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Rate limited stabilization: Sub-optimal encoder–decoder schemes

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SUMMARY

We extend in this paper results from packet-based control theory and present sufficient conditions on the rate of a packet network to guarantee asymptotic stabilizability of unstable discrete LTI systems with less inputs than states. We use a truncation-based encoder/decoder scheme and two types of network control systems are considered in the absence of communication delays, then for one of the two types, the case of a constant time delay is discussed. For one of the network types, we also propose a zoom-in-type dynamic quantizer scheme with lower data rate but a more complex encoding scheme than the truncation-based one. The new dynamic quantizer requires a lower data rate to achieve stabilization, and while it does not achieve the minimum data rate given by the Data Rate Theorem, it uses an encoding algorithm that is simpler than others reported in the literature. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Feedback control systems wherein the control loops are closed through a real-time network are called networked control systems (NCS) [1, 2]. In 1999, Wong and Brockett [3] considered a feedback system communicating through a digital channel with finite capacity, and since asymptotic stability was deemed unrealistic, the concept of containability was introduced. Mitter [4] and collaborators have contributed to the development of a new theory of NCS that matches classical control theory with traditional information theory, (see [5–8]). In [8], an efficient encoder–decoder scheme is proposed to guarantee stabilization of a class of discrete linear time-invariant (DLTI) system using the minimum rate imposed by the Data Rate Theorem [8]. Reference [9] described an encoder/decoder scheme that also achieved the minimum data rate while considering packet losses. Similarly, Reference [10] presents an encoder–decoder scheme that deals with uncertainty in the plant model. It is clear in all of these schemes that the cost of reducing the data rate implies an increase in the complexity of the stabilizability algorithm and the computational power required for the encoding/decoding operations. There may however be situations where simpler algorithms are preferred, at the expense of requiring a higher data rate. The purpose of this paper is to provide such simple encoder/decoder schemes that may require higher data rates in order to guarantee asymptotic stability.

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The first scheme presented is based on ideas proposed in [11–13]. The authors of those papers considered a general DLTI system $x(k+1)=Ax(k)+Bu(k)$ and found a sufficient rate for exponential stabilization of an unstable plant of order n, under the rather limiting assumption that the system has n inputs (where n is the number of states) and an invertible input matrix B. The work addressed finite rate issues, packet dropping, as well as uncertainties in the plant model. Moreover, the authors assumed the existence of a truncation-based encoder/decoder without providing its specific structure.

We extend the results of [11] to the case of DLTI system with m inputs such that $m\leqslant n$, where n is the order of the system. We also relax the condition of the invertibility of the B matrix, and extend the stabilizability results to systems with a constant time-delay induced by the sensor-to-controller network. Moreover, we present an easily implementable encoder/decoder structure. As was considered in [11], we discuss two types of NCS: one that includes a network between the sensors and the controller, and another that models two networks in the loop, one between the sensors and controller, and another between the controller and the actuator. Section 4 of this work is an extension to the preliminary results we presented in [14].

Finally, we also propose a zoom-in-type dynamic quantizer scheme with lower data rate but a more complex encoding scheme than the truncation-based one. The new dynamic quantizer requires a lower data rate to achieve stabilization, and while it does not achieve the minimum data rate given by the Data Rate Theorem, it uses an encoding algorithm that is simpler than others reported in [8–10]. Examples and simulations are provided in Section 8 to illustrate the results.

2. PROBLEM SETUP

We consider the two configurations for the packet-based NCS presented in [11]. The first system is referred to as *NCS Type I* and has a rate of R_{p1} packets/time-step. This packet-based network accommodates a packet size of D_{Max} bits used for data (although the protocol information requires extra bits in the packet, it is not needed for this analysis). Let us consider the discrete LTI system shown in Figure 1 and described by F1

$$x(k+1)=Ax(k)+Bu(k) \tag{1}$$

where A is $n\times n$, B is $n\times m$ and $u(k)$ is $m\times 1$. The second type of packet-based network, referred to as *NCS Type II*, consists of the same discrete LTI system given by Equation (1), but with the addition of a second network between the controller and the actuator with rate R_{p2} as shown in Figure 2. From here on, the following notations are adopted. The norm symbol ($\|.\|$) denotes the Euclidean norm and $\lceil . \rceil$ is the *ceil* function. In addition, we use the variable μ to denote the controllability index, which for multivariable linear systems [15] is defined as the least integer k such that F2

$$\mathrm{rank}[B|\ AB|\ \ldots|\ A^{k-1}B]=n \tag{2}$$

We assume that the controller does not saturate, and that the packet-network does not drop packets nor is it subjected to disturbances (noise). For both NCS types, we assume that the states may be measured. We also assume that the decoder knows exactly the encoding scheme used by the encoder at all times (equimemory property), as described in Section 3. The last assumption is that the encoder and decoder know a value $L_0>0$ such that $\|x(0)\|<L_0$ and that both have access to the control signal or can compute it as represented by a dotted line in Figures 1 and 2. The assumption of knowing the value L_0 does not constrain the applications of the scheme. It may be simply chosen as any upper bound for $x(0)$ that is logical from the physical

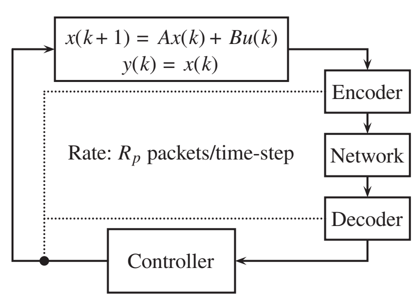

Figure 1. Closed-loop NCS: Type I.

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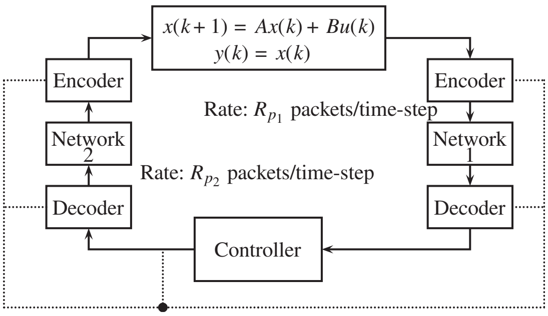

Figure 2. Closed-loop NCS: Type II.

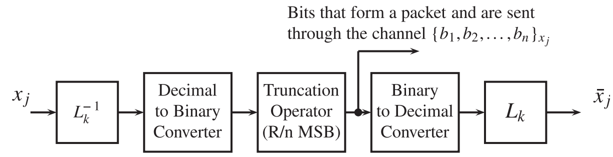

Figure 3. Encoder scheme.

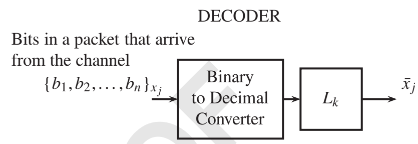

Figure 4. Decoder scheme.

constraints of the plant (for example: if $x(0)$ is the initial position of a robot within a room, then the dimensions of the room will be a suitable choose of L_0).

