Please stick the barcode label here.

Candidate Number

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Question No.	Marks	Marks
1 – 2		
3 – 4		
5 – 6		
7 – 8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
Total		

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

MATHEMATICS   Compulsory Part
PAPER 1 (Sample Paper)
Question-Answer Book

Time allowed: 2 hours 15 minutes
This paper must be answered in English.

INSTRUCTIONS

1.   Write your Candidate Number in the space provided on Page 1.

2.   Stick barcode labels in the spaces provided on Pages 1, 3, 5, 7 and 9.

3.   This paper consists of THREE sections, A(1), A(2) and B. Each section carries 35 marks.

4.   Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

5.   Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

6.   Unless otherwise specified, all working must be clearly shown.

7.   Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

8.   The diagrams in this paper are not necessarily drawn to scale.

1

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(xy)^2}{x^{-5}y^6}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of the formula $a(b+7)=a+b$. (3 marks)

Please stick the barcode label here.

3. Factorize

 (a) $3m^2 - mn - 2n^2$,

 (b) $3m^2 - mn - 2n^2 - m + n$.

(3 marks)

4. The marked price of a handbag is $\$ 560$. It is given that the marked price of the handbag is 40% higher than the cost.

 (a) Find the cost of the handbag.

 (b) If the handbag is sold at $\$ 460$, find the percentage profit.

(4 marks)

5.   In a football league, each team gains 3 points for a win, 1 point for a draw and 0 point for a loss. The champion of the league plays 36 games and gains a total of 84 points. Given that the champion does not lose any games, find the number of games that the champion wins.   (4 marks)

6.   Figure 1 shows a solid consisting of a hemisphere of radius r cm joined to the bottom of a right circular cone of height 12 cm and base radius r cm. It is given that the volume of the circular cone is twice the volume of the hemisphere.

(a)   Find r.

(b)   Express the volume of the solid in terms of π.

(4 marks)

Figure 1

7.   In Figure 2, O is the centre of the semicircle $ABCD$. If $AB \mathbin{/\mkern-5mu/} OC$ and $\angle BAD = 38°$, find $\angle BDC$.

(4 marks)
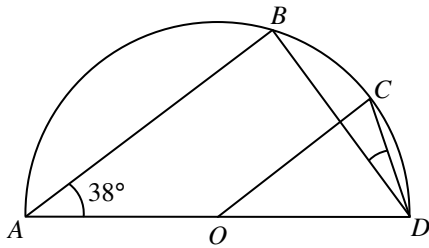
Figure 2

8.   In Figure 3, the coordinates of the point A are $(-2, 5)$. A is rotated clockwise about the origin O through $90°$ to A' . A'' is the reflection image of A with respect to the *y*-axis.

(a)   Write down the coordinates of A' and A'' .

(b)   Is OA'' perpendicular to AA' ? Explain your answer.

(5 marks)

Figure 3

9.   In Figure 4, the pie chart shows the distribution of the numbers of traffic accidents occurred in a city in a year. In that year, the number of traffic accidents occurred in District A is 20% greater than that in District B.
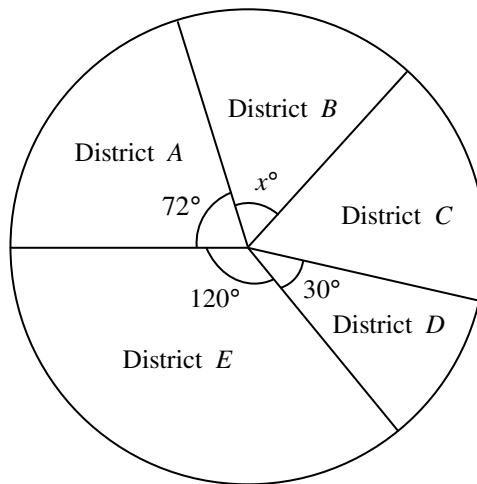
The distribution of the numbers of traffic accidents occurred in the city

Figure 4

(a)   Find x.

(b)   Is the number of traffic accidents occurred in District A greater than that in District C? Explain your answer.

(5 marks)

Please stick the barcode label here.

Section A(2) (33 marks)

10. (a) Find the quotient when $5x^3 + 12x^2 - 9x - 7$ is divided by $x^2 + 2x - 3$. (2 marks)

    (b) Let $g(x) = (5x^3 + 12x^2 - 9x - 7) - (ax + b)$, where a and b are constants. It is given that $g(x)$ is divisible by $x^2 + 2x - 3$.

      (i) Write down the values of a and b.

      (ii) Solve the equation $g(x) = 0$.

      (4 marks)

Answers written in the margins will not be marked.

Go on to the next page

11. In a factory, the production cost of a carpet of perimeter s metres is $\$C$. It is given that C is a sum of two parts, one part varies as s and the other part varies as the square of s. When $s = 2$, $C = 356$; when $s = 5$, $C = 1250$.

(a) Find the production cost of a carpet of perimeter 6 metres. (4 marks)

(b) If the production cost of a carpet is $\$539$, find the perimeter of the carpet. (2 marks)

Please stick the barcode label here.

12. Figure 5 shows the graph for John driving from town A to town D (via town B and town C) in a morning. The journey is divided into three parts: Part I (from A to B), Part II (from B to C) and Part III (from C to D).
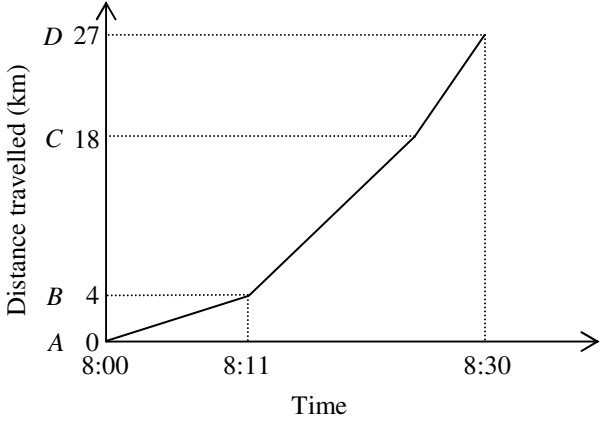

Figure 5

(a) For which part of the journey is the average speed the lowest? Explain your answer. (2 marks)

(b) If the average speed for Part II of the journey is 56 km/h , when is John at C ? (2 marks)

(c) Find the average speed for John driving from A to D in m/s . (3 marks)

Go on to the next page

13. In Figure 6, the straight line L_1: $4x - 3y + 12 = 0$ and the straight line L_2 are perpendicular to each other and intersect at A. It is given that L_1 cuts the y-axis at B and L_2 passes through the point $(4, 9)$.

Figure 6

(a) Find the equation of L_2. (3 marks)

(b) Q is a moving point in the coordinate plane such that $AQ = BQ$. Denote the locus of Q by Γ.

    (i) Describe the geometric relationship between Γ and L_2. Explain your answer.

    (ii) Find the equation of Γ. (6 marks)

10

14. The data below show the percentages of customers who bought newspaper A from a magazine stall in city H for five days randomly selected in a certain week:

62%   63%   55%   62%   58%

   (a)   Find the median and the mean of the above data. (2 marks)

   (b)   Let $a\%$ and $b\%$ be the percentages of customers who bought newspaper A from the stall for the other two days in that week. The two percentages are combined with the above data to form a set of seven data.

