# The Dynamic Effects of Public Capital: VAR Evidence for 22 OECD Countries

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#### Abstract

The issue of whether government capital is productive has received a lot of attention in the recent past. Yet, empirical analyses of public capital productivity have in general been limited to a small sample of countries for which official capital stock estimates are available. Building on a new database that provides internationally comparable capital stock estimates, this paper estimates the dynamic macroeconomic effects of public capital using the vector autoregressive (VAR) methodology for a large panel of OECD countries. The paper adds to the empirical literature by presenting results for many countries for which there is no VAR evidence so far and by proposing a new identification scheme that extends the approach proposed by Blanchard and Perotti (2002).

Keywords: Public capital; VAR model; Cointegration; Identification; OECD countries

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# **1** Introduction

Since Sims (1980) first introduced vector autoregressive (VAR) models, they have become increasingly popular. They are now one of the principal tools of macro-econometric analysis. In their survey of the VAR methodology, Stock and Watson (2001) identify four tasks that have been tackled with the help of such models: (i) data description, (ii) forecasting, (iii) structural inference, and, (iv) policy analysis. For our purposes, the last two tasks are especially relevant. While VAR models have been extensively applied to study the effects of monetary policy shocks, application of this methodology to questions related to fiscal policy in general is a relatively recent phenomenon. In particular, as discussed in the literature survey below, most VAR studies on the dynamic effects of public capital have been published over the past five years.

The VAR approach has a number of advantages over the production function approach pioneered by Aschauer (1989): (i) Whereas the production function approach assumes a causal relationship running from the three inputs to output, the VAR approach does not impose any causal links between the variables a priori. Rather, VAR models allow to test whether the causal relationship implied by the production function approach is valid or whether there are feedback effects from output to the inputs. (ii) Unlike the production function approach, the VAR approach allows for indirect links between the model variables. In the production function approach, the long-run output effect of public capital is given by the elasticity of output with respect to capital. In contrast, in the VAR approach, the long-run output effect of a change in public capital results from the interaction of the model variables. For example, it is conceivable that public capital does not directly affect output but that a change in public capital has an impact on output only indirectly via its effects on the private factors of production. The VAR approach allows to capture such indirect effects. (iii) Unlike the production function, the VAR approach does not assume that there is at most one long-run (cointegration) relationship among the four model variables. The Johansen (1988, 1991) methodology described in Section 3.2 allows to explicitly test for the cointegration rank (the number of long-run relationships) and to impose it in the estimation of the VAR model.

Estimation of VAR models is based on a reduced form. Without the prior solution of an identification problem, the VAR estimates cannot be given a structural interpretation and can

in general not be used for policy analysis.<sup>62</sup> In this paper, we consider two solutions to the identification problem. The first one, known as the recursive approach, was introduced by Sims (1980) and is standard in the related literature. This approach is applied in Section 4, presenting empirical results on the dynamic effects of public capital for 22 OECD countries. The second solution to the identification problem is, to the best of our knowledge, an addition to the literature. It extends the identification scheme proposed by Blanchard and Perotti (2002), who considered the dynamic effects of taxes and aggregate government spending, by decomposing aggregate spending into government investment and government consumption. Section 5.2 presents empirical results for this alternative identification scheme.

The paper is organized as follows. Section 2 briefly reviews recent studies that have applied the VAR approach to study the effects of public capital. Section 3 describes the econometric methodology underlying our empirical application. Section 4 presents new empirical evidence on the dynamic effects of public capital for 22 OECD countries building on capital stock estimates provided by Kamps (2004). Section 5 discusses the robustness of the empirical results to alternative identifying assumptions. The last section summarizes the main findings.

# 2 A Short Survey of the Literature

This section briefly reviews the empirical literature having applied the VAR approach to study the dynamic effects of public capital. The only survey of the VAR approach so far, Sturm et al. (1998a), traced merely four studies. Instead, Table 1 summarizes information on twenty VAR studies, witnessing the increased popularity of this approach in the very recent past. A number of interesting findings with respect to the object of investigation and model specification emerge from the table: (i) Nearly half of the considered VAR studies have investigated the effects of public capital for the United States. Moreover, only two studies, Mittnik and Neumann (2001) as well as Pereira (2001b), have extended the analysis to a group of OECD countries. (ii) The vast majority of studies has relied on annual data, due to the restriction that capital stock data are not available at higher frequency. (iii) The majority of studies has considered a model in the four variables public capital, private capital, employment and output. In the remaining cases, in general either investment has been

 $<sup>^{62}</sup>$  See Favero (2001) for an insightful treatment of the identification problem, which is by no means special to VAR models but rather a general phenomenon in econometrics.

