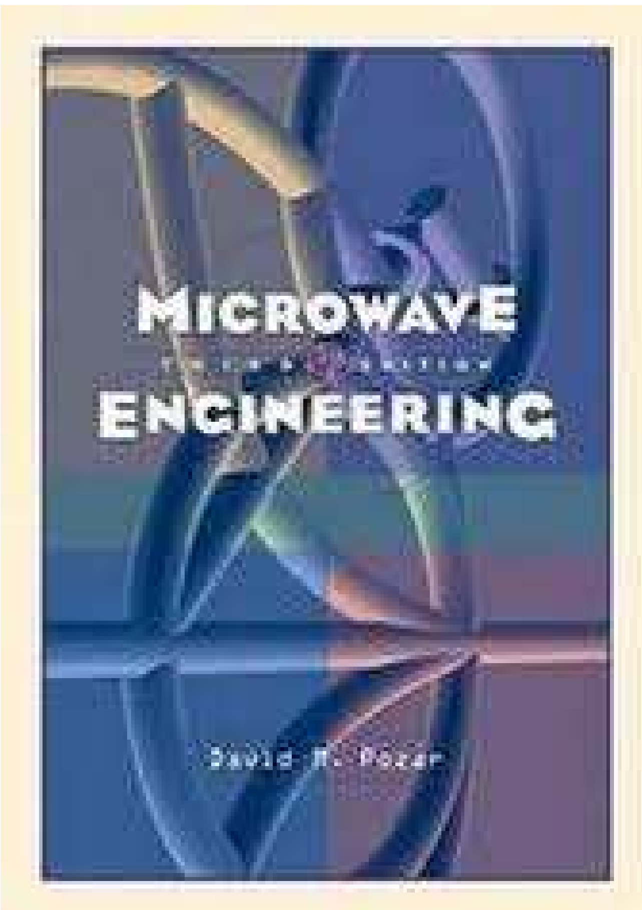
ECE 5317-6351
Microwave Engineering

Fall 2011



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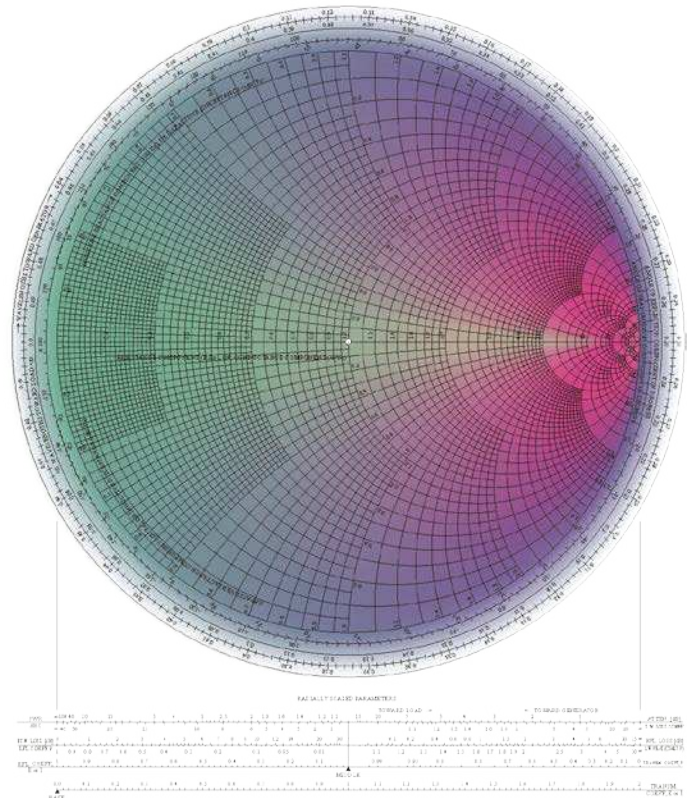
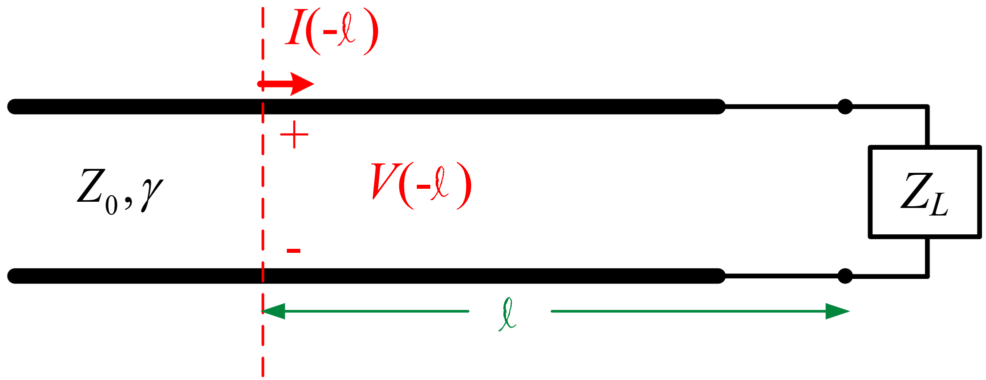
Dept. of ECE

Notes 2
Smith Charts



Generalized Reflection Coefficient



Recall,

$$V(-\ell) = V_0^+ e^{\gamma\ell}\left(1+\Gamma_L e^{-2\gamma\ell}\right) = V_0^+ e^{\gamma\ell}\left(1+\Gamma(-\ell)\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1-\Gamma_L e^{-2\gamma\ell}\right) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1-\Gamma(-\ell)\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1+\Gamma_L e^{-2\gamma\ell}}{1-\Gamma_L e^{-2\gamma\ell}}\right) = Z_0\left(\frac{1+\Gamma(-\ell)}{1-\Gamma(-\ell)}\right)$$

Generalized reflection Coefficient: $\Gamma(-\ell) = \Gamma_L\, e^{-2\gamma\ell}$

Generalized Reflection Coefficient (cont.)

$$\Gamma(-\ell) = \Gamma_L\, e^{-2\gamma\ell}$$
$$= |\Gamma_L|\, e^{j\phi_L}\, e^{-2\gamma\ell}$$
$$= \Gamma_R(-\ell) + j\Gamma_I(-\ell)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For $\mathrm{Re}\{Z_L\} \geq 0$

$$\Rightarrow |\Gamma_L| \leq 1$$

Proof:

$$\Gamma_L = \frac{(R_L + jX_L) - Z_0}{(R_L + jX_L) + Z_0} = \frac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L}$$

