2.1 ~ Quadratic Functions & Parabolas

Graphing Parabolas:

1. Standard Form: $f(x)=a(x-h)^2+k$ Find vertex (h, k) and make a t-chart.

   *Note: if $a>0$, the parabola opens _______ and the vertex is a ______________.

   if $a<0$, the parabola opens _______ and the vertex is a ______________.

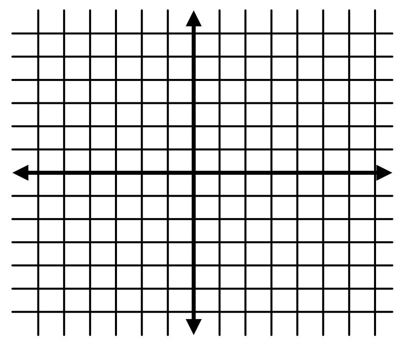
2. Quadratic Function: $f(x)=ax^2+bx+c$ Use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ to find the vertex and make a t-chart.

3. Identify the vertex and axis of symmetry. Find the x & y intercepts to use as additional points.

Ex 1: Graph $f(x)=2(x-3)^2+4$

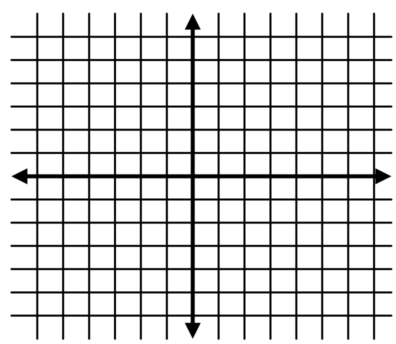
Vertex: ___________

X	Y



Ex 2: Graph $f(x)=x^2-4x+3$ use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ to find the vertex.

X	Y



Ex 3: From example 2: $f(x)=x^2-4x+3$ use vertex (____, ____) and find the following.

X-intercepts Y-intercepts Axis of symmetry

What if the equation is not in standard form?!?

** Then change it into standard form… by ___________________________________.

In examples 4-7, change each quadratic into standard form and find the vertex of each.

Ex 4: $f(x) = x^2 + 12x + 9$

Ex 5: $f(x) = 2x^2 + 8x + 7$

Ex 6: $f(x) = -x^2 + 6x - 8$

Ex 7: $f(x) = -3x^2 - 12x - 16$

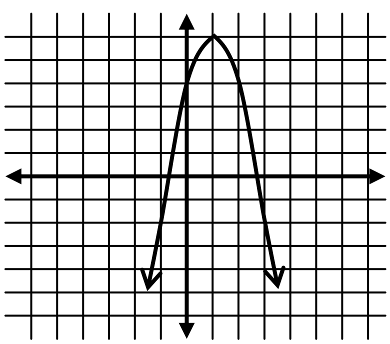
Ex 8: Write an equation of the quadratic given the vertex (3, 4) and point that passes through (1, 2).

Ex 9: Write an equation of the quadratic given the vertex (2, 3) and point that passes through (0, 2).

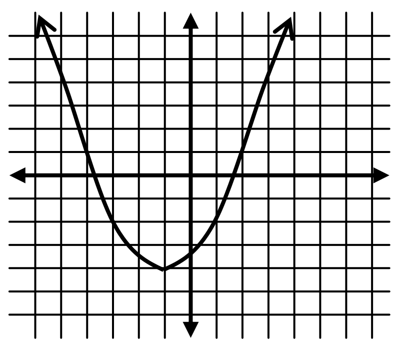
Ex 10: Write an equation of the quadratic given the vertex (-2,5) and point that passes through (-1,7).

Ex 11: Write an equation of the quadratic given the vertex (-1, 4) and point that passes through (3, 0).

Ex 12: Write the equation from the graph.



Ex 13: Write the equation from the graph.



Ex 14: **Maximum Revenue:** Find the number of units sold that yields a maximum annual revenue for a sporting goods manufacturer. The total revenue R (in dollars) is given by $R = 1000x - 0.0002x^2$, where x is the number of units sold. *Hint: To answer this question, the vertex of the parabola is needed.*

$R = 1000x - 0.0002x^2$

X= # of units sold
R = total Revenue

Ex 15: A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45° with respect to the ground. The path of the baseball is given by the function $f(x) = -0.0032x^2 + x + 3$, where $f(x)$ is the height of the baseball (in feet) and x is the horizontal distance from home plate (in feet). What is the maximum height reached by the baseball?