3. ENCODER–DECODER DESIGN

Several approaches for the design of an encoder/decoder scheme were presented in previous works. Most of them are based on some type of predictor that emulates the evolution of the plant state and the difference between this prediction and the actual state of the plant, i.e. the error. The quantized error is sent through the channel, then decoded at the receiver and used to obtain an approximation of the state, which is used to generate the control signal. In our case however, we send a quantized version of every state component rather than the error using a modified version of the encoder/decoder scheme proposed in [16]. Figures 3 and 4 illustrate our scheme, which is
F3 described next in detail. At the first instant, $k=0$, the
F4 sensor measures the state exactly. Since we assume that both the encoder and decoder know L_0, each component x_j of the measured state is divided by L_0, which gives a number x_j/L_0 that is strictly less than or equal to 1 in magnitude. We assume for now that x_j/L_0 is positive (in Section 4 we describe on how to proceed if x_j/L_0 is negative). The encoder converts this x_j/L_0 to its binary representation and keeps only the r_j most significant bits (MSB). This truncated version is labeled as $(x_j(0)/L_0)_{T_{r_j}}$, where the symbol $()_{T_{r_j}}$ denotes the truncation operation that retains the r_j MSB. The quantity r_j will be calculated in Section 4. The decimal representation of these r_j bits is multiplied by L_0 resulting in an estimate $\bar{x}_j(0) = (x_j(0)/L_0)_{T_{r_j}} L_0$, which is stored in the encoder. By grouping into a vector the j truncated state components, we obtain the state estimate $\bar{x}(0)$. The bits in each truncated state component form a packet (or packets depending on D_{Max}) that is sent through the channel. On the receiver side, the decoder receives a packet (or packets) and separates the bits that correspond to each state component. Assuming perfect transmission, the decoder then converts the binary representation of the bits received into a decimal representation and multiplies by L_0, which gives the value $\bar{x}_j(0)$. This should result in the same value stored in the encoder and, therefore, the equimemory property between encoder and decoder is preserved. Since the control signal at time $k=1$ only depends on $\bar{x}(0)$, we can show that at time $k=1$, $x_j(1)$ is bounded as follows. Using the triangle inequality and matrix norm properties we have:

$$
\begin{aligned}
\|x(1)\| &\leqslant \|Ax(0) + Bu(\bar{x}(0))\| \\
&\leqslant \|A\|\|x(0)\| + \|Bu(\bar{x}(0))\| \\
&\leqslant \|A\| L_0 + \|Bu(\bar{x}(0))\| \\
&= L_1
\end{aligned}
$$

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Since the control algorithm is predefined, the encoder and decoder can both calculate this value L_1 right after they have calculated the value $\bar{x}(0)$. The stored L_1 will then be used at instant $k=1$ to keep the ratio $|x(1)/L_1| \leqslant 1$. By carefully examining the above steps, we obtain the following scalar difference equation to bound the norm of each state component:

$$L_k = \|A\| L_{k-1} + \|Bu(\bar{x}(k-1))\| \quad \forall k = \{1, \dots, \mu\} \tag{3}$$

Since Equation (3) only depends on the terms L_{k-1} and $\bar{x}(k-1)$, all signals needed to compute this equation are available at the encoder and the decoder. In Section 4 we will see that L_k only evolves for μ time-steps, before it is reset to a new starting value for the next μ time-steps and this is the reason to limit k to a maximum of μ in Equation (3).

4. RESULTS FOR THE TRUNCATION-BASED ENCODING SCHEME

4.1. *Network control system*: *type I*

In the case of NCS Type I, the state vector $x(k)$ is given by $x(k) = [x_1(k) \;\; x_2(k) \;\; \dots \;\; x_n(k)]'$. We assume below that $x_j(k) > 0$, $\forall j$ since the sign of each state component may later be accounted for by adding n extra bits to the rate (one extra bit per each state component sign). We then obtain the following binary representation of $x(0)/L_0$ at the encoder side:

$$\frac{x(0)}{L_0} = \begin{bmatrix} \dfrac{x_1(0)}{L_0} \\ \dfrac{x_2(0)}{L_0} \\ \vdots \\ \dfrac{x_n(0)}{L_0} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{\infty} \alpha_{1i} 2^{-i} \\ \sum_{i=1}^{\infty} \alpha_{2i} 2^{-i} \\ \vdots \\ \sum_{i=1}^{\infty} \alpha_{ni} 2^{-i} \end{bmatrix} \tag{4}$$

where $\alpha_{ij} \in \{0, 1\}$. This binary representation is truncated keeping only the r_j MSB for state component x_j. The truncated representation is given by

$$\left(\frac{x(0)}{L_0}\right)_{T_{r_j}} = \begin{bmatrix} \left(\dfrac{x_1(0)}{L_0}\right)_{T_{r_j}} \\ \left(\dfrac{x_2(0)}{L_0}\right)_{T_{r_j}} \\ \vdots \\ \left(\dfrac{x_n(0)}{L_0}\right)_{T_{r_j}} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{r_1} \alpha_{1i} 2^{-i} \\ \sum_{i=1}^{r_2} \alpha_{2i} 2^{-i} \\ \vdots \\ \sum_{i=1}^{r_n} \alpha_{ni} 2^{-i} \end{bmatrix} \tag{5}$$

where $\alpha_{ij} \in \{0, 1\}$. The r_j bits per state component j are sent through the channel and, at the receiver site, the decoder transforms the bits back into decimal numbers, and multiplies them by L_0 in order to obtain $\bar{x}(0)$. With this encoding/decoding process, we guarantee that the error between the actual state component and its encoded version, $\varepsilon_j(0) = x_j(0) - \bar{x}_j(0)$, is limited by $\|\varepsilon_j(0)\| < 2^{-r_j} L_0$, $\forall j \in \{0, 1, \dots, n\}$. Using the triangle inequality, the norm of the total error is bounded by

$$\|\varepsilon(0)\| \leqslant \sqrt{\sum_{j=1}^{n} 2^{-2r_j}}\, L_0 \tag{6}$$

Let us then consider the evolution of the system starting at time $k=0$:

$$\begin{aligned} x(1) &= Ax(0) + Bu(0) \\ x(2) &= A^2 x(0) + ABu(0) + Bu(1) \\ &\;\;\vdots \\ x(l) &= A^l x(0) + \sum_{i=1}^{l} A^{l-i} Bu(i-1) \quad \forall l \geqslant 3 \end{aligned}$$

Recalling that μ represents the controllability index, after μ steps we have

$$\begin{aligned} x(\mu) = {} & A^{\mu} x(0) + A^{\mu-1} Bu(0) + A^{\mu-2} Bu(1) \\ & + \cdots + Bu(\mu-1) \end{aligned}$$

This equation may be re-arranged as $x(\mu) = A^{\mu} x(0) + \zeta_{\mu} \mathbb{U}$, where

$$\begin{aligned} \zeta_{\mu} &= [B| \;\; AB| \;\; \dots| \;\; A^{\mu-1}B] \\ &= [\delta_1| \;\; \delta_2| \;\; \dots| \;\; \delta_j| \;\; \dots| \;\; \delta_{\mu}] \end{aligned}$$