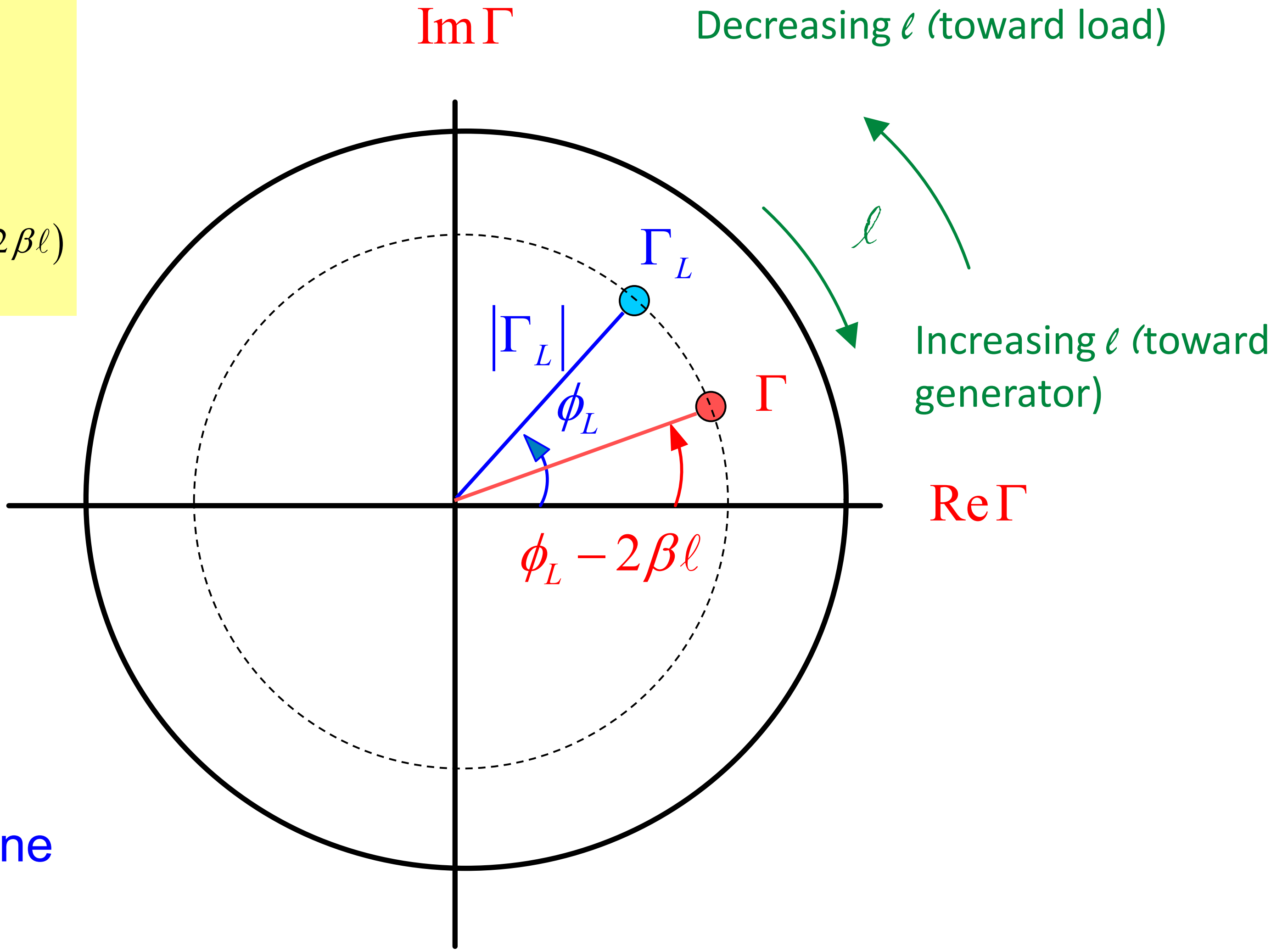
Lossless transmission line ($\alpha = 0$)

$$\Gamma(-\ell) = |\Gamma_L|\, e^{j(\phi_L - 2\beta\ell)}$$

$$\Rightarrow |\Gamma_L|^2 = \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}$$

Complex Γ Plane

$$\begin{aligned}\Gamma &= \Gamma(-\ell) \\ &= \Gamma_R + j\Gamma_I \\ &= \Gamma_L e^{j(-2\beta\ell)} \\ &= |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\end{aligned}$$



Lossless line

Impedance (Z) Chart

$$Z(-\ell) = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right) \qquad \Gamma = \Gamma(-\ell)$$

$$Z_n(-\ell) \equiv \frac{Z(-\ell)}{Z_0} = \left(\frac{1+\Gamma}{1-\Gamma}\right)$$

Define

$$Z_n = R_n + jX_n \;\; ; \;\; \Gamma = \Gamma_R + j\Gamma_I$$

Substitute into above expression for $Z_n(-\ell)$:

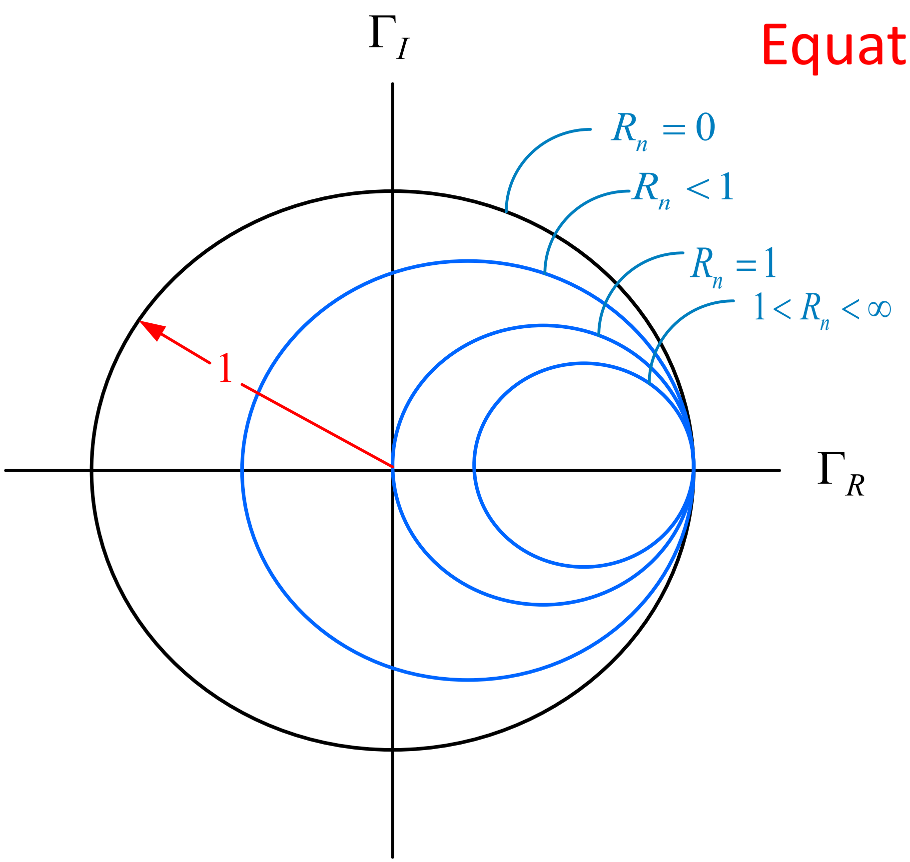
$$R_n + jX_n = \left(\frac{1+(\Gamma_R + j\Gamma_I)}{1-(\Gamma_R + j\Gamma_I)}\right)$$

