the number of rows in the second matrix.

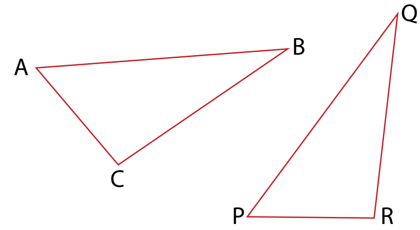
# Geometry

## Similar triangles

Properties of triangles:

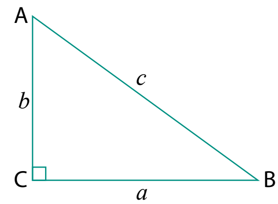
1. **Midpoint theorem** states that a line joining midpoints of two sides of a triangle is parallel to the third side.
2. **The converse**: If a line joining two points on two sides of a triangle is parallel to the line on the third side, then the two points on the sides of the triangle are midpoints of those sides.
3. **Congruency**: Two triangles are congruent if the corresponding angles are equal and the corresponding sides are the same length. In the diagrams on the right you will see that: AB = PQ, BC = QR and AC = PR and ∠ A = ∠ P, ∠ B = ∠ Q and ∠ C = ∠ R.




## Pythagoras theorem

Pythagoras theorem states that for any right-angled triangle, the sum of the squares of the length of the two shorter sides is equal to the square of the longest side.




$c^2 = a^2 + b^2$

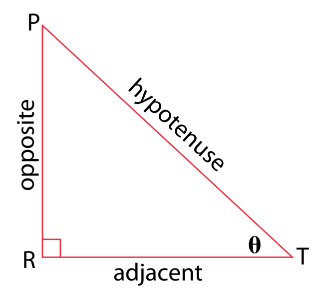
or

$c = \sqrt{a^2 + b^2}$

## Trigonometric functions

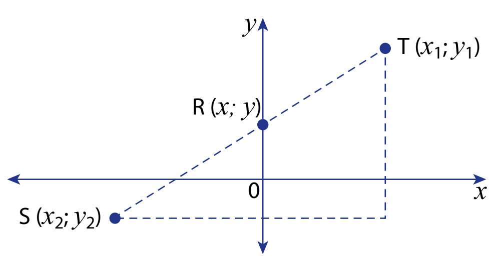
For any right-angled triangle, if two sides and one angle (other than the right angle) are known, then the rest of the angles and sides can be found using trigonometry. The trigonometric functions are:

- $\text{Sin}\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- $\text{Cos}\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
- $\text{Tan}\theta = \dfrac{\text{opposite}}{\text{adjacent}}$



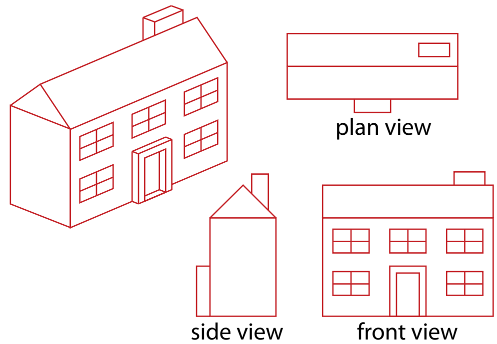

## Straight line segments

- The distance ($d$) between two points ($x_1$, $y_1$) and ($x_2$, $y_2$) on the Cartesian coordinates is given by:
  $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ or
  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The midpoint ($x$, $y$) of two points ($x_1$, $y_1$) and ($x_2$, $y_2$) on the Cartesian plane is given by the formula:
  $x = \dfrac{(x_1 + x_2)}{2}; y = \dfrac{(y_1 + y_2)}{2}$




## Plans and elevations

A three-dimensional diagram can be viewed from three different directions: the **plan view**, the **front view** and the **side view**.




## Bearings

A bearing is the distance which a body moves, measured from the North line in a clockwise direction.