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and

$$\mathbb{U}=[u(\mu-1)\ \ldots\ u(0)]'=[u_1\ \ldots\ u_j\ \ldots\ u_\mu]'$$

noting that δ_j is the jth column in ζ_μ and u_j is the jth element in the vector $\mathbb{U}$. Let us select the first n independent columns of ζ_μ and build a new matrix, called ζ_n. Let us also select the elements of $\mathbb{U}$ corresponding to the columns chosen from ζ_μ and form a new vector, called $\mathbb{U}_n$. Recalling that $x(0)=\bar{x}(0)+\varepsilon(0)$ we have $x(\mu)=A^\mu\bar{x}(0)+A^\mu\varepsilon(0)+\zeta_\mu\mathbb{U}$. If we choose the control law

$$\mathbb{U}_n=-\zeta_n^{-1}A^\mu\bar{x}(0) \tag{7}$$

we may reconstruct $\mathbb{U}$ by replacing u_j with the corresponding values of $\mathbb{U}_n$ in the proper order and letting $u_j=0$ for the remaining elements. After μ steps, and by applying the control sequence $\mathbb{U}$ we obtain

$$x(\mu)=A^\mu\varepsilon(0) \tag{8}$$

Then, from Equations (6), (8), and the properties of matrix norms, we obtain

$$\|x(\mu)\|=\|A^\mu\varepsilon(0)\|\leqslant\|A^\mu\|\|\varepsilon(0)\|\leqslant\|A^\mu\|\sqrt{\sum_{j=1}^{n}2^{-2r_j}}L_0$$

In order to force the state to decrease in the norm (after μ steps), we shrink the upper bound of the state $x(\mu)$ by forcing it to be less than a fraction of the upper bound of the state $x(0)$, i.e. $\|A^\mu\|\sqrt{\sum_{j=1}^{n}2^{-2r_j}}L_0<L_0/\delta$, for some $\delta>1$. At this point, we have to decide on the value of each r_j. This may be converted into an optimization problem whose objective is to minimize the total rate given by $\sum_{j=1}^{n}r_j$. In other words, let us consider the optimization problem:

$$\min_{r_j}\ \sum_{j=1}^{n}r_j \tag{9}$$

$$\text{subject to}\quad \sqrt{\sum_{j=1}^{n}2^{-2r_j}}<\frac{1}{\delta\|A^\mu\|}=C_* \tag{10}$$

This problem may be solved by applying the Karush–Kuhn–Tucker (KKT) conditions [17] to the Lagrangian function $L(r_1,r_2,\ldots,r_n,l)$ with Lagrange multiplier l as given by

$$L=r_1+r_2+\cdots+r_n$$
$$-l(C_*-\sqrt{2^{-2r_1}+2^{-2r_2}+\cdots+2^{-2r_n}})$$

The KKT conditions are then:

$$\frac{\partial L}{\partial r_i}=1-l\frac{2^{-2r_i}\ln(2)}{\sqrt{2^{-2r_1}+2^{-2r_2}+\cdots+2^{-2r_n}}}=0$$
$$\forall i\in\{1,2,\ldots,n\}$$

Solving this system of n equations, we obtain: $r_1=r_2=\cdots=r_j=\cdots=r_n$. Therefore, an equal allocation of bits per each state component actually guarantees the minimum total rate. Using the constraint (10) we obtain the optimal rate allocation $r_n>\lceil\log_2(\|A^\mu\|)+\frac{1}{2}\log_2(n)+\log_2(\delta)\rceil$. We notice that δ is a parameter that determines the fraction by which the upper bound of $\|x(0)\|$ is shrinking. Therefore, it is sufficient to consider the *infimum* of this quantity to obtain $r_n>\lceil\log_2((\|A^\mu\|))+\frac{1}{2}\log_2(n)\rceil$. Note that the $\lceil.\rceil$ function was introduced since r_n must be an integer denoting the number of bits for each state component. We can therefore define the total R bits in a packet (or packets) as $R=nr_n+n$ where the second n term may be used to code the sign of each state component.

For the next μ steps, we repeat the same steps as above but using $x(\mu)$ as the initial condition. To stop the growth of L_k, and noting that $\|x(\mu)\|<n\|A^\mu\|2^{-r_n}L_0$, we assign $L_\mu=n\|A^\mu\|2^{-r_n}L_0$ as the new L_0 for the next μ time steps in Equation (3). We repeat this procedure every μ steps. Using the same algorithm to generate the control sequence and the same rate R, the state $x(2\mu)$ will be a shrunken version of $x(\mu)$. Proceeding in the same manner, $x(t\mu)$ will tend to zero as $t\in\mathbb{N}$ grows and, therefore, the state x will tend to zero and asymptotic stabilizability will be achieved. Note that R is the sufficient number of effective bits that we need to transmit for the whole state to guarantee stabilization, but since a packet has a maximum length D_{Max}, if $R\leqslant D_{\text{Max}}$, we need a packet rate of $R_p=1$ packet/sample-time. If on the other hand, $R>D_{\text{Max}}$ then, a minimum of $\lceil R/D_{\text{Max}}\rceil$ packets/time-step are needed. Note that the last expression actually covers both cases, since $R/D_{\text{Max}}<1$

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gives a 1 packet/sample-time when the ceil function is applied.

This analysis may be summarized in the following theorem.

Theorem 4.1
Assuming an equal allocation of bits per state component, a network rate R_p packets/time-step, and assuming that (A, B) is a controllable pair with controllability index μ, a sufficient condition for system (1) to be asymptotically stabilizable is $R_p = \lceil R/D_{\text{Max}} \rceil$, where $R > n\lceil \log_2(\|A^{\mu}\|) + \frac{1}{2}\log_2(n) \rceil + n$ and every state allocates R/n bits/time-step.

An immediate consequence of Theorem 4.1 in the specific case of a single input system is given in the following corollary.

Corollary 4.1
Assuming an equal allocation of bits per state component, a network rate R_p packets/time-step, (A, B) is a controllable pair, and B is $n \times 1$, a sufficient condition for system (1) to be asymptotically stabilizable is $R_p = \lceil R/D_{\text{Max}} \rceil$, where $R > n\lceil \log_2(\|A^{n}\|) + \frac{1}{2}\log_2(n) \rceil + n$ and every state allocates R/n bits/sample.

Proof
The proof is the same as that of Theorem 4.1. If B is $n \times 1$ and $u(k)$ is 1×1, then $\mu = n$. Substituting μ in R in the proof of Theorem 4.1, we obtain the rate given by the corollary. □

Although the proof of Theorem 4.1 relies on a specific control law, Reference [18] shows the data rate condition using a truncation-based scheme with the given control law $u(k) = -K\bar{x}(k)$. It is clear for the simulations in [18] that the rates there are much higher than the ones obtained using the control law in Equation (7).

4.2. Network control system Type I with time delay

One of our motivations for extending the results of [11] was to account for the effects of time delays that may be present in the network. As mentioned earlier, even for the scalar case, the invertibility requirement of B would not allow the traditional augmentation of the state by its delayed versions. Let us consider the modified NCS

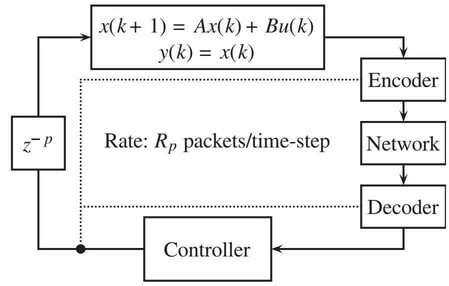

Figure 5. Closed-loop NCS Type I with time-delay.

type I shown in Figure 5 and the DLTI system given by the following equation: F5

$$x(k+1) = Ax(k) + Bu(k-p) \tag{11}$$

where A is $n \times n$, B is $n \times 1$ and $u(k)$ is 1×1. We assume here that the control signal to actuator delay is a constant equal to $p \in \mathbb{N}$ time-steps. Under such conditions, we obtain the following theorem:

Theorem 4.2
Assuming an equal allocation of bits per state component, a network rate of $R_p = \lceil R/D_{\text{Max}} \rceil$ packets/time-step, and

$$\mathbb{A} = \begin{bmatrix} A & B & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ & & & & 1 \\ 0 & 0 & \vdots & \dots & 0 \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

such that $(\mathbb{A}, \mathbb{B})$ is a controllable pair. A sufficient condition for system (11) to be asymptotically stabilizable is $R_p = \lceil R/D_{\text{Max}} \rceil$, where $R > (n+p)\lceil \log_2(\|\mathbb{A}^{n+p}\|) + \frac{1}{2}\log_2(n+p) \rceil + (n+p)$, and each state component of the augmented system allocates $R/(n+p)$ bits/time-step.