Next, multiply both sides by the RHS denominator term and equate real and imaginary parts. Then solve the resulting equations for Γ_R and Γ_I in terms of R_n and X_n. This gives two equations.

Impedance (Z) Chart (cont.)

1) Equation #1:

$$\left(\Gamma_R - \frac{R_n}{1+R_n}\right) + \Gamma_I^2 = \left(\frac{1}{1+R_n}\right)^2$$



Equation for a circle in the Γ plane

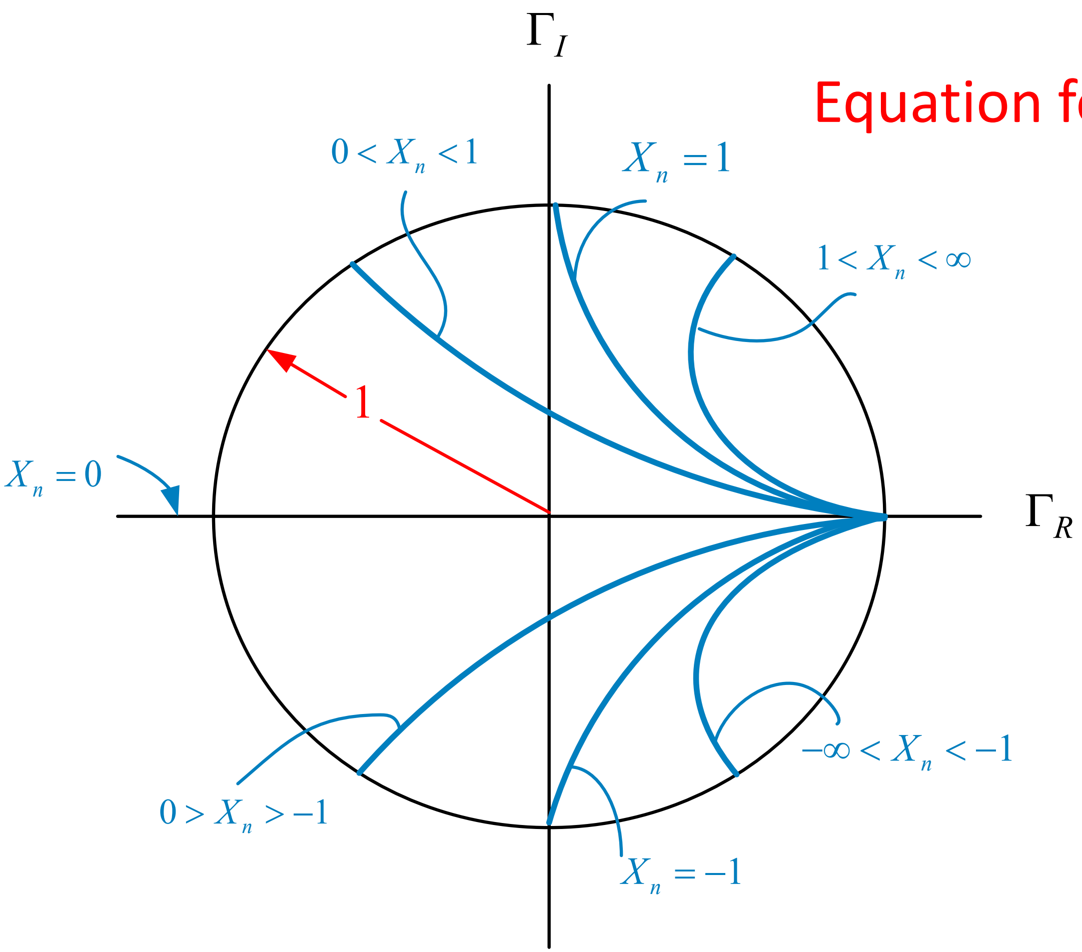
$$\text{center} = \left(\frac{R_n}{1+R_n}, 0\right)$$

$$\text{radius} = \frac{1}{1+R_n}$$

Impedance (Z) Chart (cont.)

2) Equation #2:

$$(\Gamma_R - 1)^2 + \left(\Gamma_I - \frac{1}{X_n}\right)^2 = \left(\frac{1}{X_n}\right)^2$$



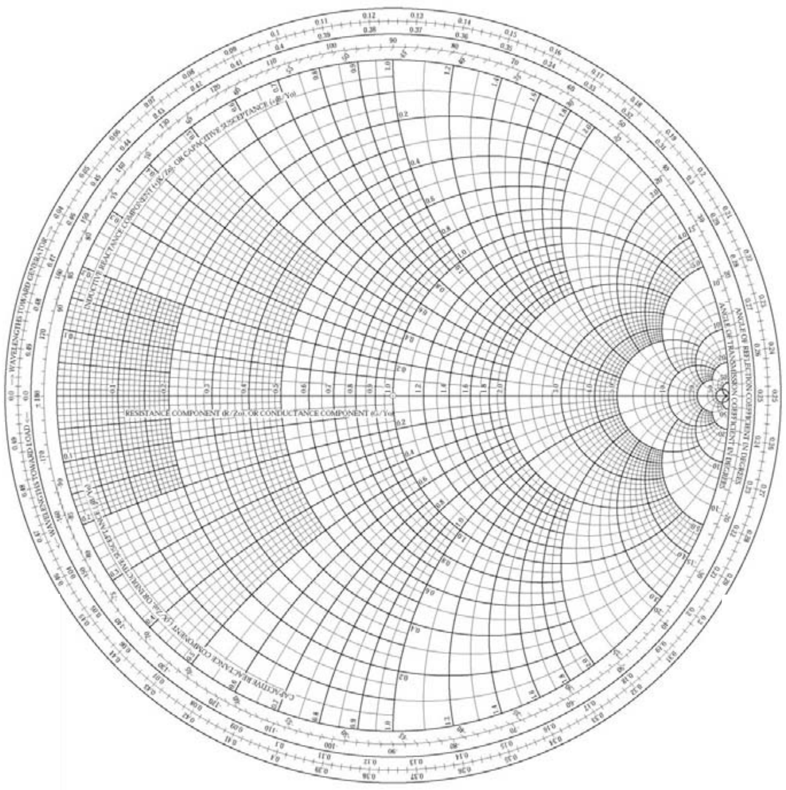
Equation for a circle in the Γ plane:

$$\text{center} = \left(1, \frac{1}{X_n}\right)$$

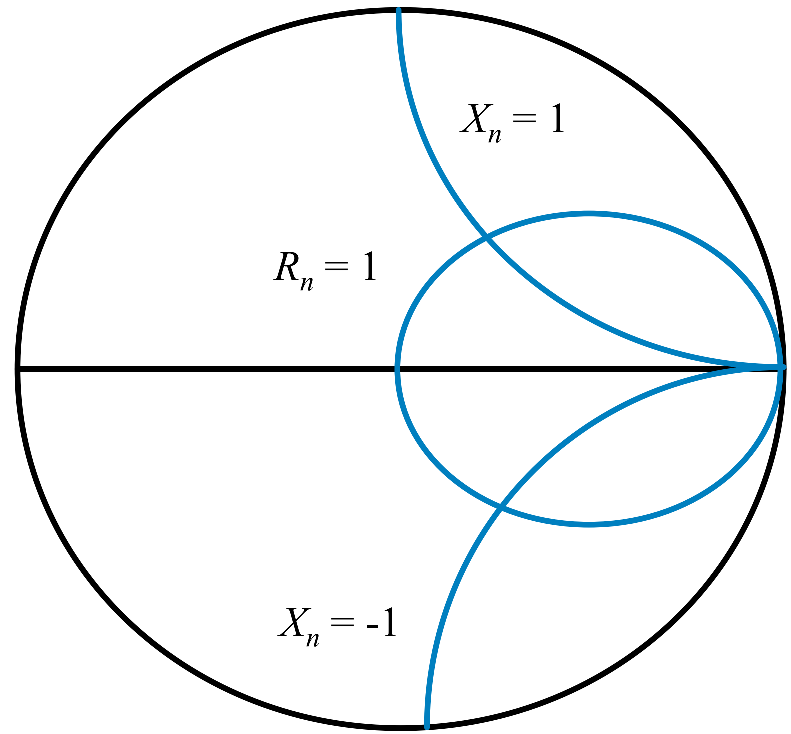
$$\text{radius} = \frac{1}{|X_n|}$$

Impedance (Z) Chart (cont.)

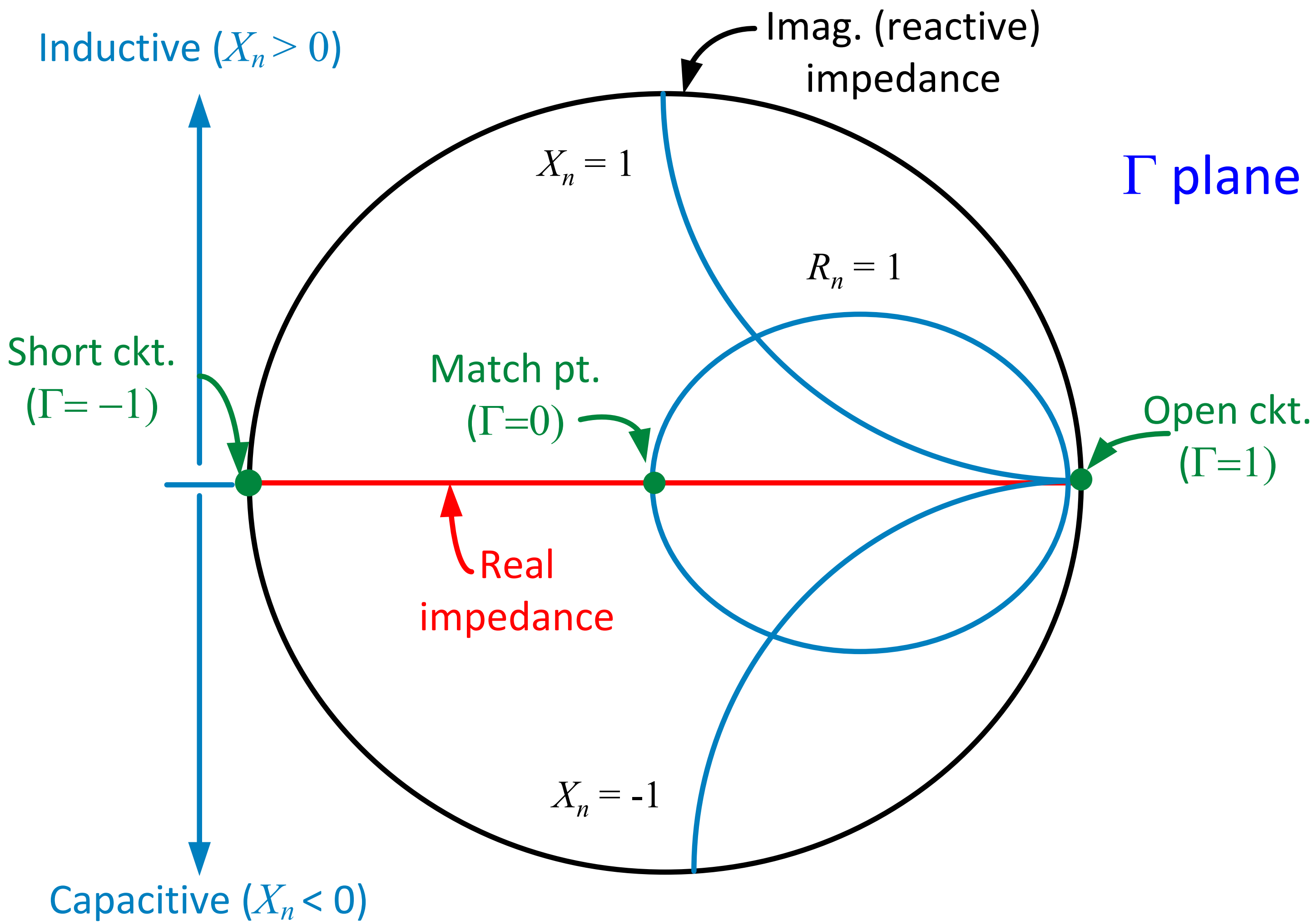
Smith Chart
(Z-Chart)



Short-hand version



Impedance (Z) Chart (cont.)