Ex 16: A small local soft-drink manufacturer has daily production costs of $C = 70,000 - 120x + 0.075x^2$ where *C* is the total cost (in dollars) and *x* is the number of units produced. How many units should be produced each day to yield a minimum cost?

2.2 ~ Polynomial Functions

I. **Polynomial Functions**:

- Must be ________________________ (no breaks, holes or gaps in the graph)
- Would be classified as __________ or _________ depending on the degree* of the polynomial.

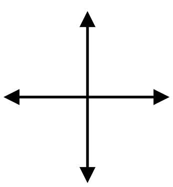
    1. Even functions

    2. Odd functions

II. **Leading Coefficient Test:**

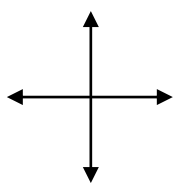
*You will look at the leading coefficient to classify the function as even or odd, then determine what the graph will resemble depending on if *a* is positive or negative.

a) $f(x)=x^2$



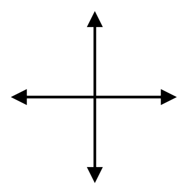
degree: *even*
a is ________
starts____ ends ____

b) $f(x)=-x^2$



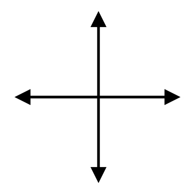
degree: *even*
a is _________
starts_____ ends ____

c) $f(x)=x^3$



degree: *odd*
a is _________
starts ____ ends ____

d) $f(x)=-x^3$



degree: *odd*
a is ________
starts ____ ends____

* *Remember that all even functions will resemble the ______________, and odd functions will resemble the ________.

III. **Zeros of a Polynomial Function:**

* The zeros of a function are the __________________ or the __________________.

* The degree of your polynomial helps you to know…

***Multiplicity** –

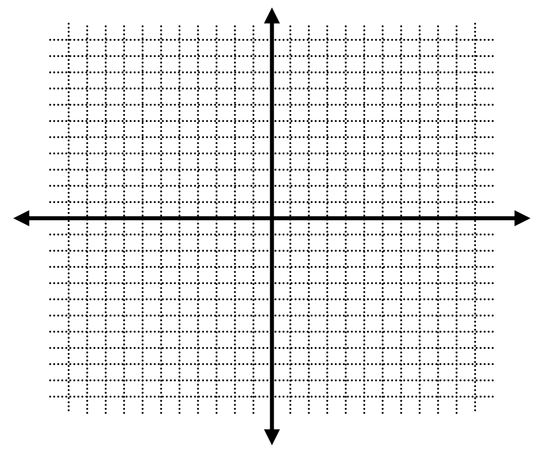
- Solutions that have *even multiplicity* will ____________ the *x*- axis at that point and bounce away.
- Solutions that have *odd multiplicity* will _____________ the *x*- axis at that point.

Ex 1: Find the zeros of: $f(x) = -2x^4 + 2x^2$ **Determine the multiplicity of each zero and whether it touches or crosses the graph.**

Steps to Sketch a Polynomial

1. Apply the leading coefficient test.
2. Find the zeros by factoring.
3. Plot a few additional points-use points that are between each of the zeros.
4. Sketch the graph.

Ex 2: Sketch the graph of $f(x) = -2x^4 + 2x^2$



For examples 3-6, find the zeros. Then determine the multiplicity of each zero and whether it touches or crosses the graph.

Ex 3: $f(x) = 2x^2 + 11x - 21$

Ex. 4: $f(x) = t^3 - 3t$

Ex. 5 $f(x) = x(x+3)^2$

Ex. 6 $f(x) = x^4 - 3x^3 - 2x^2$

Ex 7: Find a polynomial function that has the given zeros.

a) -4 and 5

b) 0, 2, and 5

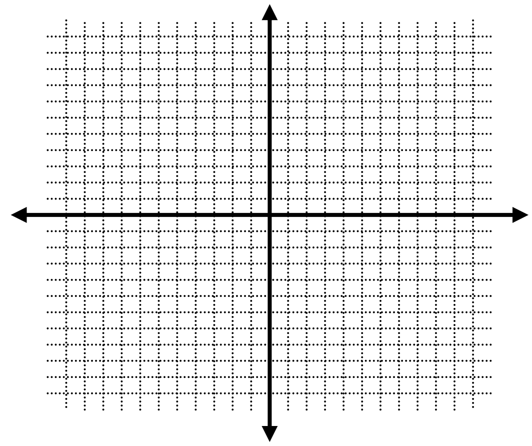
Ex 8: Find a polynomial of <u>degree 3</u> with given zeros:

a) -2, 3, and 5

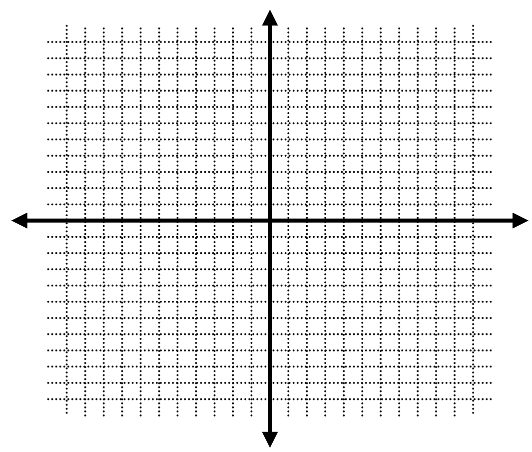
b) 3

For example 9 and 10, determine the multiplicity of each zero and whether it touches or crosses the graph. Then sketch each graph.

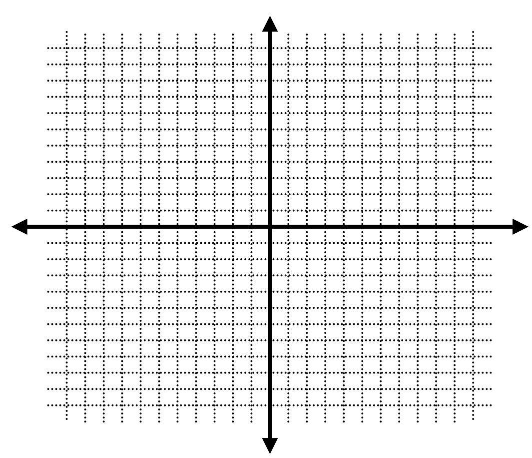
Ex 9: $f(x) = 2x^3 - 6x^2 + \frac{9}{2}x$



Ex 10: $f(x) = x^4 - x^3 - 2x^2$



Ex 11: $f(x) = x^5 + x^3 - 6x$



2.3 ~ Polynomial and Synthetic Division

**Use synthetic division or long division to show that x is a zero of the equation.

Ex 1: Use Long Division: a) $\dfrac{4x^3-8x^2+x+3}{2x-3}$ b) $\dfrac{3x^3-16x^2-72}{x-6}$

Ex 2: Use synthetic division: a) $\dfrac{4x^3+16x^2-23x-15}{x+\frac{1}{2}}$ b) $\dfrac{5-3x+2x^2-x^3}{x+1}$

****NOTE**: You will not always get zero as a remainder! You only get zero if it is a ___________ .

Ex 3: Use synthetic division to find each function value of $f(x)=x^6-4x^4+3x^2+2$

a) $f(2)=$ _____ *b)* $f(-1)=$ _____

.

Ex 4: List **all** the zeros of $6x^3-19x^2+16x-4=0$ *given* $x=2$.

Zeros: ____________

Ex 5: List **all** the zeros of $x^3 - x^2 - 13x - 3 = 0$ *given* $x = 2 - \sqrt{5} \rightarrow$ __________ *(always comes with conjugate)*

Ex 6: A. Verify the given factors of the function f.
B. Find the remaining factors
C. List all real zeros of f.

Function: $f(x) = 6x^3 + 23x^2 - 39x - 140$

Factors: $(2x-5)(x+4)$

Factors: ________________
Zeros: _________________

****<u>NOTE:</u>** If a quadratic does not factor, we should use the ___________________________________ .

X =

Ex 7: Find the remaining zeros given **one** zero. $x^3 - 28x - 48 = 0$ $x = 6$

Zeros: _____________

2.4 ~ Complex Numbers

Imaginary Unit " i": $i =$ _____ $i^2 =$ _____

Ex 1: Complete the following. What is the pattern that you find?