Proof
Similar to works [19, 20], we start out by augmenting the state vector, considering as new states the last p

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previous inputs. We then obtain

$$\mathbb{X}(k+1)=\begin{bmatrix}\mathbf{x}(k+1)\\ x_{n+1}(k+1)\\ x_{n+2}(k+1)\\ \vdots\\ x_{n+p}(k+1)\end{bmatrix}$$

$$=\begin{bmatrix}A & B & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ 0 & 0 & 0 & \dots & 0\\ & & & & 1\\ 0 & 0 & \vdots & \dots & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}(k)\\ x_{n+1}(k)\\ x_{n+2}(k)\\ \vdots\\ x_{n+p}(k)\end{bmatrix}$$

$$+\begin{bmatrix}0\\0\\0\\ \vdots\\ 1\end{bmatrix}u(k)$$

This may be written as $\mathbb{X}(k+1)=\mathbb{A}\mathbb{X}(k)+\mathbb{B}u(k)$. We now have a system similar to the one treated in Corollary 4.1 with a state dimension $n+p$ instead of n. Therefore, in order to shrink the upper bound of the state $\mathbb{X}(k+n+p)$ we need a rate R given by

$$R/(n+p)>\lceil \log_2(\|\mathbb{A}^{n+p}\|)+\tfrac{1}{2}\log_2(n+p)\rceil+1$$

Similar to previous proofs, we find a minimum rate of $R_p=\lceil R/D_{\text{Max}}\rceil$ packets/time-step. □

4.3. *Network control system: Type II*

We now consider an NCS Type II and show the following result.

Theorem 4.3

Assume an equal allocation of bits per state component, network rates of $R_{p1}=\lceil (R_1+n)/D_{\text{Max}}\rceil$ packets/time-step and $R_{p2}=\lceil (R_2+1)/D_{\text{Max}}\rceil$ packets/time-step for network 1 and 2, respectively. Assuming also that (A,B) is a controllable pair, where B is $n\times 1$, the controllability matrix is given by $\zeta=[B|\ AB|\ \dots|\ A^{n-1}B]$, a sufficient condition for system (1) to be asymptotically stabilizable is

$$n\|A^n\|2^{-(R_1/n+1)}+\|\zeta\|\|\zeta^{-1}A\|2^{-(R_2+1)}<1$$

Proof

Since there is now a rate constraint from the controller to the plant actuators, we can no longer apply the calculated control signal $u(k)$ directly to the plant. Instead, only the bits encoding $u(k)$ according to the available rate R_2 may be used. This encoded control signal $\widetilde{u}(k)$ is the one that is received by the plant. We then have $x(k+1)=Ax(k)+B\widetilde{u}(k)$. Let us assume that we have exactly the same encoding and decoding schemes used in Theorem 4.1. The evolution of the system in the first n time steps is given by $x(n)=A^nx(0)+\zeta\widetilde{\mathbb{U}}$, where $\widetilde{\mathbb{U}}=[\widetilde{u}(n-1)\ \dots\ \widetilde{u}(0)]'$. If we choose the control signal $\mathbb{U}=-\zeta^{-1}A^n\bar{x}(0)$, then $\|\mathbb{U}\|\leqslant\|\zeta^{-1}A^nL_0\|\leqslant\|\zeta^{-1}A^n\|L_0=L2_0$. For other time k, the normalization value that is kept in the memory of the encoder/decoder of network II, i.e. $L2_k$, is given by $L2_k=\|\zeta^{-1}A^n\|L_k$. Since $\widetilde{u}(k)$ represents the R_2 MSB of $u(k)$ we know that

$$\|\mathbb{U}-\widetilde{\mathbb{U}}\|\leqslant\|\zeta^{-1}A^n\|L_02^{-R_2} \tag{12}$$

From Equation (12) and recalling that $x(0)=\bar{x}(0)+\varepsilon(0)$ and $\|\varepsilon(0)\|<\sqrt{n}L_02^{-R_1/n}$, we have

$$\begin{aligned}\|x(n)\| &= \|A^n\bar{x}(0)+A^n\varepsilon(0)+\zeta\widetilde{\mathbb{U}}\|\\ &= \|\zeta(\zeta^{-1}A^n\bar{x}(0)+\widetilde{\mathbb{U}})+A^n\varepsilon(0)\|\\ &\leqslant \|\zeta\|\|\mathbb{U}-\widetilde{\mathbb{U}}\|+\|A^n\varepsilon(0)\|\\ &\leqslant \|\zeta\|\|\zeta^{-1}A\|L_02^{-R_2}+\sqrt{n}\|A^n\|L_02^{-R_1/n}\\ &< \frac{L_0}{\delta}\end{aligned}$$

To guarantee the decrease of $x(n)$, we enforce that $\|\zeta\|\|\zeta^{-1}A\|L_02^{-R2}+\sqrt{n}L_0\|A^n\|2^{-R_1/n}<L_0$, i.e. $\sqrt{n}\|A^n\|2^{-R_1/n}+\|\zeta\|\|\zeta^{-1}A\|2^{-R_2}<1$. As in previous proofs, we now select $x(n)$ as the new initial condition and using the same control law and rates, R_1 and R_2, the state $x(2n)$ will be a shrunken version of $x(n)$. Continuing in the same manner, $x(tn)$ will

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tend to zero as $t \in \mathbb{N}$ grows and, therefore, $x(k)$ will tend to zero and asymptotic stability is achieved. To take into account the sign of the state, we add n bits to R_1, one per state component. We will need a minimum of $R_{p1} = \lceil R_1/D_{\text{Max}} \rceil$ packets/time-step for the sensor-controller network and a minimum of $R_{p2} = \lceil R_2/D_{\text{Max}} \rceil$ packets/time-step in the controller–actuator network. □

We remark that Theorem 4.3 can be easily extended to the multidimensional if we assume equal allocation of bits for every input (a slight modification of the upper bound in Equation (12) will give the extension). However, the equal allocation of bits cannot be assumed as the optimal allocation before solving an optimization problem similar to the one in Equations (9) and (10).

