Today we will revisit Parametric equations and learn how to use them with Conics.

X-y plane Unit Circle

Cartesian Form

$x^2 + y^2 = 1$

Parametric Form

$x = \cos t$
$y = \sin t$

Where $0 \leq t \leq 2\pi$

Keep Calc in Radians

Graph the following conic section: circle

$x = 2 + \cos t$
$y = -5 + \sin t$

center $(2, -5)$

Cartesian

$$(x-2)^2 + (y+5)^2 = 1$$

Graph the following conic section: ellipse

$x = 2\cos t$
$y = 5\sin t$

Center $(0, 0)$

Stretch vert. by 5
Stretch horz by 2

Cartesian

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$

$x = \boxed{4} + \underline{3}\cos t$
$y = \boxed{-1} + 5\sin t$

center

center: $(4, -1)$

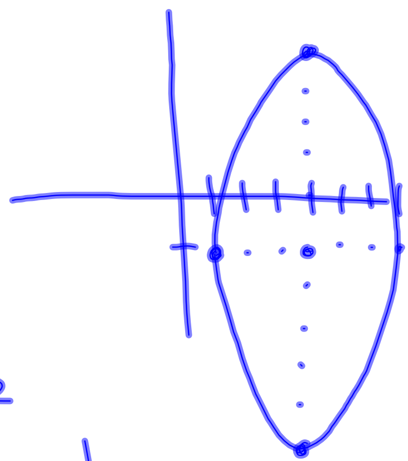
horizontal radius: 3

vertical radius: 5

Cartesian equation:

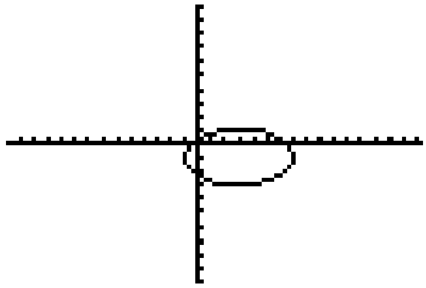
$$\left(\frac{x-4}{3}\right)^2 + \left(\frac{y+1}{5}\right)^2 = 1$$

Sketch the graph



$$\left(\frac{x-3}{4}\right)^2+\left(\frac{y+1}{2}\right)^2=1$$

Sketch the graph



center: $(3,-1)$

horizontal radius: 4

vertical radius: 2

Parametric equation: $x=3+4\cos t$

$y=-1+2\sin t$

General parametric form for an ellipse or circle.

$$x=h+a\cos t$$

$$y=k+b\sin t$$

center: (h,k)

horizontal radius: a

vertical radius: b

Unit Hyperbola

Cartesian Form		Parametric Form
$x^2 - y^2 = 1$ or $x^2 = 1 + y^2$	opens right + left	$x = \sec t$ $y = \tan t$
$-x^2 + y^2 = 1$	opens up and down	$x = \tan t$ $y = \sec t$

Where $0 \le t \le 2\pi$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

$x = 3 + 1\sec t$

$y = -2 + 1\tan t$

center: $(3, -2)$

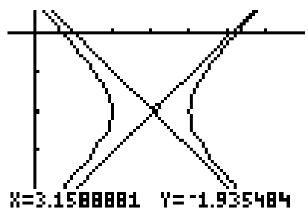
slopes of asymptotes: ± 1

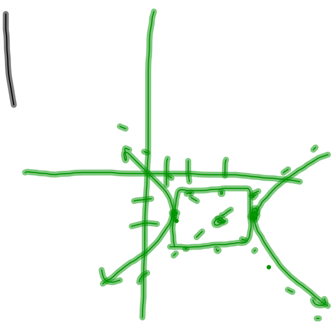
opens: right + left

Cartesian equation:

$$(x-3)^2 - (y+2)^2 = 1$$

Sketch the graph

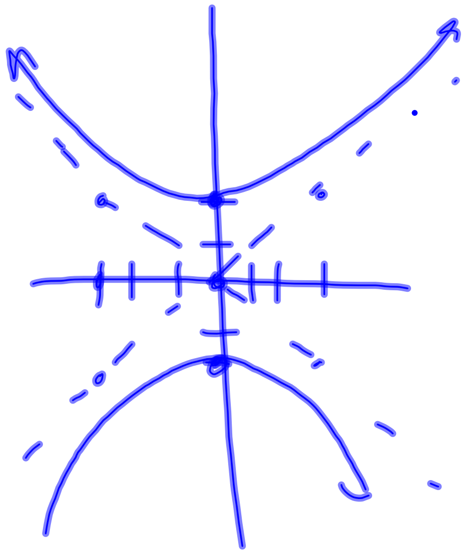




$x = 3\tan t$

$y = 2\sec t$

Sketch the graph



center: $(0,0)$

slopes of asymptotes: $\pm 2/3$

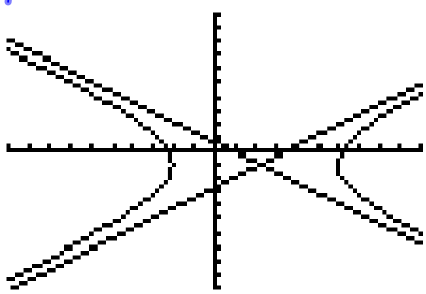
opens: up + down

Cartesian equation:

$$-\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$\left(\frac{x-2}{4}\right)^2 - \left(\frac{y+1}{3}\right)^2 = 1$$

Sketch the graph



center: $(2,-1)$

slopes of asymptotes: $\pm 3/4$

opens: right/left

Parametric equation:

$x = 2 + 4\sec t$

$y = -1 + 3\tan t$

General parametric form for a hyperbola.

$$x = h + a\sec t \qquad\qquad x = h + a\tan t$$

or

$$y = k + b\tan t \qquad\qquad y = k + b\sec t$$

center: (h, k)

slopes of asymptotes: $\pm b/a$

opens: right left

opens: up + down

Solve for cos*t* and sin*t*, and then use a Pythagorean identity to show how the parametric equation transforms into the Cartesian form.

$x = 3 + 2\cos t \rightarrow \left(\frac{x-3}{2}\right)^2 = (\cos t)^2$

$y = -1 + 5\sin t \rightarrow \left(\frac{y+1}{5}\right)^2 = (\sin t)^2$

Add

$$\left(\frac{x-3}{2}\right)^2 + \left(\frac{y+1}{5}\right)^2 = \cos^2 t + \sin^2 t$$

$$= 1$$

Conic Section	Standard Form	Parametric Form
ellipse	$\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$	$x = h + a\cos t$ $y = k + b\sin t$
hyperbola	$\left(\frac{x-h}{a}\right)^2-\left(\frac{y-k}{b}\right)^2=1$ opens horizontally	$x = h + a\sec t$ $y = k + b\tan t$ opens horizontally
	$-\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$ opens vertically	$x = h + a\tan t$ $y = k + b\sec t$ opens vertically

Homework:
Section 12.2
pp. 505-506
13-20, 21-24 b-c, 26, 28