$i^0 = 1$ $i^1 =$ _________ $i^2 =$ _________ $i^3 =$ _________

$i^4 =$ _________ $i^5 =$ _________ $i^6 =$ _________ $i^7 =$ _________

$i^8 =$ _________ $i^9 =$ _________ $i^{10} =$ _________ $i^{11} =$ _________

Simplify the following: *a)* $i^{23} =$ _________ *b)* $i^{37} =$ _________ *c)* $i^{32} =$ _________

Complex numbers:

Standard Form:

Ex 2: Adding and Subtracting Complex Numbers

a) $(4+7i)+(1-6i)$ b) $(1+2i)-(4+2i)$ c) $3i-(-2+3i)-(2+5i)$

Ex 3: Multiplying Complex Numbers

a) $4(-2+3i)$ b) $(2+i)(4+3i)$ c) $(3+2i)^2$ d) $(3+2i)(3-2i)$

Complex Conjugates:

Ex 4: Writing the quotient of complex numbers in standard form ($a+bi$).

a) $\frac{6}{i}$

b) $\frac{2}{3-i}$

c) $\frac{2+3i}{4-2i}$

d) $\frac{2}{1+i}-\frac{3}{1-i}$

Complex Solutions of Quadratic Equations

In section 2.2 we used the quadratic formula to find x–intercepts and often obtained a solution such as $\sqrt{-5}$, which meant no "*real*" solutions. By factoring out $i=\sqrt{-1}$, you can write the number in standard form:

$$\sqrt{-5} \quad = \quad \sqrt{5(-1)} \quad = \quad \sqrt{5} \quad \sqrt{-1} \quad = \quad 5i$$

5i is considered the principal square root

Ex 5: Write the complex numbers in standard form.

a) $\sqrt{-3}\sqrt{-12}$

b) $\sqrt{-48}-\sqrt{-27}$

c) $(-1+\sqrt{-3})^2$

Ex 6) Use the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve the quadratic equation $5x^2-4x+7=0$.

2.5 ~ Zeros of Polynomial Functions

I. Writing the equation of functions given the roots: R_1 and R_2.

Method 1: *-OR-* **Method 2:**

$(x-R_1)(x-R_2)=0$ *foil to find polynomial*

opposite of the sum ↓ *product* ↓

$x^2 + \_\_\_\_\_ x + \_\_\_\_\_ = 0$

Ex 1: Roots: 3, -2

$(x- \quad)(x- \quad)=0$

-OR-

sum: ____ + ____ = ____

product: (   )(   )= _____

Ex 2: Roots: $\frac{2}{3}, \frac{-3}{8}$

Ex 3: Roots: $1+2i$, $1-2i$

EX 4: Roots: $1+\sqrt{3},\quad 1-\sqrt{3},\quad -2$

<u>Complex Roots come in Conjugate Pairs</u>*:* $1+\sqrt{2}$ *and* _________ $4-2i$ *and*_________*, etc.*

II. The Zeros of the Polynomial

To determine the possible Rational Zeros = $\frac{p}{q}$ = $\frac{\textit{all the factors of the constant term}}{\textit{all the factors of the leading coefficient}}$

Ex 5: $f(x)=2x^3+3x^2-8x+3$ *degree=*_____ *, which means there are* ___________ *.*

$\frac{p}{q}$ =

Ex 6: $f(x)=-10x^3+15x^2+16x-12$ *degree=*_____*, which means*_______________________ *.*

Ex 7: Find all the zeros, given that *1 + 3i*, is one of the given roots of: $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$

If 1 + 3i is a factor, so is ___________. *How many zeros are we looking for?*________

Zeros: ___________________________

Ex 8: Find the zeros: $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$ a) List all the factors and b) list all the zeros.

Zeros: ___________________________

Ex 9: Find all the zeros, given that $1 - i\sqrt{3}$, is one of the given roots of: $f(x) = 3x^3 - 4x^2 + 8x + 8$

Remember that once the depressed polynomial is a quadratic, it can be solved using traditional methods such as factoring, completing the square, or quadratic formula.

Ex 10: Find the zeros: $f(x) = x^4 - 625$ **a) List all the factors and b) list all the zeros.**

Ex 11: Find the zeros: $f(x) = x^3 - 3x^2 + 4x - 2$ **a) List all the factors and b) list all the zeros.**

Proving Upper and Lower Bounds

Ex. 12 Use synthetic division to verify the upper and lower bounds of the real zeros of *f*.

$$f(x) = 2x^3 - 3x^2 - 12x + 8$$

a) Upper x=4 b) lower x= –3

Ex. 13 Use synthetic division to verify the upper and lower bounds of the real zeros of *f*.

$$f(x) = 2x^4 - 8x + 3$$

b) Upper x=3 b) lower x= –4

2.6 ~ Rational Functions

Rational function:

Ex 1: Find the domain of the following rational functions:

A. $f(x)=\dfrac{x+4}{x^2-9}$

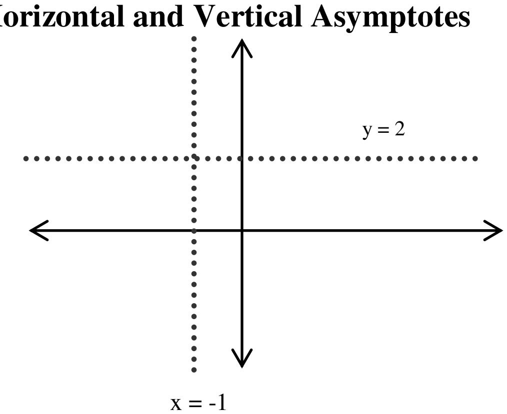
B. $f(x)=\dfrac{x^2+5x+4}{x^3+3x^2-4x-12}$

Asymptotes of a Rational Function

Let *f* be the rational function given by: $f(x) = \dfrac{N(x)}{D(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + ...}{b_d x^d + b_{d-1}x^{d-1} + ...}$

1. The graph of *f* has ***vertical asymptotes*** at the zeros of *D(x)*.
2. The graph of *f* has either one or no ***horizontal asymptote*** determined by comparing the degrees of *N(x)* and *D(x)*.
   a. If the degree of the numerator is the same as the degree of the denominator, the graph of *f* has the line $y=\frac{a_n}{b_d}$ as a horizontal asymptote. (The coefficients of leading term.)
   b. If the degree of the numerator is less than the degree of the denominator, then the graph of *f* has the line *y = 0* as a horizontal asymptote. (The x-axis.)
   c. If the degree of the numerator is the greater than the degree of the denominator, the graph has no horizontal asymptote. (It may have a slant asymptote)

Horizontal and Vertical Asymptotes



For the graph at the left: $f(x)=\dfrac{2x+1}{x+1}$, there are two asymptotes.

Vertical Asymptote (Domain) ______________________

Horizontal Asymptote $\dfrac{\text{Degree of top}}{\text{Degree of bottom}}$ (look at leading coefficients) = _____ so y = _____

Ex 2: Find the domain, list any holes, horizontal and vertical asymptotes of the graph of each rational function.

A. $f(x) = \dfrac{3x+1}{x^2+x-2}$

Domain: ______________________________

Vertical Asymptote(s):__________________

Horizontal Asymptote:__________________

B. $f(x) = \dfrac{3x^2-5}{2x^2-9x+7}$

Domain: __________________________________

Vertical Asymptote(s):_______________________

Horizontal Asymptote:_______________________

C. $f(x) = \dfrac{2x-4}{x^2-4}$

Domain: ______________________________

Vertical Asymptote(s):__________________

Horizontal Asymptote:__________________

Hole(s): ______________________________

D. $f(x) = \dfrac{3x^2+5x}{1x-x^2}$

Domain: __________________________________

Vertical Asymptote(s):_______________________

Horizontal Asymptote:_______________________

Hole(s): __________________________________

Oblique (Slant) Asymptote

State the domain of the function and identify any holes, vertical, horizontal, or slant asymptotes.