5. REMOVING THE RATE DEPENDENCY ON $\|A\|$

The result of Theorem 4.1 (as well as Corollary 4.1 and Theorem 4.2) established a sufficient rate in terms of the norm of A. For different matrices A with the same eigenvalues, however, this may lead to very different rates, some of which may also be very large compared with the minimum rates specified by the Data Rate Theorem. One way to remove this disadvantage is to modify the control law used in the proof of Theorem 4.1. Instead of trying to asymptotically stabilize the state x, we attempt to stabilize the state $z = \Phi^{-1}x$, where Φ is a linear transformation such that $\Phi^{-1}A\Phi$ is the diagonal matrix equivalent to A (or more generally the Jordan-block matrix). The error $\varepsilon_z(0)$ in the z space is given by $\Phi^{-1}(x_j(0) - \bar{x}_j(0))$. For stabilization analysis purposes, designing a control law to stabilize the state z is equivalent to stabilizing x since $z \to 0$ implies $x \to 0$. There will however be a difference in the transient response as we will see later. The change of variable implies that the control law in Equation (7) no longer depends on the controllability matrix of the pair (A, B), i.e. ζ_μ. But will instead depend on the controllability matrix of the pair $(\Phi^{-1}A\Phi, \Phi B)$, denoted by ζ_{Φ_μ}. Therefore, the new control law is given by

$$\mathbb{U}_n = -\zeta_{\Phi_\mu}^{-1}(\Phi^{-1}A\Phi)^\mu \Phi^{-1}\bar{x}(0) \tag{13}$$

and in the z space, after μ time-steps, we will have

$$z(\mu) = (\Phi^{-1}A\Phi)^\mu \varepsilon_z(0) \tag{14}$$

Then, from Equations (6) and (14), and using the properties of matrix norms, we obtain

$$\begin{aligned}\|z(\mu)\| &= \|(\Phi^{-1}A\Phi)^\mu \varepsilon_z(0)\| \leqslant \|(\Phi^{-1}A\Phi)^\mu\| \|\varepsilon_z(0)\| \\ &\leqslant \sqrt{n} 2^{-r_n} \|(\Phi^{-1}A\Phi)^\mu\| \|\Phi^{-1}\| L_0\end{aligned}$$

Similarly, in order to force the state z to decrease in the norm (after μ steps), we shrink the upper bound of the state $z(\mu)$ by forcing it to be less than the lower bound of the state $z(0)$, i.e. $2^{-R_n}\sqrt{n}\|(\Phi^{-1}A\Phi)^\mu\|\|\Phi^{-1}\|L_0 < \|\Phi^{-1}\|L_0$. However, if $\Phi^{-1}A\Phi$ is a diagonal matrix then $\|(\Phi^{-1}A\Phi)^\mu\| = |\rho(A)|^\mu$, where $\rho(A)$ is the spectral radius of A. We can then replace in Theorem 4.1 the expression $R > n\lceil \log_2(\|A^\mu\|) + 1/2\log_2(n) \rceil + n$ with

$$R > n\lceil \log_2(\rho(A)^\mu) + \tfrac{1}{2}\log_2(n) \rceil + n \tag{15}$$

If matrix $\Phi^{-1}A\Phi$ is a Jordan-block matrix (for the case of repeated eigenvalues of A), we also know that $\|(\Phi^{-1}A\Phi)^\mu\| \approx |\rho(A)|^\mu$. This quantity, in general, is less than $\|A^\mu\|$. We can consider as an approximation that the rate is no longer a function of the norm of A but rather a function of $\rho(A)$. Therefore, this leads to a lower sufficient rate for stabilizability, but with the possible deterioration in the transient response.

6. A NEW ENCODER/DECODER DESIGN: A ZOOM-IN-TYPE DYNAMIC QUANTIZER

In the previous sections we obtained sufficient stabilization rates with an easily implementable encoder/decoder scheme for Network Type I. Although such rates are larger than the ones given by the Data Rate Theorem, the implementation of the truncation-based scheme requires less computational power than other published schemes. Specifically, the evolution of the quantizer in our scheme uses one scalar equation (Equation (3)). On the other hand, encoder–decoder schemes such as the ones proposed in [8, 9] or achieve the minimum rate established by the Data Rate Theorem at the expense of a higher computational cost since they require state-space predictors, the use

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similarity transformation (to undo the rotations caused by the A matrix), and the calculation of the centroid of the region that traps the state-space variables. In some scenarios, both the computational power and the rate may be constrained. Our purpose in this section is to design an encoder–decoder scheme that achieves a rate close to that provided by the Data Rate Theorem, while using less computational power. The following builds upon ideas described in [8–10].

6.1. Encoder–decoder design

Let the initial state be bounded by some value L_0, i.e. $\|x(0)\| \leqslant L_0$. This equal-length side n-cube region will have 2^n vertices. The set of 2^n vertices is denoted by V_0, and each vertex is denoted by, v_0. We introduce a matrix Q_R:

$$Q_R = \text{diag}\left\{\frac{1}{2^{r_1}}, \frac{1}{2^{r_2}}, \ldots, \frac{1}{2^{r_n}}\right\} \tag{16}$$

Moreover, we will assume that $r_1, r_2, \ldots, r_n$ are such that the matrix $A_Q = AQ_R$ is a stable matrix (we will show later how to accomplish this goal). In the following steps, we focus on the analysis problem and assume that the plant is deterministic and undriven as described by $x(k+1) = Ax(k)$. The controller design problem will be discussed in Section 6.2. The first step is to generate an n-dimensional cube centered at the origin with sides of length $2L_0$. The center of this first quantizer will be labeled $C_Q(0)$. The uncertainty region is divided in 2^{r_1} subregions in the x_1 direction, 2^{r_2} subregions in the x_2 direction, and so on until we obtain 2^{r_n} subregions in the x_n direction. After one time step, the state will land in one of these smaller n-dimensional cubes and the total of small cubes will be $2^{r_1+r_2+\cdots+r_n}$. Therefore, the number of bits needed to represent all the cube centroids is $R = r_1 + r_2 + \cdots + r_n$, which is the actual rate in bits/time-step. After determining in which cube the state has landed, we calculate the centroid of this smaller cube. This centroid will be chosen by the encoder as the estimate of the state, $\bar{x}(0)$. The binary symbol, s, that represents $\bar{x}(0)$ is transmitted to the receiver. Note that the error between the state and the state estimate, $\varepsilon(0)$, lies in the region $\{[-L_0/2^{r_1}, L_0/2^{r_1}], [-L_0/2^{r_2}, L_0/2^{r_2}], \ldots, [-L_0/2^{r_n}, L_0/2^{r_n}]\}$. This is the key property of this quantizer.

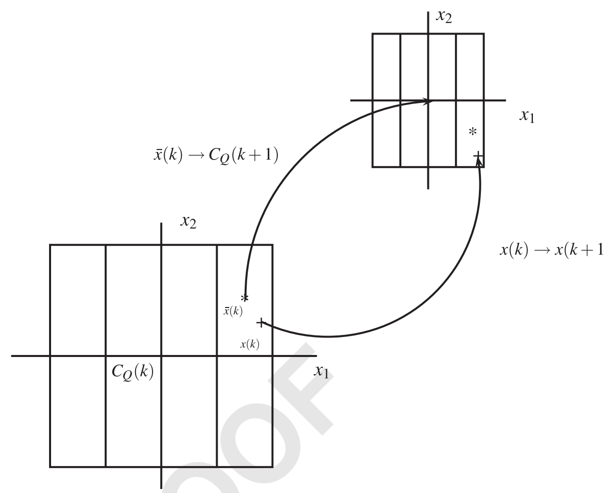

Figure 6. Quantizer evolution sample: centroid, state and state estimator.