      (i)   Write down the least possible value of the median of the combined set of seven data.

      (ii)   It is known that the median and the mean of the combined set of seven data are the same as that found in (a). Write down one pair of possible values of a and b.
(3 marks)

   (c)   The stall-keeper claims that since the median and the mean found in (a) exceed 50%, newspaper A has the largest market share among the newspapers in city H. Do you agree? Explain your answer. (2 marks)

SECTION B (35 marks)

15. The seats in a theatre are numbered in numerical order from the first row to the last row, and from left to right, as shown in Figure 7. The first row has 12 seats. Each succeeding row has 3 more seats than the previous one. If the theatre cannot accommodate more than 930 seats, what is the greatest number of rows of seats in the theatre?

Figure 7

(4 marks)

16. A committee consists of 5 teachers from school A and 4 teachers from school B. Four teachers are randomly selected from the committee.

    (a) Find the probability that only 2 of the selected teachers are from school A. (3 marks)

    (b) Find the probability that the numbers of selected teachers from school A and school B are different. (2 marks)

17. A researcher defined Scale A and Scale B to represent the magnitude of an explosion as shown in the following table:

Scale	Formula
A	$M = \log_4 E$
B	$N = \log_8 E$

It is given that M and N are the magnitudes of an explosion on Scale A and Scale B respectively while E is the relative energy released by the explosion. If the magnitude of an explosion is 6.4 on Scale B, find the magnitude of the explosion on Scale A. (5 marks)

18. In Figure 8(a), ABC is a triangular paper card. D is a point lying on AB such that CD is perpendicular to AB. It is given that $AC = 20$ cm , $\angle CAD = 45°$ and $\angle CBD = 30°$.
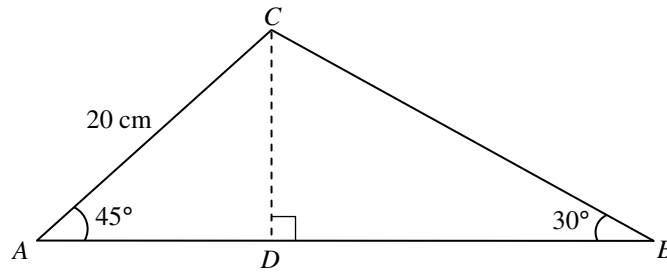
Figure 8(a)

(a) Find, in surd form, BC and BD . (3 marks)

(b) The triangular paper card in Figure 8(a) is folded along CD such that ΔACD lies on the horizontal plane as shown in Figure 8(b).
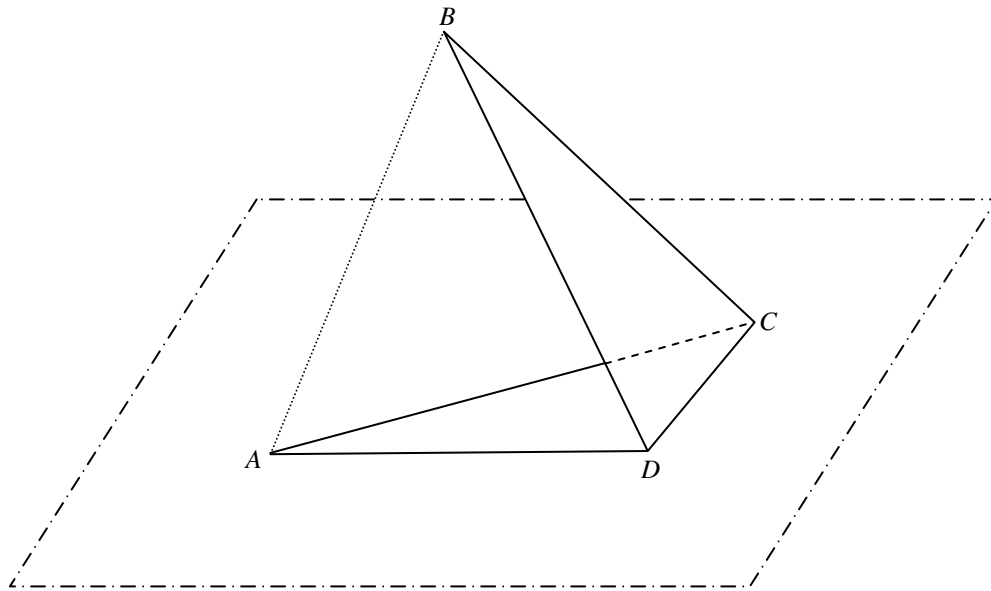
Figure 8(b)

(i) If the distance between A and B is 18 cm , find the angle between the plane BCD and the horizontal plane.

(ii) Describe how the volume of the tetrahedron $ABCD$ varies when $\angle ADB$ increases from $40°$ to $140°$. Explain your answer.

(5 marks)

17

19. In Figure 9, the circle passes through four points A , B , C and D . PQ is the tangent to the circle at C and is parallel to BD . AC and BD intersect at E . It is given that $AB = AD$.
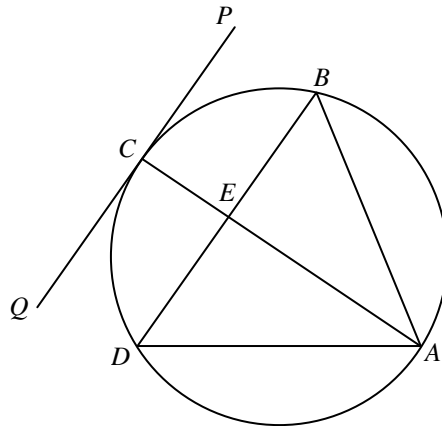
Figure 9

(a) (i) Prove that $\triangle ABE \cong \triangle ADE$.

(ii) Are the in-centre, the orthocentre, the centroid and the circumcentre of $\triangle ABD$ collinear? Explain your answer.

(6 marks)

(b) A rectangular coordinate system is introduced in Figure 9 so that the coordinates of A , B and D are $(14, 4)$, $(8, 12)$ and $(4, 4)$ respectively. Find the equation of the tangent PQ . (7 marks)

END OF PAPER

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

MATHEMATICS   Compulsory Part
PAPER 2   (Sample Paper)

Time allowed: 1 hour 15 minutes

1. Read carefully the instructions on the Answer Sheet. Stick a barcode label and insert the information required in the spaces provided.