Admittance (Y) Calculations

Note:

$$Y(-\ell) = \frac{1}{Z(-\ell)} = \frac{1}{Z_0}\left(\frac{1-\Gamma(-\ell)}{1+\Gamma(-\ell)}\right)$$

$$= Y_0\left(\frac{1+\left(-\Gamma(-\ell)\right)}{1-\left(-\Gamma(-\ell)\right)}\right) \qquad Y_0 = \frac{1}{Z_0}$$

$$\Rightarrow Y_n(-\ell) = \frac{Y(-\ell)}{Y_0} = \left(\frac{1+\left(-\Gamma(-\ell)\right)}{1-\left(-\Gamma(-\ell)\right)}\right) = G_n(-\ell) + jB_n(-\ell)$$

Define: $\Gamma' = -\Gamma$

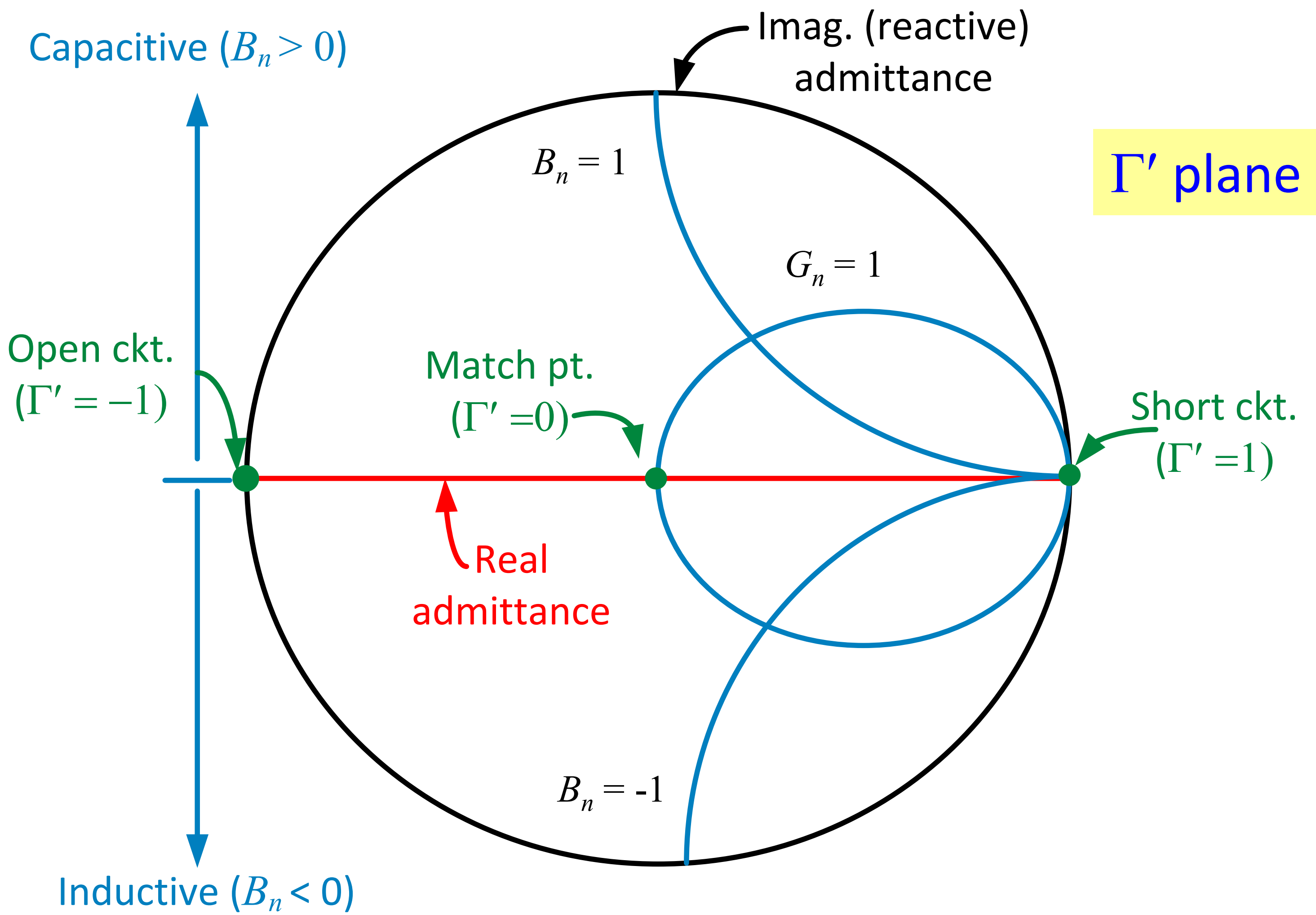
$$Y_n(-\ell) = \left(\frac{1+\Gamma'}{1-\Gamma'}\right)$$

Conclusion: The same Smith chart can be used as an admittance calculator.

Same mathematical form as for Z_n:

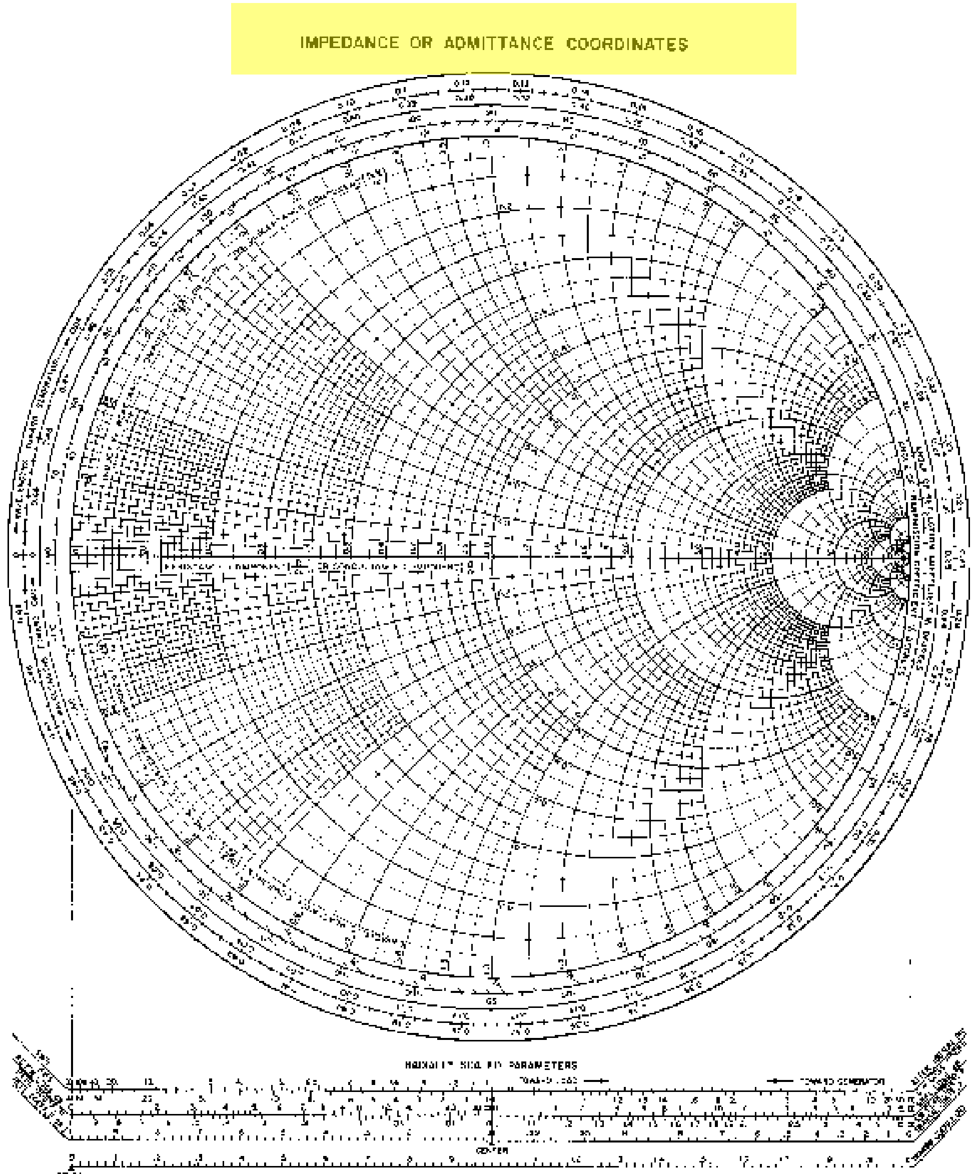
$$Z_n(-\ell) = \left(\frac{1+\Gamma}{1-\Gamma}\right)$$

Admittance (Y) Calculations (cont.)



Impedance or Admittance (Z or Y) Calculations

The Smith chart can be used for either impedance or admittance calculations, as long as we are consistent.



Admittance (Y) Chart

As an alternative, we can continue to use the original Γ plane, and add admittance curves to the chart.

$$Y_n(-\ell) = \left(\frac{1+\left(-\Gamma(-\ell)\right)}{1-\left(-\Gamma(-\ell)\right)} \right) = G_n(-\ell) + jB_n(-\ell)$$

Compare with previous Smith chart derivation, which started with this equation:

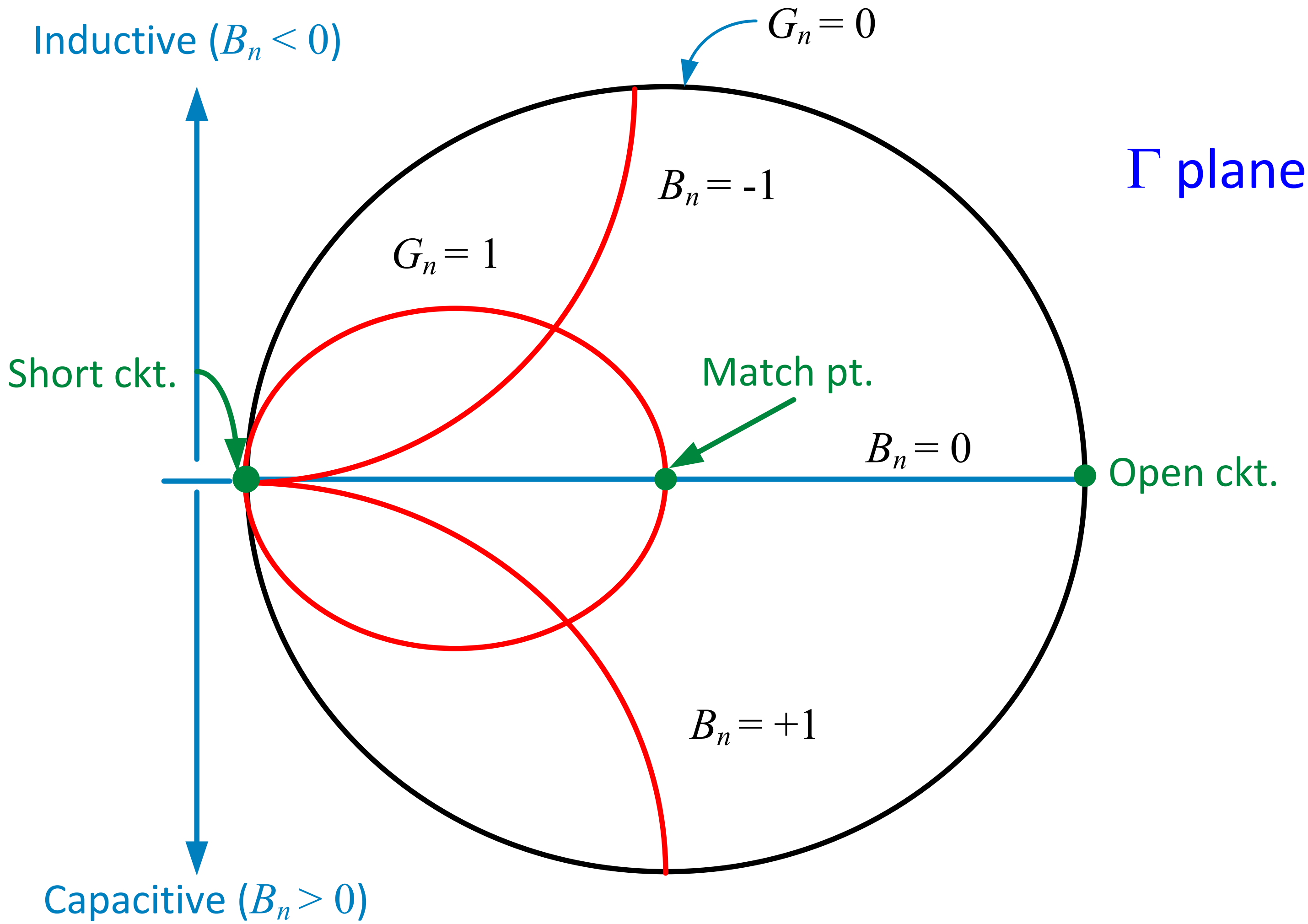
$$Z_n(-\ell) = \left(\frac{1+\left(\Gamma(-\ell)\right)}{1-\left(\Gamma(-\ell)\right)} \right) = R_n(-\ell) + jX_n(-\ell)$$

If $(R_n\ X_n) = (a,\ b)$ is some point on the Smith chart corresponding to $\Gamma = \Gamma_0$,
Then $(G_n\ B_n) = (a,\ b)$ corresponds to a point located at $\Gamma = -\ \Gamma_0$ (180^o rotation).