Figure 6 shows an example of a two-dimensional quantizer with $r_1 = 2$ and $r_2 = 1$. The encoder and decoder will evolve the center of the quantizer, C_Q at time $k+1$: F6

$$C_Q(k+1) = A\bar{x}(k) \tag{17}$$

This new center is used to generate an uncertainty region that may be divided into another $2^{r_1+r_2+\cdots+r_n}$ subregions with the same 2^{r_i} subregions in the x_i direction as explained before. At time $k+1$, the length of each of the sides parallel to x_i is determined by the quantity Δ_{x_i}. These Δ_{x_i} quantities are determined using the matrix A_Q and the vertices v_0 of the original uncertain n-dimensional cube and given by

$$\Delta_{x_i} = \max_{v_0} |(A_{Q,i})^{k+1} v_0| \quad \forall v_0 \in V_0 \tag{18}$$

where $A_{Q,i}$ is the 'ith' row of matrix A_Q. Equation (18) evaluates the maximum over absolute values, therefore, we can guarantee that the state $x(k+1)$ at time $k+1$ will land in an n-dimensional box (not necessarily a cube) that is centered on $C_Q(k+1)$ and with sides of length $2\Delta_{x_i}$ in the x_i direction. In other words, the hyper-planes that are perpendicular to the x_i component direction will be located at $-\Delta_{x_i}$ and Δ_{x_i} units from $C_Q(k+1)$ in the x_i direction. The new uncertainty box will again be divided into $2^{r_1+r_2+\cdots+r_n}$ boxes with 2^{r_i} in

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the x_i direction. We label these small boxes with binary symbols (a total of $2^{r_1+r_2+\cdots+r_n}$ binary symbols). We then determine in which of these boxes the actual state, $x(k+1)$, lies and use the centroid of this specific box as the state estimate $\bar{x}(k+1)$ at time $k+1$. We again transmit the binary symbol, s, that corresponds to the box where the state lies. Because of the way we have constructed this quantizer and since A_Q was assumed to be stable, the uncertainty box keeps on shrinking as k tends to infinity, which guarantees that our state estimate reaches the actual state and that $\|\varepsilon\|$ tends to zero. Note that both encoder and decoder must know the original size L_0 of the uncertainty as well as the exact dynamics of the plant. In addition, both encoder and decoder must be able to compute Equations (17) and (18). This guarantees the equimemory property. The only remaining issue is to guarantee that A_Q is stable. This may be done by the following procedure:

1. Set $r_i = \lceil \log_2(|\lambda_i|) \rceil$ $\forall i \in \{1, 2, \ldots, n\}$, where λ_i is any of the n eigenvalues of A such that for all $i \neq j$ the eigenvalue chosen is different. For the particular case where A is diagonal or a Jordan block matrix, then r_i is chosen to be $r_i \geqslant \lceil \log_2(|\lambda_i|) \rceil$, where λ_i is the eigenvalue associated with the state-space component x_i.
2. Using rates r_i, we form the matrix Q_R and obtain the eigenvalues of $A_Q = AQ_R$.
3. Check that all such eigenvalues are inside the unit circle, i.e. $|\lambda_{A_Q}| < 1$.
4. If $|\lambda_{A_Q}| < 1$, stop and use the rates r_i for transmission. If for any eigenvalue of A_Q we have $|\lambda_{A_Q}| \geqslant 1$, then we look for the largest r_i in Q_R such that $r_i < \lceil \log_2(\rho(A)) \rceil$, and replace it by $r_i + 1$ and return to step 2.

We note that when A is not in Jordan form, there is a degree of freedom in the way we allocate the bits for every x_i; i.e. what eigenvalue is picked for every r_i. Therefore, the rate given by this algorithm is no unique and optimizing this allocation is a part of a future work.

6.2. Adding a controller for stabilization

We consider the system described by Equation (1). Let us include this system in the encoder/decoder computations and modify Equations (17) and (18) accordingly. The new equations are

$$C_Q(k+1) = A\bar{x}(k) + Bu(k) \tag{19}$$

$$\Delta_{x_i} = \max_{v_0} |(A_{Q,i})^{k+1} v_0| \quad \forall v_0 \in V_0 \tag{20}$$

where $A_{Q,i}$ is the 'ith' row of matrix A_Q. We assume that the encoder/decoder has access to the control signal or that it may be computed locally. The derivations of the previous subsection remain valid since the addition of the control law only represent a *translation* of the centroid of the quantizer. At this point the simplest controller is the estimated state linear feedback controller, $u(k) = -K_c\bar{x}(k) = -K_c(x(k) - \varepsilon(k))$, which is motivated by the following lemma found in [8].

Lemma 6.1 (*Tatikonda and Mitter* [*8*])
Let A_s be a stable matrix. Let Bs_m a set of matrices such that $\|Bs_m\| \leqslant M$, $M \in \mathbb{R}$, $\forall m$, and the limit $\lim_{m\to\infty} Bs_m \to 0$. Let $S_k = \sum_{m=0}^{k-1} A_s^{k-1-m} Bs_m$ then $\lim_{k\to\infty} S_k \to 0$.

We will use this Lemma 6.1 as follows. If a K_c is found such that $A - BK_c$ is stable, then we can solve iteratively $x(k+1) = Ax(k) + B(-K_c\bar{x}(k)) = Ax(k) + B(-K_c(x(k) - \varepsilon(k)))$ with initial condition $\bar{x}(0)$:

$$x(k) = (A - BK_c)^k \bar{x}(0) + \sum_{m=0}^{k-1} (A - BK_c)^{k-1-m} BK_c \varepsilon(m) \tag{21}$$

Our encoder/decoder scheme guarantees that $\|\varepsilon(m)\| \leqslant \sup_k \|(A_{Q,i})^{k+1} v_0\|$ and that $\|\varepsilon(m)\|$ tends to zero when m grows. Moreover, since A_Q is stable, we know that $\sup_k \|(A_{Q,i})^{k+1} v_0\| \leqslant \infty$. If we let $A_s = A - BK_c$ and $Bs_m = BK_c\varepsilon(m)$, we then may apply Lemma 6.1. We see that any stabilizing K_c asymptotically stabilize the system using the rates obtained earlier since the first additive term in Equation (21) tends to zero (since $A - BK_c$ is stable), and the second additive term tends to zero by Lemma 6.1.

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7. COMPARISON BETWEEN BOTH ENCODING SCHEMES

The truncation-based scheme requires a larger data rate than the dynamic quantizer. To prove this fact, we note that the worst data rate that is required in the dynamic quantizer is when $r_i = \lceil \log_2(\rho(A)) \rceil$, $\forall i$. Let $r_\rho = \lceil \log_2(\rho(A)) \rceil$, then $Q_R = (1/2^{r_\rho})\mathbb{I}_{n\times n}$. This is the worst case since A_Q is guaranteed to be stable for this particular Q_R. This is easily proven since $A_Q = AQ_R = (1/2^{r_\rho})A$. From Linear Algebra, we know that the eigenvalues of A_Q are the eigenvalues of A multiplied by $1/2^{r_\rho}$. From the definition of r_ρ, the eigenvalues of A_Q are inside the unit circle. We note that for the worst case, the rate given by the dynamic quantizer is $R = nr_\rho = n\lceil \log_2(\rho(A)) \rceil$. The best case for the truncation-based encoding scheme is $R = n\lceil \log_2(\rho(A)^\mu) + \frac{1}{2}\log_2(n) \rceil + n$ according to Equation (15). It is then obvious that the dynamic quantizer achieves lower rate than the truncation-based one. In terms of the computational cost of both schemes, we note that the truncation-based only needs to compute the scalar equation (3) in order to decode correctly the transmitted signal. The dynamic quantizer however has to compute two Equations, (19) and (20). Moreover, once the quantizer evolves from k to $k+1$ we need to compute in which of the $2^{r_1+r_2+\cdots+r_n}$ boxes the state is located, and this requires several comparison operations.