2. When told to open this book, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You should use an HB pencil to mark all your answers on the Answer Sheet. Wrong marks must be completely erased with a clean rubber.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

Not to be taken away before the
end of the examination session

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $(3a)^2 \cdot a^3 =$

      A. $3a^5$.

      B. $6a^6$.

      C. $9a^5$.

      D. $9a^6$.

2. If $5 - 3m = 2n$, then $m =$

      A. n .

      B. $\dfrac{2n-5}{3}$.

      C. $\dfrac{-2n+5}{3}$.

      D. $\dfrac{-2n+15}{3}$.

3. $a^2 - b^2 + 2b - 1 =$

      A. $(a-b-1)(a+b-1)$.

      B. $(a-b-1)(a+b+1)$.

      C. $(a-b+1)(a+b-1)$.

      D. $(a-b+1)(a-b-1)$.

2

4.   Let p and q be constants. If $x^2 + p(x+5) + q \equiv (x-2)(x+5)$, then $q =$

      A. -25 .

      B. -10 .

      C. 3 .

      D. 5 .

5.   Let $f(x) = x^3 + 2x^2 - 7x + 3$. When $f(x)$ is divided by $x+2$, the remainder is

      A. 3 .

      B. 5 .

      C. 17 .

      D. 33 .

6.   Let a be a constant. Solve the equation $(x-a)(x-a-1) = (x-a)$.

      A. $x = a+1$

      B. $x = a+2$

      C. $x = a$ or $x = a+1$

      D. $x = a$ or $x = a+2$

7.   Find the range of values of k such that the quadratic equation $x^2 - 6x = 2 - k$ has no real roots.

      A. $k < -7$

      B. $k > -7$

      C. $k < 11$

      D. $k > 11$

3

8.　In the figure, the quadratic graph of $y = f(x)$ intersects the straight line L at $A(1, k)$ and $B(7, k)$. Which of the following are true?

I.　The solution of the inequality $f(x) > k$ is $x < 1$ or $x > 7$.

II.　The roots of the equation $f(x) = k$ are 1 and 7.

III.　The equation of the axis of symmetry of the quadratic graph of $y = f(x)$ is $x = 3$.

A.　I and II only

B.　I and III only

C.　II and III only

D.　I, II and III
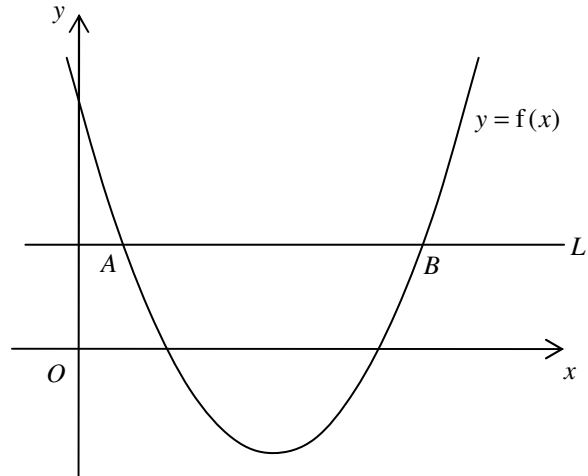
9.　The solution of $5 - 2x < 3$ and $4x + 8 > 0$ is

A.　$x > -2$.

B.　$x > -1$.

C.　$x > 1$.

D.　$-2 < x < 1$.

10.　Mary sold two bags for $\$240$ each. She gained 20% on one and lost 20% on the other. After the two transactions, Mary

A.　lost $\$20$.

B.　gained $\$10$.

C.　gained $\$60$.

D.　had no gain and no loss.

4

11. Let a_n be the nth term of a sequence. If $a_1 = 4$, $a_2 = 5$ and $a_{n+2} = a_n + a_{n+1}$ for any positive integer n, then $a_{10} =$

    A.   13 .

    B.   157 .

    C.   254 .

    D.   411 .

12. If the length and the width of a rectangle are increased by 20% and $x\%$ respectively so that its area is increased by 50% , then $x =$

    A.   20 .

    B.   25 .

    C.   30 .

    D.   35 .

13. If x , y and z are non-zero numbers such that $2x = 3y$ and $x = 2z$, then $(x + z) : (x + y) =$

    A. $3 : 5$.

    B. $6 : 7$.

    C. $9 : 7$.

    D. $9 : 10$.

14. It is given that z varies directly as x and inversely as y . When $x = 3$ and $y = 4$, $z = 18$. When $x = 2$ and $z = 8$, $y =$

    A.   1 .

    B.   3 .

    C.   6 .

    D.   9 .

15. The lengths of the three sides of a triangle are measured as 15 cm , 24 cm and 25 cm respectively. If the three measurements are correct to the nearest cm , find the percentage error in calculating the perimeter of the triangle correct to the nearest 0.1% .

     A.   0.8%

     B.   2.3%

     C.   4.7%

     D.   6.3%

16. In the figure, O is the centre of the circle. C and D are points lying on the circle. OBC and BAD are straight lines. If $OC = 20$ cm and $OA = AB = 10$ cm , find the area of the shaded region BCD correct to the nearest cm^2 .

     A.   214 cm^2

     B.   230 cm^2

     C.   246 cm^2

     D.   270 cm^2
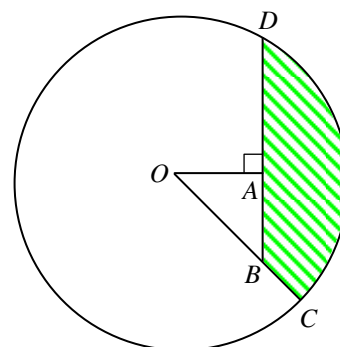
17. The figure shows a right circular cylinder, a hemisphere and a right circular cone with equal base radii. Their curved surface areas are a cm^2, b cm^2 and c cm^2 respectively.
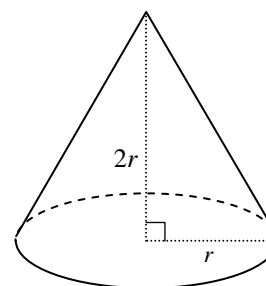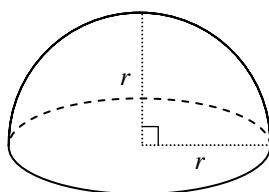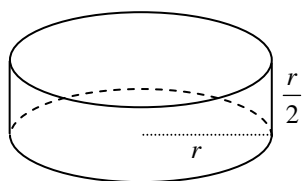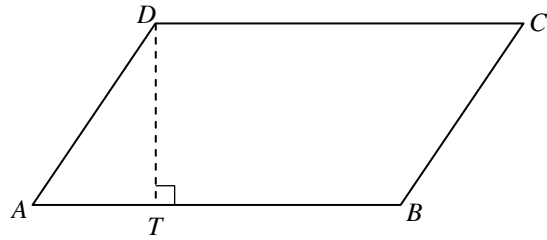

Which of the following is true?

     A. $a < b < c$

     B. $a < c < b$

     C. $c < a < b$

     D. $c < b < a$

18. In the figure, $ABCD$ is a parallelogram. T is a point lying on AB such that DT is perpendicular to AB. It is given that $CD = 9$ cm and $AT : TB = 1 : 2$. If the area of the parallelogram $ABCD$ is 36 cm^2, then the perimeter of the parallelogram $ABCD$ is

    A.   26 cm .
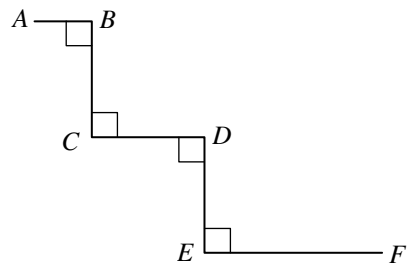
    B.   28 cm .

    C.   30 cm .

    D.   32 cm .

19. $\dfrac{\sin\theta}{\cos 60°} + \dfrac{\cos(270° - \theta)}{\tan 45°} =$

    A. $\sin\theta$.

    B. $3\sin\theta$.

    C. $2\sin\theta - \cos\theta$.

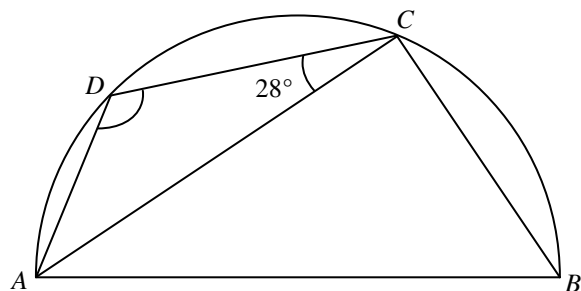   D. $2\sin\theta + \cos\theta$.