# Measures

## Area and perimeter

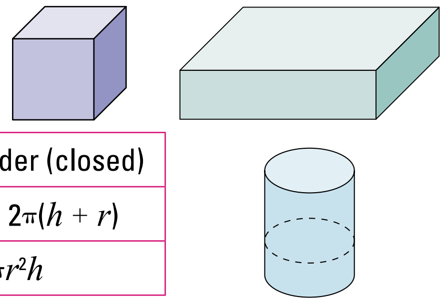
Formulae for the area and perimeter of some shapes:

| Shape | Square | Rectangle | Triangle | Circle | Rhombus | Parallelogram |
|---|---|---|---|---|---|---|
| Area | A = $s^2$ | A = $lb$ | A = $\frac{1}{2}(bh)$ | A = $\pi r^2$ | A = $bh$ | A = $bh$ |
| Perimeter | P = $4s$ | P = $2(l + b)$ | P = $a + b + c$ | C = $2\pi r$ | P = $4s$ | P = $2(a + b)$ |

## Ratios of areas of 2-D shapes

For any two-dimensional shape, the ratio of areas of two similar shapes is the square of the ratio of length. If the ratio of lengths is $(\frac{a}{b})$ then the ratio of areas is $(\frac{a}{b})^2$.

## Surface area and volume of solids



Surface area (SA) is the sum of the areas of the faces of a solid.
Surface areas and volumes (V) of some solids:

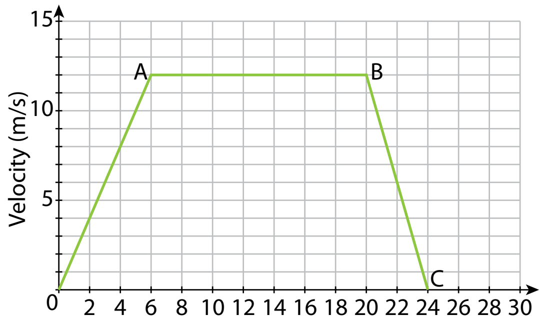
| Solid | Cube | Rectangular prism (cuboid) | Sphere | Cylinder (closed) |
|---|---|---|---|---|
| SA | SA = $6s^2$ | SA = $2(lb + bh + lh)$ | SA = $4\pi r^2$ | SA = $2\pi(h + r)$ |
| V | V = $s^3$ | V = $l \times b \times h$ | V = $(\frac{4}{3})\pi r^3$ | V = $\pi r^2 h$ |

## Ratio of length to surface area to volume

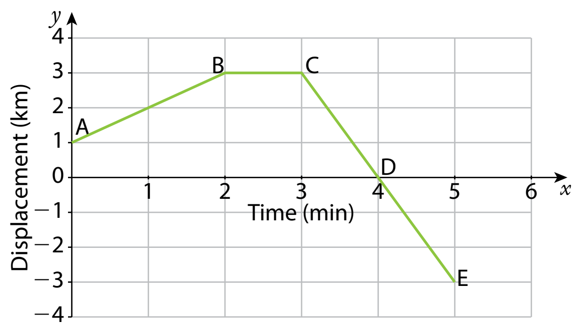
If the ratio of the lengths of two solids is $(\frac{a}{b})$, then the ratio of the surface areas is $(\frac{a}{b})^2$, and the ratio of the volumes is $(\frac{a}{b})^3$.

## Travel graphs

There are four types of travel graphs: Distance-time graph, speed-time graph, displacement-time graph and velocity-time graph.

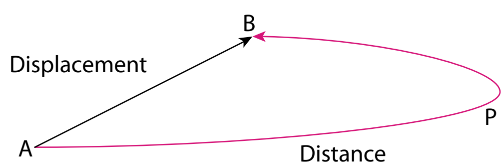



Velocity-time graph




Displacement-time graph

**Displacement** is the shortest distance between two places. The straight line AB in the diagram to the right shows displacement.




**Distance** is the actual path travelled between A and B. The path from A to B through P is the distance travelled.
**Speed** is the distance travelled in a given time.
**Velocity** is the displacement in a given time.

# Algebra

## Variables

Factorise quadratic expressions:

$x^2 + bx + c$

Example: $x^2 - 4x - 21$

$b = -4$ and $c = -21$

1. Find the factors of $c$ that have the sum $b$.
   These are the factors of $c$:
   $(-1 \times 21), (-21 \times 1), (-3 \times 7), (-7 \times 3)$.
   The factors that add up to $b$ are $(-7 \times 3)$.