We can summarize that while our scheme saves the matrix transformation step, it requires some additional bits, the question of when to use one or the other depends on the quality of the network (channel capacity) versus the quality of the processor power. Whichever is more limited will determine what scheme is more suitable. On the other hand, if we compare the schemes in [8, 10], with the truncation-based scheme proposed in this work, it will be difficult to predict the performance of the closed-loop system. While our scheme uses more bits, the fixed structure of our controller limits the performance as compared with an optimal choice of the gain in the state feedback controllers used in [8, 10]. However, since the rate for our scheme depends solely on the norm of A, it is easier to incorporate other issues such as unmodeled dynamics or saturation in the controllers (see [11]). Moreover, even if we have an unconstrained network (high bandwidth), another advantage of our scheme is that it can be easily used for bit-limited acquisition systems (which by design truncates the measured signals).

8. SIMULATIONS

To verify some of the results derived in the previous sections, we present several numerical examples using Matlab®. We consider a DTLI plant, so that, $x(k)$ exists only at the time instants $k = \{0, 1, 2, \ldots\}$. We do not consider the discretization of a continuous time system; hence, the sampling time is not specified in the simulations. However, in all the plots, $x(k)$ was interpolated between sampling times for ease of visualization. We intentionally omit the packet maximum length $D_{\max}$; hence, we can compare the rates in bit/time-step and not in packet/time-step, which is equivalent to assuming that $D_{\max} = 1$ bit/packet. The value L_0 that is known *a priori* by the encoder–decoder scheme was selected in the simulations to be $L_0 = 2\|x(0)\|$.

8.1. Example 1

We tested the results of Theorem 4.1 for the system

$$x(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} u(k)$$

Let $L_0 = 71.68$ and we assume the initial condition $x(0) = [-16.333\ 30.768\ 8.44]'$, such that, $\|x(0)\| \leqslant L_0$. Since for this example $n = 3$ and $\mu = 2$, the rate obtained according to Theorem 4.1 is $R = 18$ bit/time-step and the simulation is shown in Figure 7. F7
Note that asymptotic stability is indeed achieved. We note that for this system, the Data Rate Theorem gives 3.58 bit/time-step (or more accurately 4) while the dynamic quantizer requires a rate larger than 4 bit/time-step.

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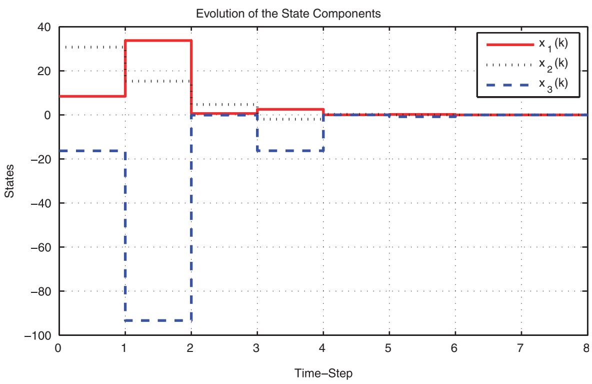

Figure 7. Truncation-based scheme: closed-loop NCS (Type I) using $R=18$ bits/time-step.

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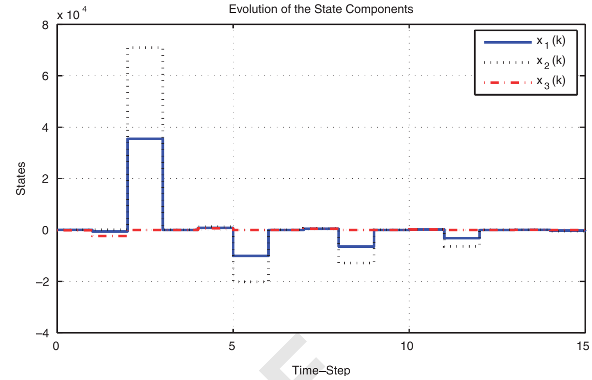

Figure 8. Truncation-based scheme: closed-loop NCS (Type I) using $R=51$ bit/time-step.

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8.2. Example 2

To test the conservativeness of Corollary 4.1, we considered a single-input system given by

$$x(k+1)=\begin{bmatrix}20 & 0 & 10\\ 0 & 10 & 0\\ 0 & 10 & 30\end{bmatrix}x(k)+\begin{bmatrix}1\\1\\1\end{bmatrix}u(k)$$

Let $L_0=166.45$ and we assume initial condition $x(0)=[16.333\ \ 13.768\ \ -80.44]'$. Since for this example $n=\mu=3$, the rate obtained using Corollary 4.1 is
F8 $R=51$ bit/time-step. We then verify in Figure 8 the asymptotic stability claim of the corollary. Since our results provide sufficient conditions only, we tried for smaller values of R and found out that for this particular example, $R=42$ bit/time-step leads to instability,
F9 see Figure 9. We note that for this system, the Data Rate Theorem gives 12.55 bit/time-step while the dynamic quantizer requires a rate of 15 bit/time-step.

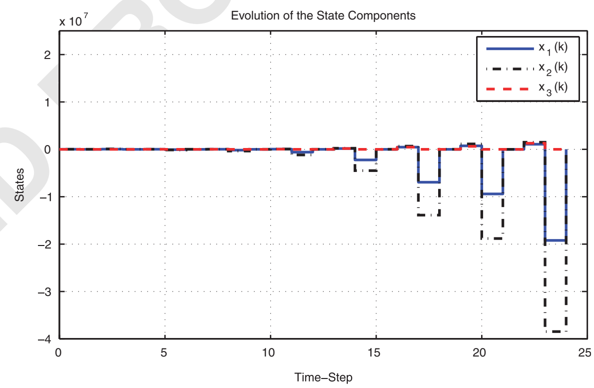

Figure 9. Truncation-based scheme: closed-loop NCS (Type I) using $R=42$ bits/time-step.

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8.3. Example 3

Consider a second-order system ($n=2$) with time-delay $p=2$ evolving according to the following dynamics:

$$x(k+1)=\begin{bmatrix}2 & 0\\ 0 & 1.5\end{bmatrix}x(k)+\begin{bmatrix}1\\1\end{bmatrix}u(k-2)$$

with the initial condition state vector $x(0)=[-16.333\ \ 30.768]'$. Assuming $L_0=69.66$, the rate

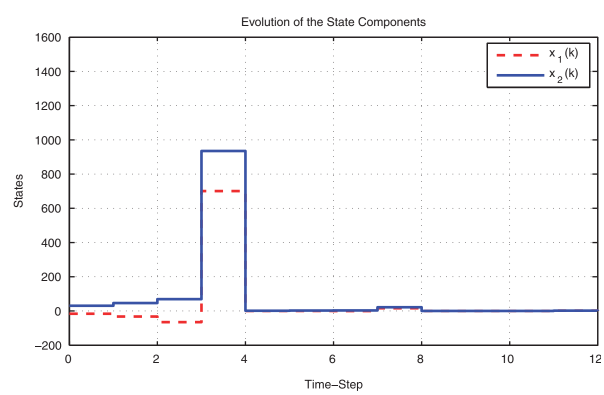

Figure 10. Closed-loop NCS with time-delay.