20. In the figure, $AB = 1$ cm , $BC = CD = DE = 2$ cm and $EF = 3$ cm . Find the distance between A and F correct to the nearest 0.1 cm .

    A.   7.2 cm
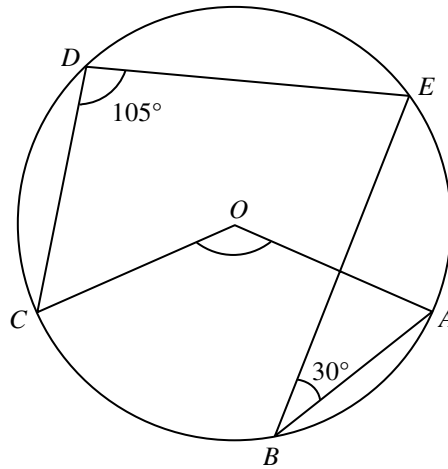
    B.   7.4 cm

    C.   8.0 cm

    D.   8.1 cm

21. In the figure, $ABCD$ is a semi-circle. If $BC = CD$, then $\angle ADC =$

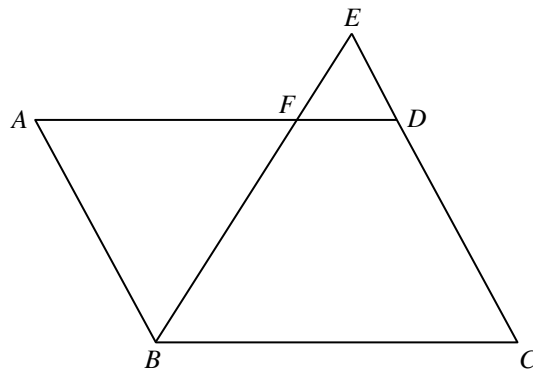   A.   118° .

    B.   121° .

    C.   124° .

    D.   126° .

7

22. In the figure, O is the centre of the circle $ABCDE$. If $\angle ABE = 30°$ and $\angle CDE = 105°$, then $\angle AOC =$

    A.   120° .

    B.   135° .

    C.   150° .

    D.   165° .
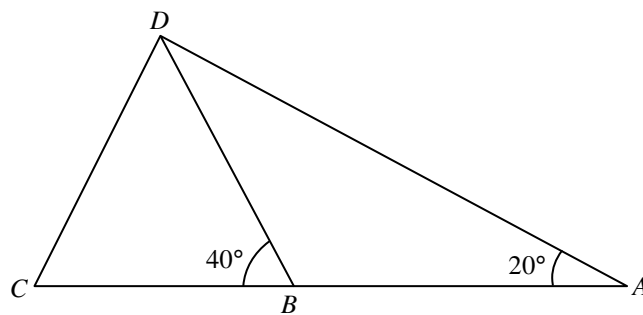

23. In the figure, $ABCD$ is a parallelogram. F is a point lying on AD. BF produced and CD produced meet at E. If $CD:DE = 2:1$, then $AF:BC =$

    A. $1:2$.

    B. $2:3$.

    C. $3:4$.
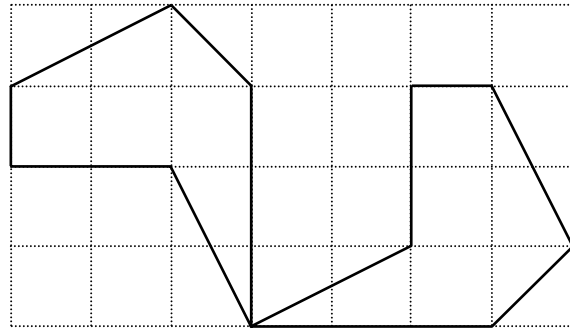
    D. $8:9$.

24. In the figure, ABC is a straight line. If $BD = CD$ and $AB = 10\,\text{cm}$, find BC correct to the nearest cm.

    A.   8 cm

    B.   13 cm

    C.   14 cm

    D.   15 cm

8

25. In the figure, the two 6-sided polygons show

    A.   a rotation transformation.

    B.   a reflection transformation.

    C.   a translation transformation.

    D.   a dilation transformation.

26. If the point $(-4,3)$ is rotated anti-clockwise about the origin through $180°$, then the coordinates of its image are

    A. $(-3,-4)$.

    B. $(3,4)$.

    C. $(-4,-3)$.

    D. $(4,-3)$.

27. The box-and-whisker diagram below shows the distribution of the scores (in marks) of the students of a class in a test.
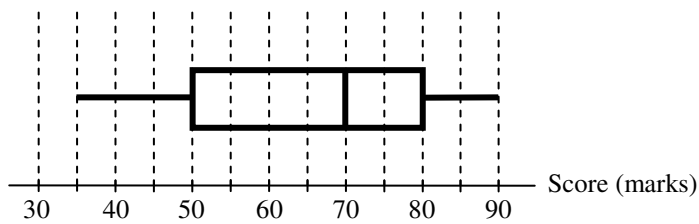
If the passing score of the test is 50 marks, then the passing percentage of the class is

    A.   25% .

    B.   50% .

    C.   70% .

    D.   75% .

9

28. The stem-and-leaf diagram below shows the distribution of heights (in cm) of 23 staff members in an office.

Stem (tens)	Leaf (units)
15	3 3 4 5 6 7 9
16	1 2 2 3 5 6 6 8
17	1 2 6 7 9
18	2 6 7

Find the median of the distribution.

    A.   164 cm

    B.   165 cm

    C.   165.5 cm

    D.   166 cm

29. $\{\, a-7\,,\ a-1\,,\ a\,,\ a+2\,,\ a+4\,,\ a+8\,\}$ and $\{\, a-9\,,\ a-2\,,\ a-1\,,\ a+3\,,\ a+4\,,\ a+6\,\}$ are two groups of numbers. Which of the following is/are true?

    I.   The two groups of numbers have the same mean.
    II. The two groups of numbers have the same median.
    III. The two groups of numbers have the same range.

    A.   I only

    B.   II only

    C.   I and III only

    D.   II and III only

30. The students' union of a school of 950 students wants to investigate the opinions of students in the school on the services provided by the tuck shop. A questionnaire is designed by the students' union and only the chairperson and vice-chairperson of the students' union are selected as a sample to fill in the questionnaire. Which of the following are the disadvantages of this sampling method?

    I.   The sample size is very small.
    II. Not all students in the school are selected.
    III. Not all students in the school have an equal chance of being selected.