$x^2 - 7x + 3x - 21$

2. Write $bx$ as expressions of the factors of $b$.
   Factorise the two sides: $x(x - 7) + 3(x - 7)$.
   Factorise further until you do not have common factors. The answer is $(x + 3)(x - 7)$.

$ax^2 + bx + c$

Example: $3x^2 - 8x + 4$

$a = 3, b = -8$ and $c = 4$

3. Find the factors of $a \times c$ that have the sum $b$.
   These are the factors of $a \times c$:
   $(-1 \times -12), (-3 \times -4), (-2 \times -6)$.
   The factors that add up to $b$ are $(-2 \times -6)$.

$3x^2 - 2x - 6x + 4$

4. Write $bx$ as the sum of two expressions.
   Factorise the two factors: $x(3x - 2) - 2(3x - 2)$.
   Factorise further: $(3x - 2)(x - 2)$.

## Formulae

Changing the subject of the formula means rewriting the formula with a different letter as the subject.

For instance, in the formula $T_n = (n - 1)d + c$, $T_n$ is the subject. To make $d$ the subject we first subtract $c$, then divide by $(n - 1)$ so that we have $d$ on its own on one side of the equal sign.

The new formula is: $d = \frac{(T_n - c)}{(n - 1)}$.

## Quadratic equations

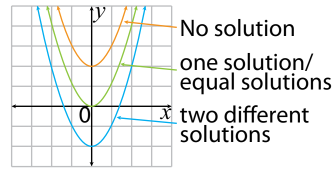
Solving quadratic equations by factorising:

This is based on the fact that if you multiply two numbers and you get zero, then one (or both) of the numbers is zero.

If $(ax + b)(x + c) = 0$,
then either $ax + b = 0 \therefore x = (\frac{-b}{a})$
or $x + c = 0$ » $x = -c$

## Quadratic graphs

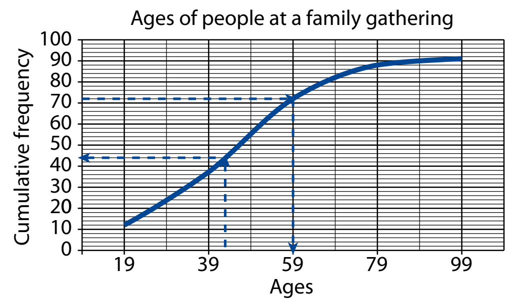
The solution of the quadratic equation $ax^2 + bx + c = 0$ is the $x$-values where the graph cuts the $x$-axis. The following diagrams show the types of quadratic graphs and the implications of the solutions.




# Statistics and Probability

## Cumulative frequency graphs

Help work out the mean, modal and median classes of a grouped frequency distribution. The cumulative graph is drawn by using the upper boundaries of given classes and the added on numbers of the frequency. This is an example of a cumulative frequency graph.

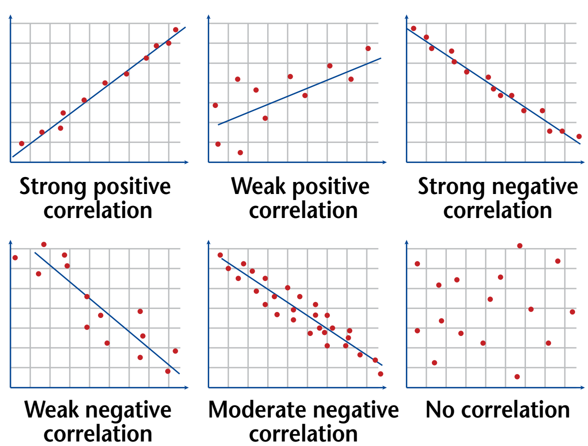


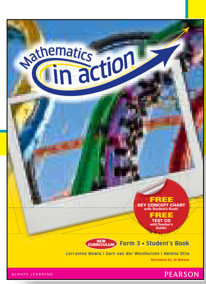

## Scatter graphs

These are used to compare two sets of data to find out if the two are somehow related. The following diagrams show scatter graphs and their implications.

**Scatterplots and Correlation**

Correlation - indicates a relationship (connection) between two sets of data.





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