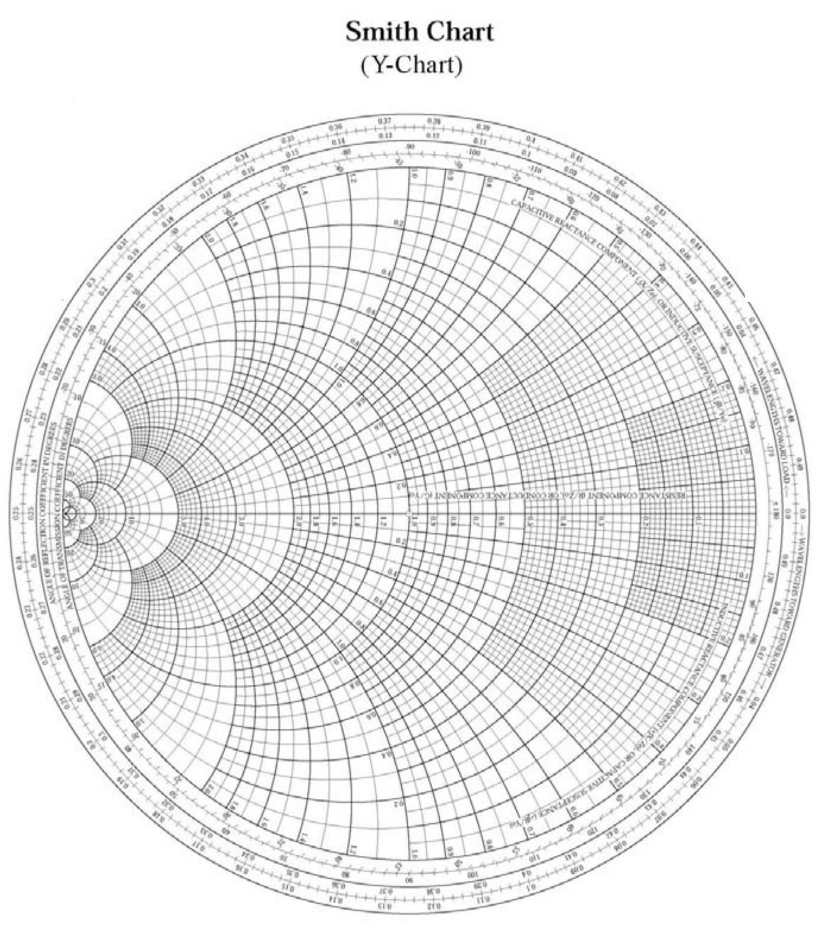
$\Rightarrow$ $R_n = a$ circle, rotated 180^o, becomes $G_n = a$ circle.
and $X_n = b$ circle, rotated 180^o, becomes $B_n = b$ circle.

Side note: A 180^o rotation on a Smith chart makes a normalized impedance become its reciprocal.

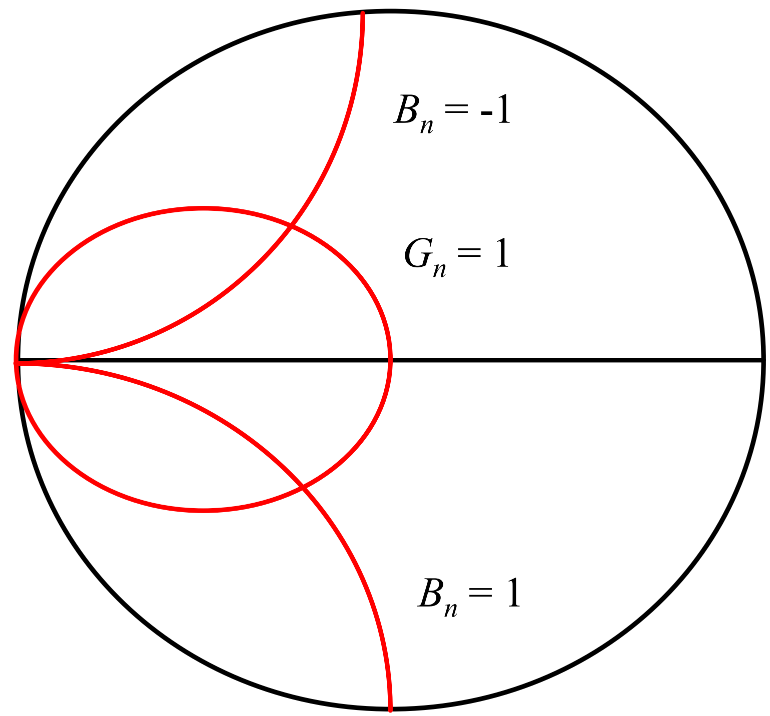
Admittance (Y) Chart (cont.)



Admittance (Y) Chart (cont.)