    A.   I and II only

    B.   I and III only
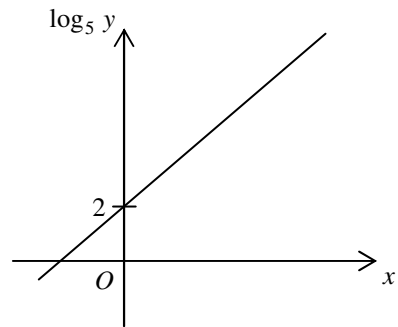
    C.   II and III only

    D.   I , II and III

Section B

31. $\dfrac{1}{2-x}+\dfrac{x-1}{(x-2)^2}=$

      A. $\dfrac{-3}{(2-x)^2}$.

      B. $\dfrac{1}{(2-x)^2}$.

      C. $\dfrac{-2x+3}{(2-x)^2}$.

      D. $\dfrac{2x-3}{(2-x)^2}$.

32. The graph in the figure shows the linear relation between x and $\log_5 y$. If $y = ab^x$, then $a =$

      A.   1 .

      B.   2 .

      C.   5 .

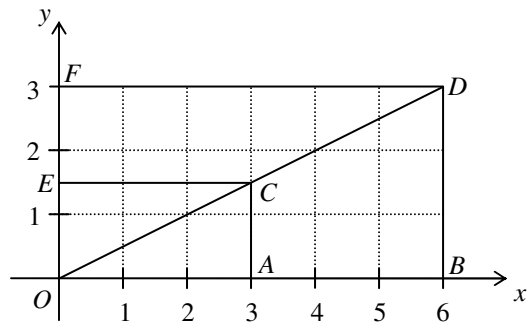   D.   25 .

33. $1010010001001_2 =$

      A. $2^{12}+2^{10}+137$.

      B. $2^{12}+2^{10}+273$.

      C. $2^{13}+2^{11}+137$.

      D. $2^{13}+2^{11}+273$.

34. If k is a real number, then $4k - \dfrac{6+ki}{i} =$

      A. $3k+6i$.

      B. $3k-6i$.

      C. $5k+6i$.

      D. $5k-6i$.

35. Which of the triangular regions in the figure may represent the solution of $\begin{cases} 0 \le x \le 6 \\ 0 \le y \le 3 \\ x \le 2y \end{cases}$?

    A. $\triangle OAC$

    B. $\triangle OBD$

    C. $\triangle OCE$

    D. $\triangle ODF$

36. If the 3rd term and the 6th term of an arithmetic sequence are 18 and -6 respectively, then the 2nd term of the sequence is

    A. -8 .

    B. 10 .

    C. 26 .

    D. 34 .

37. If the figure shows the graph of $y = f(x)$ and the graph of $y = g(x)$ on the same rectangular coordinate system, then

    A. $g(x) = f(x-2) - 3$.

    B. $g(x) = f(x-2) + 3$.

    C. $g(x) = f(x+2) - 3$.

    D. $g(x) = f(x+2) + 3$.
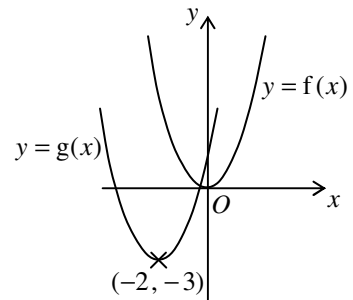
38. In the figure, $y =$

    A. $\dfrac{x \sin 77°}{\sin 56°}$.

    B. $\dfrac{x \sin 47°}{\sin 56°}$.

    C. $\dfrac{x \sin 56°}{\sin 77°}$.

    D. $\dfrac{x \sin 77°}{\sin 47°}$.

12
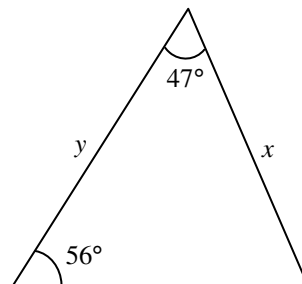
39. Peter invests P at the beginning of each month in a year at an interest rate of 6% per annum, compounded monthly. If he gets $10\,000$ at the end of the year, find P correct to 2 decimal places.

    A.   806.63

    B.   829.19

    C.   833.33

    D.   882.18

40. The figure shows a cuboid $ABCDEFGH$. If the angle between the triangle ACE and the plane $ABCD$ is θ, then $\tan\theta =$

    A. 2.

    B. $\dfrac{3}{2}$.

    C. $\dfrac{5}{2}$.

    D. $\dfrac{12}{5}$.
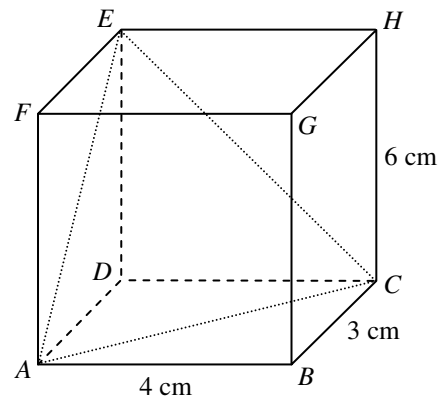
41. In the figure, A, B and C are points lying on the circle. TA is the tangent to the circle at A. The straight line CBT is perpendicular to TA. If $BC = 6\,\text{cm}$, find the radius of the circle correct to the nearest $0.1\,\text{cm}$.

    A.   3.2 cm

    B.   3.9 cm

    C.   4.2 cm

    D.   4.7 cm
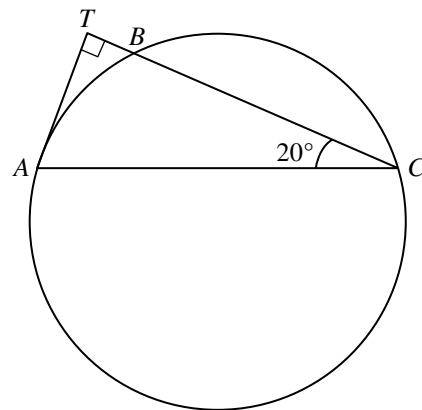
42. Let a be a constant and $-90° < b < 90°$. If the figure shows the graph of $y = a\cos(x° + b)$, then

    A. $a = -3$ and $b = -40°$.

    B. $a = -3$ and $b = 40°$.

    C. $a = 3$ and $b = -40°$.

    D. $a = 3$ and $b = 40°$.

13
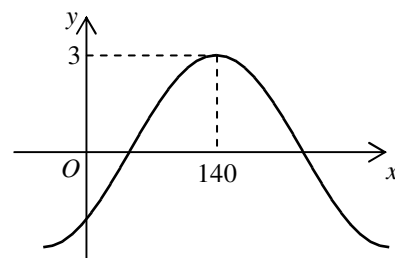
43. Bag *A* contains 2 red balls , 3 green balls and 4 white balls while bag *B* contains 2 red balls , 3 green balls and 4 yellow balls. If one ball is drawn randomly from each bag, then the probability that the two balls drawn are of different colours is

    A. $\dfrac{13}{81}$.

    B. $\dfrac{29}{81}$.

    C. $\dfrac{52}{81}$.

    D. $\dfrac{68}{81}$.

44. If 2 girls and 5 boys randomly form a queue, find the probability that the two girls are next to each other in the queue.