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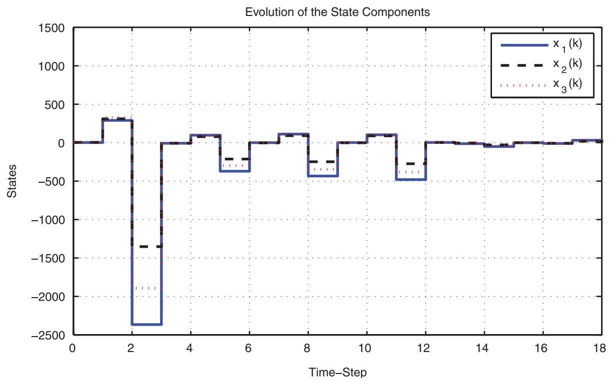

Figure 11. Truncation-based scheme: closed-loop NCS (Type II).

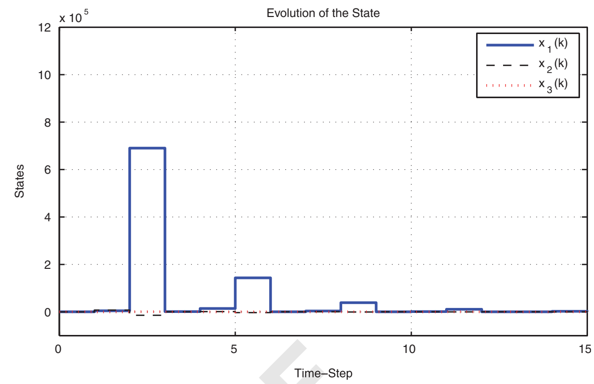

Figure 12. Closed-loop NCS using $R=42$ bit/time-step.

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F10 obtained using Theorem 4.2 is $R=28$ bit/time-step. The corresponding simulation is shown in Figure 10. For this particular example we do not compare with the Data Rate Theorem or our dynamic quantizer since neither of those consider a delayed system.

8.4. Example 4

Consider a third-order system ($n=3$) evolving according to the following dynamics:

$$x(k+1)=\begin{bmatrix}4 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 5\end{bmatrix}x(k)+\begin{bmatrix}1\\1\\1\end{bmatrix}u(k)$$

with the initial condition state vector $x(0)=[1.33\ \ 3.768\ \ 8.44]'$. We assume that this plant is a part of a Network Type II and we also assume $L_0=18.67$. The network rates obtained using Theorem 4.3 are $R_1=30$ bit/time-step and $R_2=10$ bit/time-step and
F11 the simulation is shown in Figure 11. For this particular example we do not compare with the Data Rate Theorem since this last one considers a Network Type I and not a Type II as in this example.

8.5. Example 5

The following simulation shows the evolution of x when using the control law given in Equation (13) with the rate given by $R=n\lceil\log_2(|\lambda_{\max}|^{\mu})+\frac{1}{2}\log_2(n)\rceil+n$. Let us consider the following system:

$$x(k+1)=\begin{bmatrix}2 & 100 & 100\\ 0 & 4 & 100\\ 0 & 1 & 4\end{bmatrix}x(k)+\begin{bmatrix}1\\1\\1\end{bmatrix}u(k)$$

Let the initial condition be $x(0)=[16.333\ \ 13.768\ \ -80.44]'$ and $L_0=166.45$. Using Equation (15), we find that $R=42$ bit/time-step is now sufficient for stabilization. This was not the case using the control law depending on the controllability matrix of the pair (A,B). The simulation using this control law is shown in Figure 12. We also show in Figure 13 the simulation F12
using the results of Theorem 4.1 and the rate was F13
$R=57$ bit/time-step. The tradeoff is evident when comparing the two simulations: although a lower rate is needed in the simulation in Figure 12, the transient response (overshoot, settling time) in Figure 13 is actually better.

8.6. Example 6

We present next an example considering the following system:

$$x(k+1)=Ax(k)+Bu(k)=\begin{bmatrix}2 & 0.5\\ 3 & 4\end{bmatrix}x(k)+\begin{bmatrix}1\\1\end{bmatrix}u(k)$$

$$u(k)=-K_c x(k)=-[2.533\ \ 2.566]\,x(k)$$

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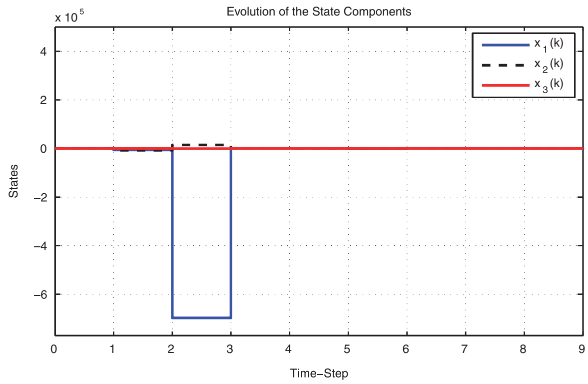

Figure 13. Closed-loop NCS using $R=57$ bit/time-step.

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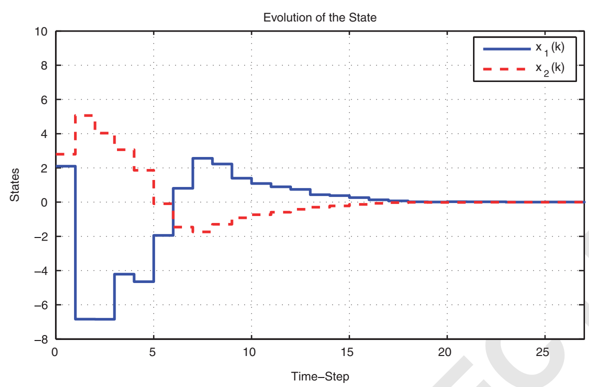

Figure 14. State evolution in NCS Type I using $R=5$ bits/time-step.

With this K_c, the poles of $(A-BK_c)$ are located at 0.5 and 0.4. We assume that the initial condition state vector $x(0)=[2.1\ \ 2.8]'$ and $L_0=7$. We calculate the rates to stabilize A_Q are $r_1=2$ and $r_2=3$. This gives a total rate of $R=5$ bit/time-step. Using the dynamic quantizer scheme we obtain the plots in Figure 14. F14

9. CONCLUSIONS AND FUTURE WORK

This paper has extended previous results for determining the sufficient rate for stabilization of a packet-based networked control system (NCS). While the rates obtained for Network Type I are higher that the limits set by the Data Rate Theorem, the computational cost of our scheme is lower than earlier proposed schemes. In this setup we were able to treat the case of a constant time-delay in the network.

We also obtained sufficient rates for stabilizing a system using a Network Type II. In order to lower the required transmission rates, we proposed a more complex encoder/decoder scheme that achieves rates close to those specified by the Data Rate Theorem.

Future work will include the inclusion of time delays in an NCS Type II, and the extension of the general case of m inputs of this type of closed-loop system. Other ideas for future work include dealing with noise in the loop and the generalization to the case of packet drops and saturation in the control signal.

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