A. $f(x) = \dfrac{x^3+x^2}{x^2-1}$

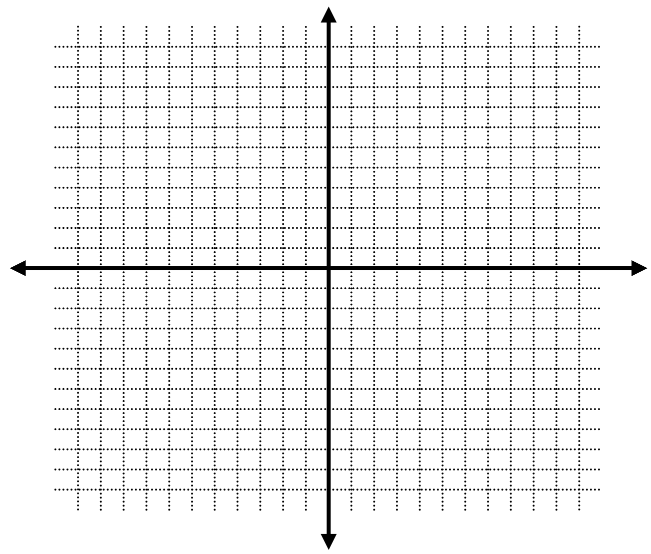
B. $f(x) = \dfrac{x^2-x+4}{x+1}$

2.6 ~ Graphing Rational Functions

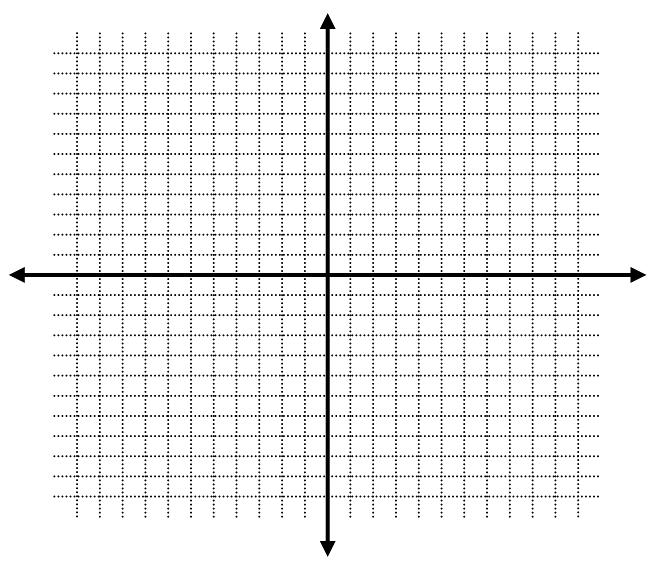
For each function below:

(a) State the domain
(b) Identify any vertical, horizontal, or slant asymptotes
(c) Identify all intercepts
(d) Plot additional solutions as needed to sketch the graph

A) $f(x) = \dfrac{t^2 - 2t - 8}{t^2 - 9}$

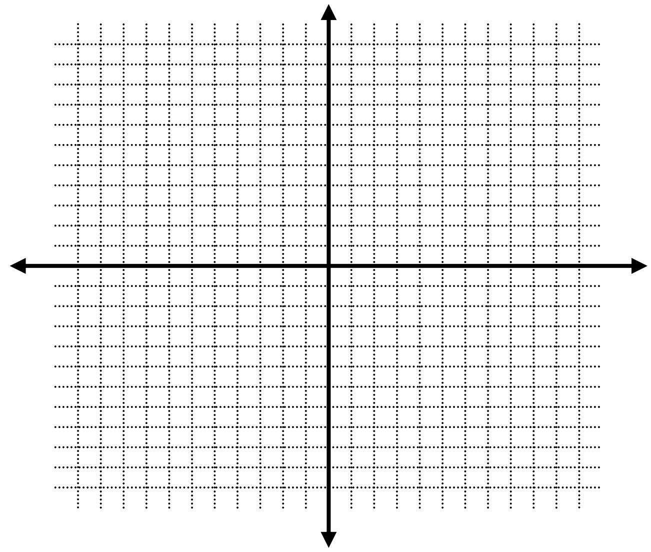


B) $f(x) = \dfrac{2x^2 - 5x - 3}{x - 2}$



Ex 5: Graph the following rational function:

A) $f(x)=\frac{x^2-4}{x^2-3x+2}$



Domain: ______________________

x –intercepts: _____________________,

y- intercepts: _____________

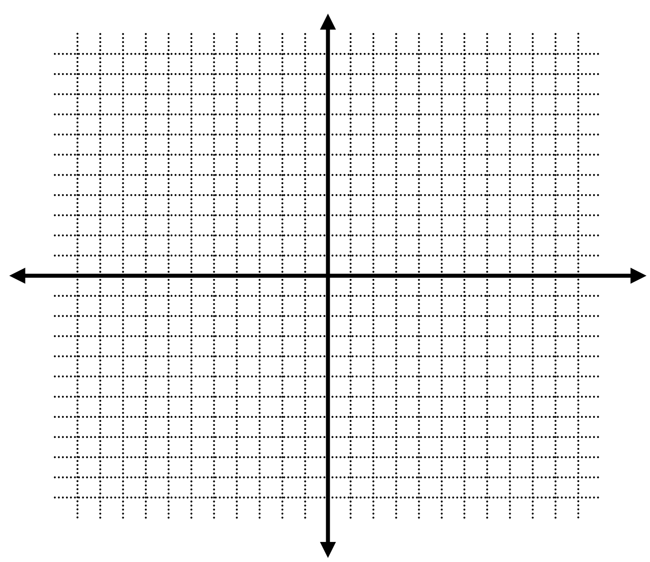
Vert:_________________________

Horizontal: ____________________

Slant: _________________________

Holes: ________________________

B) $f(x)=\frac{x^2-16}{x-4}$



Domain: ______________________

x –intercepts: _____________________,

y- intercepts: _____________

Vert:_________________________

Horizontal: ____________________

Slant: _________________________

Holes: ________________________