    A. $\dfrac{1}{7}$

    B. $\dfrac{2}{7}$

    C. $\dfrac{6}{7}$

    D. $\dfrac{1}{21}$

45. A set of numbers has a mode of 32 , an inter-quartile range of 27 and a variance of 25 . If 3 is added to each number of the set and each resulting number is then doubled to form a new set of numbers, find the mode, the inter-quartile range and the variance of the new set of numbers.

	Mode	Inter-quartile range	Variance
A.	64	60	50
B.	70	60	100
C.	70	54	50
D.	70	54	100

END OF PAPER

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

MATHEMATICS
Extended Part
Module 1 (Calculus and Statistics)

(Sample Paper)

Time allowed: 2 hours 30 minutes

This paper must be answered in English

INSTRUCTIONS

1. This paper consists of Section A and Section B. Each section carries 50 marks.

2. Answer **ALL** questions in this paper.

3. Write your answers in the AL(A) answer book. For Section A, there is no need to start each question on a fresh page.

4. All working must be clearly shown.

5. Unless otherwise specified, numerical answers must be either exact or correct to 4 decimal places.

Section A (50 marks)

1. Expand the following in ascending powers of x as far as the term in x^2 :

   (a) e^{-2x} ;

   (b) $\dfrac{(1+2x)^6}{e^{2x}}$.

   (4 marks)

2. When a hot air balloon is being blown up, its radius $r(t)$ (in m) will increase with time t (in hr). They are related by $r(t) = 3 - \dfrac{2}{2+t}$, where $t \geq 0$. It is known that the volume $V(r)$ (in m³) of the balloon is given by $V(r) = \dfrac{4}{3}\pi r^3$.

   Find the rate of change, in terms of π, of the volume of the balloon when the radius is 2.5 m.

   (4 marks)

3. A political party studied the public view on a certain government policy. A random sample of 150 people was taken and 57 of them supported this policy.

   (a) Estimate the population proportion supporting this policy.

   (b) Find an approximate 90% confidence interval for the population proportion.

   (4 marks)

4. The percentage of local Year One students in a certain university is 90% , among whom 5% are enrolled with a scholarship. For non-local Year One students, 35% of them are enrolled **without** a scholarship.

   (a) If a Year One student is selected at random, find the probability that the student is enrolled with a scholarship.

   (b) Given that a selected Year One student is enrolled with a scholarship, find the probability that this student is a non-local student.

   (4 marks)

5. A manufacturer produces a large batch of light bulbs, with a mean lifetime of 640 hours and a standard deviation of 40 hours. A random sample of 25 bulbs is taken. Find the probability that the sample mean lifetime of the 25 bulbs is greater than 630 hours.

   (5 marks)

2

6. Let $u = \sqrt{\dfrac{2x+3}{(x+1)(x+2)}}$, where $x > -1$.

   (a) Use logarithmic differentiation to express $\dfrac{du}{dx}$ in terms of u and x .

   (b) Suppose $u = 3^{y}$, express $\dfrac{dy}{dx}$ in terms of x .

   (5 marks)

7. The random variable X has probability distribution $P(X = x)$ for $x = 1$, 2 and 3 as shown in the following table.

x	1	2	3
$P(X = x)$	0.1	0.6	0.3

   Calculate

   (a) $E(X)$,

   (b) $Var(3 - 2X)$.

   (5 marks)

8. The monthly number of traffic accidents occurred in a certain highway follows a Poisson distribution with mean 1.7 . Assume that the monthly numbers of traffic accidents occurred in this highway are independent.

   (a) Find the probability that at least four traffic accidents will occur in this highway in the first quarter of a certain year.

   (b) Find the probability that there is exactly one quarter with at least four traffic accidents in a certain year.

   (6 marks)

9. L is the tangent to the curve $C : y = x^{3} + 7$ at $x = 2$.

   (a) Find the equation of the tangent L .

   (b) Using the result of (a), find the area bounded by the y-axis, the tangent L and the curve C .

   (7 marks)

3

10. The number $N(t)$ of fish, which are infected by a certain disease in a pool, can be modelled by

$$N(t) = \frac{500}{1 + ae^{-kt}} \ ,$$

where a, k are positive constants and t is the number of days elapsed since the outbreak of the disease.

The values of $N(t)$ when $t = 5, 10, 15, 20$ are as follows:

t	5	10	15	20
$N(t)$	13	34	83	175

(a) Express $\ln\left(\dfrac{500}{N(t)} - 1\right)$ as a linear function of t.

(b) Using the graph paper on page 10, estimate graphically the values of a and k (correct your answers to 1 decimal place).

(c) How many days after the outbreak of the disease will the number of fish infected by the disease reach 270 ?

(6 marks)

Section B (50 marks)

11. The manager, Mary, of a theme park starts a promotion plan to increase **the daily number of visits** to the park. The rate of change of **the daily number of visits** to the park can be modelled by

$$\frac{dN}{dt} = \frac{k(25-t)}{e^{0.04t} + 4t} \qquad (t \geq 0),$$

where N is **the daily number of visits** (in hundreds) recorded at the end of a day, t is the number of days elapsed since the start of the plan and k is a positive constant.

Mary finds that at the start of the plan, $N = 10$ and $\dfrac{dN}{dt} = 50$.

(a) (i) Let $v = 1 + 4te^{-0.04t}$, find $\dfrac{dv}{dt}$.

   (ii) Find the value of k, and hence express N in terms of t.

   (7 marks)

(b) (i) When will **the daily number of visits** attain the greatest value?

   (ii) Mary claims that there will be more than 50 hundred visits on a certain day after the start of the plan. Do you agree? Explain your answer.

   (3 marks)

(c) Mary's supervisor believes that **the daily number of visits** to the park will return to the original one at the start of the plan after a long period of time. Do you agree? Explain your answer.
   (Hint: $\lim\limits_{t \to \infty} te^{-0.04t} = 0$.)

   (2 marks)

Go on to the next page

12. (a) Let $f(t)$ be a function defined for all $t \ge 0$. It is given that

$$f'(t) = e^{2bt} + ae^{bt} + 8 \ ,$$

where a and b are negative constants and $f(0) = 0$, $f'(0) = 3$ and $f'(1) = 4.73$.

(i) Find the values of a and b.

(ii) By taking $b = -0.5$, find $f(12)$.

(5 marks)

(b) Let $g(t)$ be another function defined for all $t \ge 0$. It is given that

$$g'(t) = \frac{33}{10} te^{-kt} \ ,$$

where k is a positive constant. Figure 1 shows a sketch of the graph of $g'(t)$ against t. It is given that $g'(t)$ attains the greatest value at $t = 7.5$ and $g(0) = 0$.
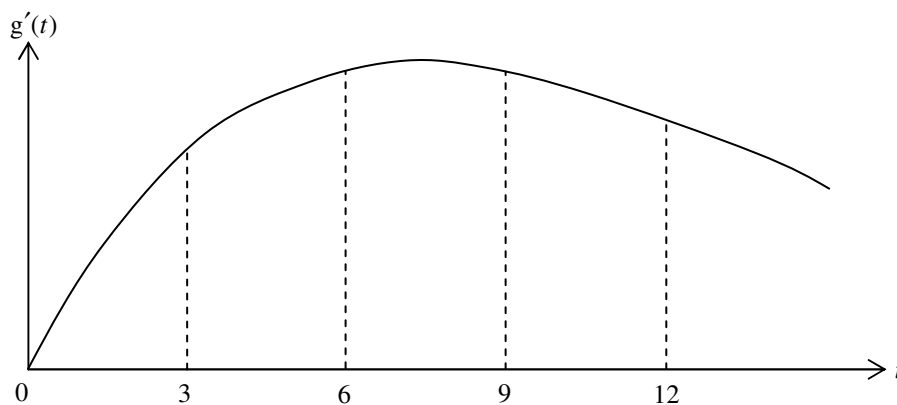
Figure 1

(i) Find the value of k.