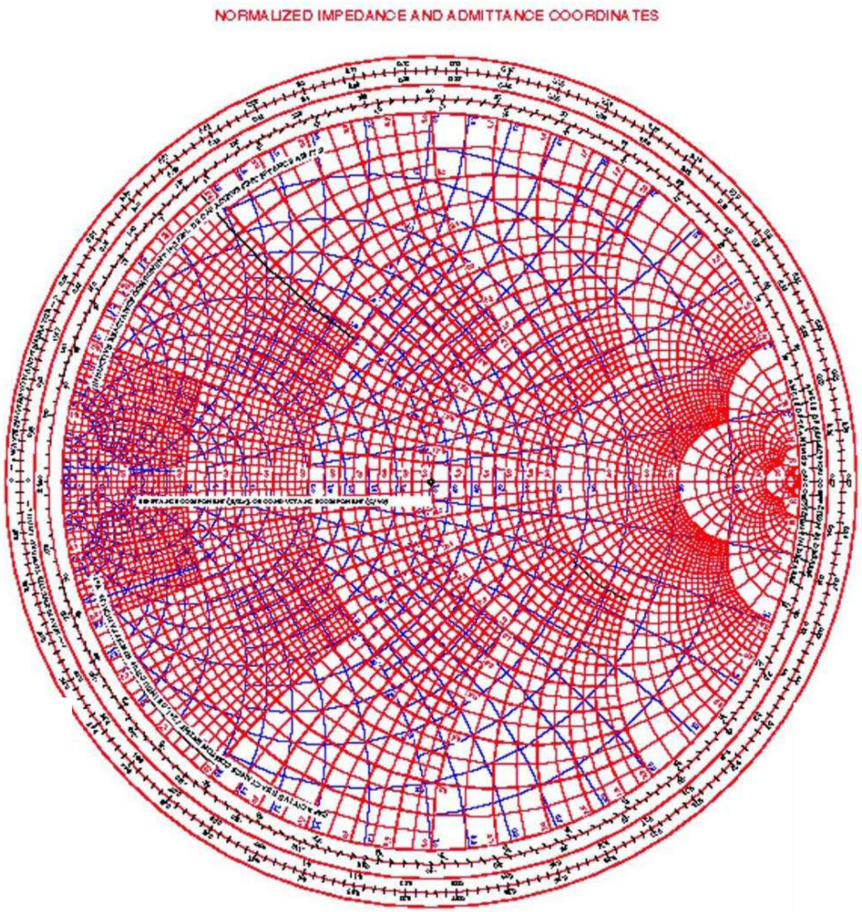


Short-hand version

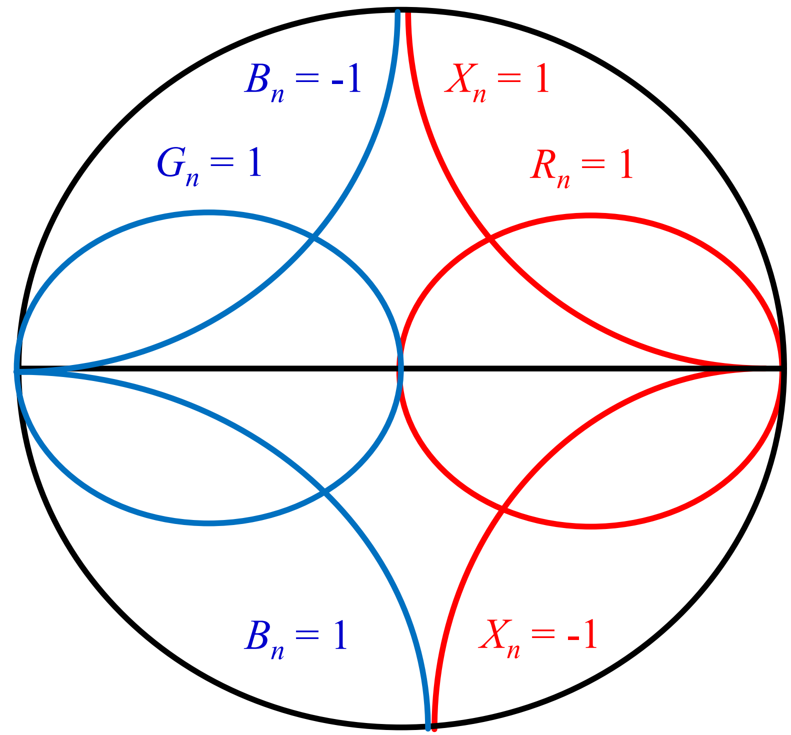


Γ plane

Impedance and Admittance (ZY) Chart

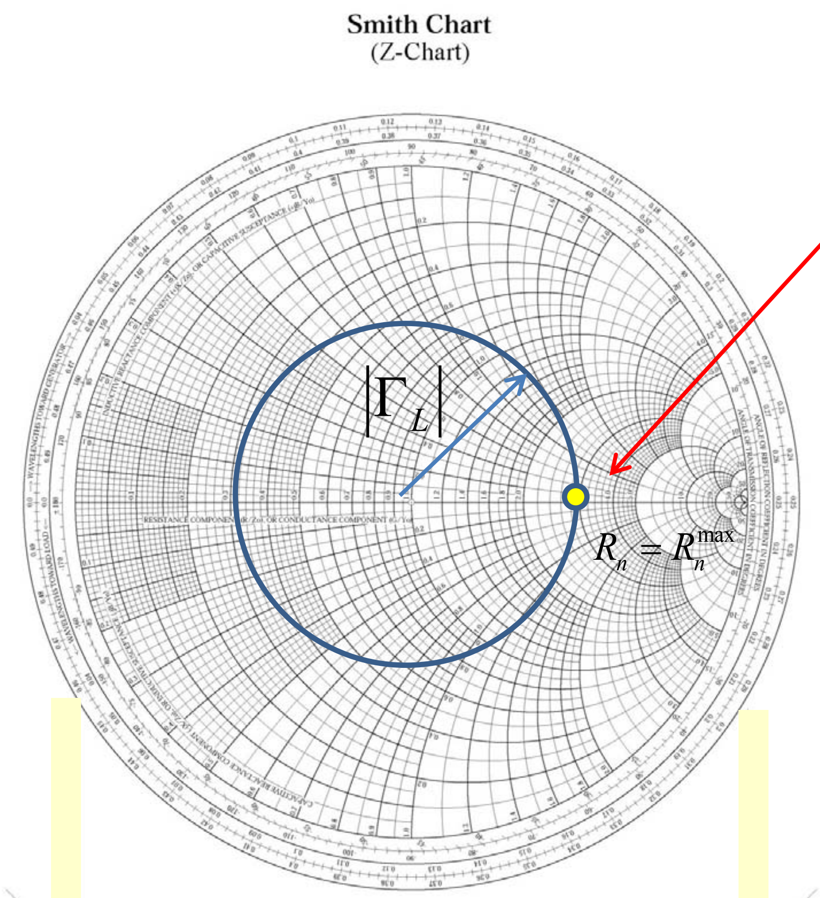


Short-hand version



Γ plane

Standing Wave Ratio



The SWR is given by the value of R_n on the positive real axis of the Smith chart.

Proof:

$$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$Z_n = \frac{1+\Gamma(-\ell)}{1-\Gamma(-\ell)} = \frac{1+\Gamma_L e^{-2j\beta\ell}}{1-\Gamma_L e^{-2j\beta\ell}} = \frac{1+|\Gamma_L| e^{j\phi_L} e^{-2j\beta\ell}}{1-|\Gamma_L| e^{j\phi_L} e^{-2j\beta\ell}}$$

$$\Rightarrow R_n^{\max} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Electronic Smith Chart

At this link

http://www.sss-mag.com/topten5.html

Download the following .zip file

smith_v191.zip

Extract the following files

smith.exe smith.hlp smith.pdf

This is the application file