(ii) Use the trapezoidal rule with four sub-intervals to estimate $g(12)$.

(iii) From the estimated value obtained in (b)(ii) and Figure 1, Jenny claims that $g(12) > f(12)$. Do you agree? Explain your answer.

(8 marks)

6

13. There are 80 operators in an emergency hotline centre. Assume that the number of incoming calls for the operators are independent and the number of incoming calls for each operator is distributed as Poisson with mean 6.2 in a ten-minute time interval (TMTI). An operator is said to be *idle* if the number of incoming calls received is less than three in a certain TMTI.

 (a) Find the probability that a certain operator is *idle* in a TMTI.

(3 marks)

 (b) Find the probability that there are at most two *idle* operators in a TMTI.

(3 marks)

 (c) A manager, Calvin, checks the numbers of incoming calls of the operators one by one in a TMTI. What is the least number of operators to be checked so that the probability of finding an *idle* operator is greater than 0.9 ?

(4 marks)

Go on to the next page

14. The Body Mass Index (BMI) value (in kg/m^2) of children aged 12 in a city are assumed to follow a normal distribution with mean μ kg/m^2 and standard deviation 4.5 kg/m^2.

   (a) A random sample of nine children aged 12 is drawn and their BMI values (in kg/m^2) are recorded as follows:

   $$16.0,\ 18.3,\ 15.2,\ 17.8,\ 19.5,\ 15.9,\ 18.6,\ 22.5,\ 23.6$$

   (i) Find an unbiased estimate for μ.

   (ii) Construct a 95% confidence interval for μ.

   (3 marks)

   (b) Assume $\mu = 18.7$. If a random sample of 25 children aged 12 is drawn and their BMI values are recorded, find the probability that the sample mean is less than 17.8 kg/m^2.

   (4 marks)

   (c) A child aged 12 having a BMI value greater than 25 kg/m^2 is said to be *overweight*. Children aged 12 are randomly selected one after another and their BMI values are recorded until two *overweight* children are found. Assume that $\mu = 18.7$.

   (i) Find the probability that a selected child is *overweight*.

   (ii) Find the probability that more than eight children have to be selected in this sampling process.

   (iii) Given that more than eight children will be selected in this sampling process, find the probability that exactly ten children are selected.

   (8 marks)

END OF PAPER

8

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note :   An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.
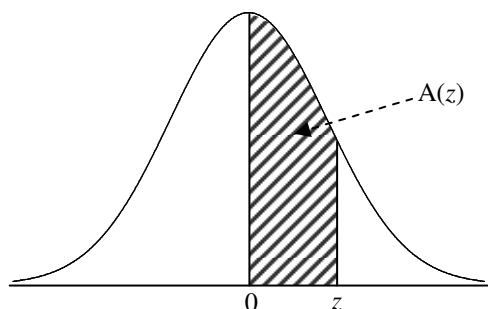
$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \, dx$$

10. (Continued)   **Put this sheet INSIDE your answer book.**
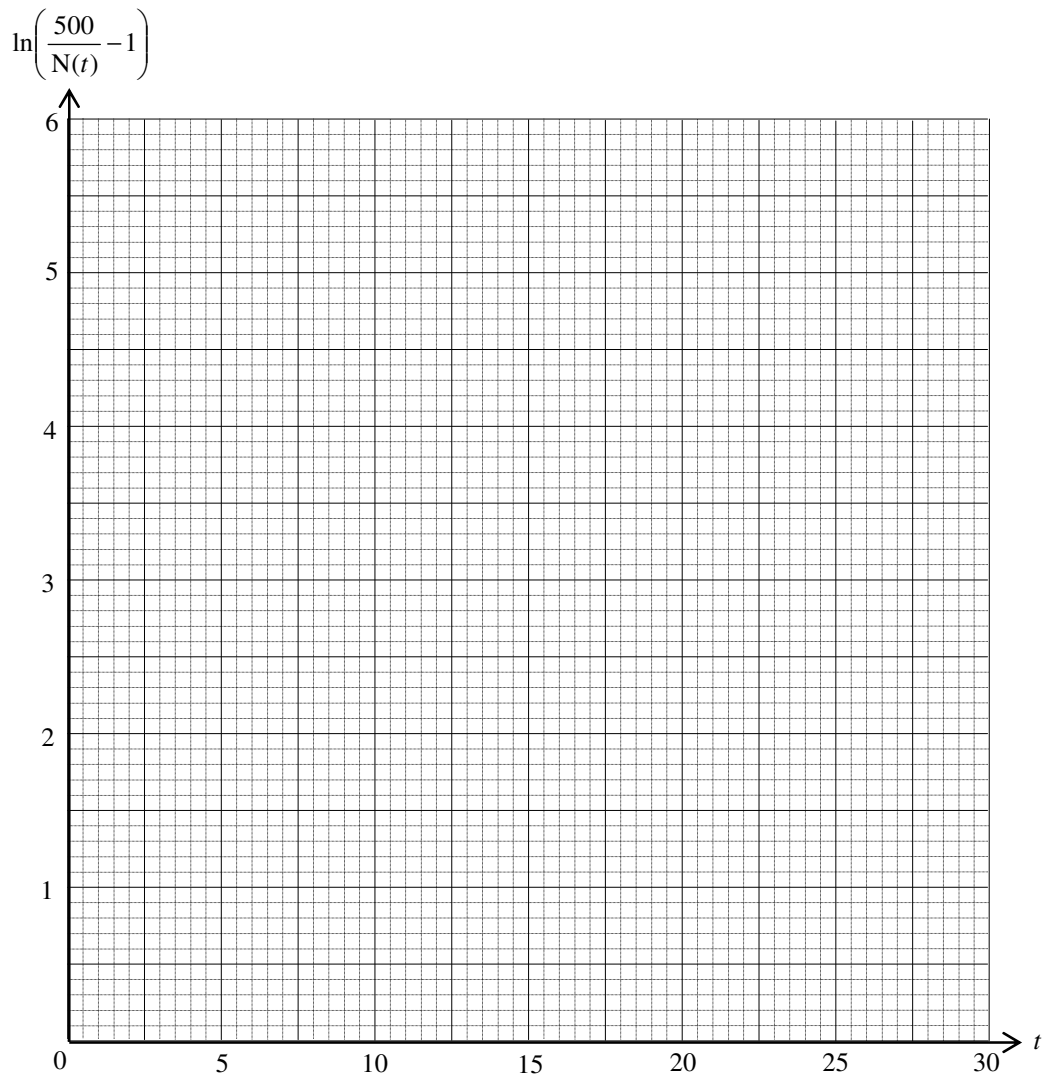
$\ln\left(\dfrac{500}{\text{N}(t)} - 1\right)$

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

MATHEMATICS
Extended Part
Module 2 (Algebra and Calculus)

(Sample Paper)

Time allowed: 2 hours 30 minutes

This paper must be answered in English

INSTRUCTIONS

1. This paper consists of Section A and Section B. Each section carries 50 marks.

2. Answer **ALL** questions in this paper.

3. Write your answers in the AL(A) answer book. For Section A, there is no need to start each question on a fresh page.

4. All working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

Not to be taken away before the end of the examination session

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

Section A (50 marks)

1. Find $\dfrac{d}{dx}(\sqrt{2x})$ from first principles.

   (4 marks)

2. A snowball in a shape of sphere is melting with its volume decreasing at a constant rate of $4\,\text{cm}^3\text{s}^{-1}$. When its radius is 5 cm, find the rate of change of its radius.

   (4 marks)

3. The slope at any point (x, y) of a curve is given by $\dfrac{dy}{dx} = 2x\ln(x^2 + 1)$. It is given that the curve passes through the point $(0, 1)$.

   Find the equation of the curve.

   (4 marks)

4. Find $\displaystyle\int \left(x^2 - \dfrac{1}{x}\right)^4 dx$.

   (4 marks)

5. By considering $\sin\dfrac{\pi}{7}\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{3\pi}{7}$, find the value of $\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{3\pi}{7}$.

   (4 marks)

6. Let C be the curve $3e^{x-y} = x^2 + y^2 + 1$.

   Find the equation of the tangent to C at the point $(1, 1)$.

   (5 marks)

2

7. Solve the system of linear equations

$$\begin{cases} x + 7y - 6z = -4 \\ 3x - 4y + 7z = 13 \\ 4x + 3y + z = 9 \end{cases}.$$

(5 marks)

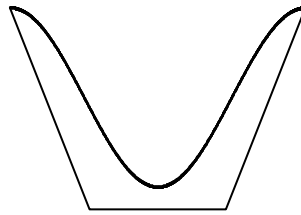8. (a) Using integration by parts, find $\int x \cos x \, dx$.

(b)

Figure 1

The **inner surface** of a container is formed by revolving the curve $y = -\cos x$ (for $0 \le x \le \pi$) about the y-axis (see Figure 1). Find the capacity of the container.
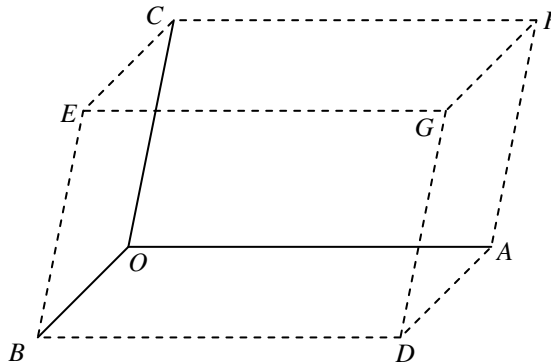
(6 marks)

9.

Figure 2

Let $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} + \mathbf{j} + 5\mathbf{k}$. Figure 2 shows the parallelepiped $OADBECFG$ formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$.

(a) Find the area of the parallelogram $OADB$.

(b) Find the volume of the parallelepiped $OADBECFG$.

(c) If C' is a point different from C such that the volume of the parallelepiped formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC'}$ is the same as that of $OADBECFG$, find a possible vector of $\overrightarrow{OC'}$.

(6 marks)

3

10. Let $0° < \theta < 180°$ and define $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

   (a) Prove, by mathematical induction, that
   $$A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$$
   for all positive integers n .

   (b) Solve $\sin 3\theta + \sin 2\theta + \sin\theta = 0$.

   (c) It is given that $A^3 + A^2 + A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$.
   Find the value(s) of a .

   (8 marks)

Section B (50 marks)

11. Let $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

   (a) Let I and O be the 3×3 identity matrix and zero matrix respectively.

      (i) Prove that $P^3 - 2P^2 - P + I = O$.

      (ii) Using the result of (i), or otherwise, find P^{-1} .

      (5 marks)

   (b) (i) Prove that $D = P^{-1}AP$.

      (ii) Prove that D and A are non-singular.

      (iii) Find $(D^{-1})^{100}$.
         Hence, or otherwise, find $(A^{-1})^{100}$.

      (7 marks)

4

12. Let $f(x) = \dfrac{4}{x-1} - \dfrac{4}{x+1} - 1$, where $x \ne \pm 1$.

    (a) (i) Find the x- and y-intercept(s) of the graph of $y = f(x)$.

        (ii) Find $f'(x)$ and prove that

$$f''(x) = \frac{16(3x^2 + 1)}{(x-1)^3 (x+1)^3}$$

            for $x \ne \pm 1$.

        (iii) For the graph of $y = f(x)$, find all the extreme points and show that there are no points of inflexion.

                                                                                       (6 marks)

    (b) Find all the asymptote(s) of the graph of $y = f(x)$.

                                                                                              (2 marks)

    (c) Sketch the graph of $y = f(x)$.

                                                                                               (3 marks)

    (d) Let S be the area bounded by the graph of $y = f(x)$, the straight lines $x = 3$, $x = a$ $(a > 3)$ and $y = -1$.

        Find S in terms of a .

        Deduce that $S < 4\ln 2$.

                                                                                                (3 marks)

5

13. (a) Let $a > 0$ and $f(x)$ be a continuous function.

Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Hence, prove that $\int_0^a f(x)\,dx = \frac{1}{2}\int_0^a [f(x) + f(a-x)]\,dx$.
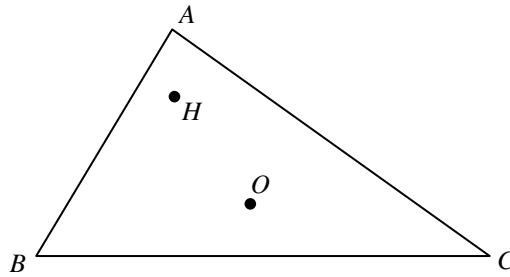
(3 marks)

(b) Show that $\int_0^1 \frac{dx}{x^2 - x + 1} = \frac{2\sqrt{3}\pi}{9}$.

(5 marks)

(c) Using (a) and (b), or otherwise, evaluate $\int_0^1 \frac{dx}{(x^2 - x + 1)(e^{2x-1} + 1)}$.

(6 marks)

14.

Figure 3

In Figure 3, $\triangle ABC$ is an acute-angled triangle, where O and H are the circumcentre and orthocentre respectively. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OH} = \mathbf{h}$

(a) Show that $(\mathbf{h} - \mathbf{a}) /\!/ (\mathbf{b} + \mathbf{c})$.

(3 marks)

(b) Let $\mathbf{h} - \mathbf{a} = t(\mathbf{b} + \mathbf{c})$, where t is a non-zero constant.
Show that

(i) $t(\mathbf{b} + \mathbf{c}) + \mathbf{a} - \mathbf{b} = s(\mathbf{c} + \mathbf{a})$ for some scalar s ,

(ii) $(t-1)(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{c} - \mathbf{a}) = 0$.

(5 marks)

(c) Express $\mathbf{h}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

(2 marks)

END OF PAPER

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