Table 1: Studies using the VAR approach

Study	Country	Sample	Model	Variables	Output effect of public capital <sup>a</sup>
Cullison (1993)	United States	1955–1992 (A)	VAR (FD)	$I^G, G^D, B^G, Y, M$	insignificant
McMillin & Smyth (1994)	United States	1952–1990 (A)	VAR (L, FD)	$E, \pi, K^G / K^P, N / K^P, Y / K^P$	insignificant
Crowder and Himarios (1997)	United States	1947–1989 (A)	VECM	$K^G, K^P, Y, N, E$	n.a.
Batina (1998)	United States	1948–1993 (A)	VECM, VAR (L)	$K^G, Y, N, K^P$	positive <sup>b</sup>
Pereira & Flores de Frutos (1999)	United States	1956–1989 (A)	VAR (FD)	$K^G, K^P, N, Y$	positive <sup>b</sup>
Pereira (2000)	United States	1956–1997 (A)	VAR (FD)	$I^G, I^P, N, Y$	positive <sup>b</sup>
Pereira (2001a)	United States	1956–1997 (A)	VAR (FD)	$I^G, I^P, N, Y$	n.a.
Pereira & Andraz (2001)	United States	1956–1997 (A)	VAR (FD)	$I^G, I^P, N, Y$	positive <sup>b</sup>
Flores de Frutos et al. (1998)	Spain	1964–1992 (A)	VARMA (L)	$K^G, K^P, N, Y$	positive <sup>b</sup>
Pereira & Roca Sagales (1999)	Spain	1970–1989 (A)	VAR (FD)	$K^G, K^P, N, Y$	positive <sup>b</sup>
Pereira & Roca Sagales (2001)	Spain	1970–1993 (A)	VAR (FD)	$K^G, K^P, N, Y$	positive <sup>b</sup>
Pereira & Roca Sagales (2003)	Spain	1970–1995 (A)	VAR (FD)	$K^G, K^P, N, Y$	positive <sup>b</sup>
Otto and Voss (1996)	Australia	1959–1992 (Q)	VAR (L)	$K^G, K^P, N, Y$	insignificant <sup>c</sup>
Everaert (2003)	Belgium	1953–1996 (A)	VECM	$K^G, K^P, Y$	n.a.
Mamatzakis (1999)	Greece	1959–1993 (A)	VECM	$K^G, K^P, N, Y$	n.a.
Sturm et al. (1999)	Netherlands	1853–1913 (A)	VAR (L)	$I^G, I^P, Y$	insignificant <sup>c</sup>
Ligthart (2002)	Portugal	1965–1995 (A)	VAR (L)	$K^G, K^P, N, Y$	insignificant
Voss (2002)	United States, Canada	1947–1996 (Q)	VAR (FD)	$Y, p^G, p^P, r, I^G / Y, I^P / Y$	n.a.
Mittnik & Neumann (2001)	6 OECD countries	1955–1994 (Q)	VAR (FD), VECM	$I^G, C^G, I^P, Y$	insignificant <sup>c</sup> / positive
Pereira (2001b)	12 OECD countries	1960–1990 (A)	VAR (FD), VECM	$I^G, I^P, N, Y$	positive <sup>b</sup>

*Notes:* A = annual data. Q = quarterly data. VAR = vector autoregression. VECM = vector error correction model. VARMA = vector autoregressive moving average model. FD = model in (log) first differences. L = model in (log) levels. Y = output. N = employment.  $K^P$  = private capital.  $K^G$  = public capital.  $I^P$  = private investment.  $I^G$  = public investment.  $C^G$  = public consumption.  $G^D$  = government defense spending.  $B^G$  = government debt. M = money supply. E = energy price.  $\pi$  = inflation.  $p^G$  = relative price of public investment.  $p^P$  = relative price of private investment. r = real interest rate.

<sup>b</sup> Study does not report any measure of the statistical significance of the estimated effect. – <sup>c</sup> Positive and statistically significant short-run effect.

substituted for capital or additional variables have been included in the model. (iv) There is a wide variety of model specifications as regards the (non-)consideration of cointegration. Some studies, such as Cullison (1993), specify VAR models in first differences without testing for cointegration. This way of proceeding seems dubious since it neglects potential long-run relationships between the levels. Other studies, such as Ligthart (2002), specify VAR models in levels based on the result of Sims et al. (1990) that ordinary least squares estimates of VAR coefficients are consistent even if the variables are non-stationary and possibly cointegrated. Unfortunately, the consistency of VAR coefficient estimates does not carry over to estimates of impulse response functions as discussed in the next section. Finally, some studies, such as Pereira (2000), test for cointegration using the Engle-Granger approach, thus neglecting the possibility that there may be more than one cointegration relationship in higher-dimensional systems.

The last column of Table 1 reports the main conclusions of the considered studies regarding the long-run output effects of public capital.<sup>63</sup> As can be seen in the majority of studies the long-run response of output to a shock to public capital is positive. In general, the effects are considerably smaller than those reported in the literature applying the production function approach (see, e.g., Pereira (2000)). However, almost all of these studies fail to provide any measure of the uncertainty surrounding the impulse response estimates so that it is impossible to judge the statistical significance of the results. For those studies for which such measures are provided, the long-run output effect is in general insignificant. Another important result emerging from this literature is that many studies find evidence for reverse causation, i.e., feedback from output to public capital and vice versa (see, e.g., Batina (1998)). This suggests that it is indeed important to treat public capital as endogenous variable.

Our study can be viewed as both a reassessment of and an addition to the existing empirical literature: (i) We reassess the empirical literature by carefully addressing the important issue of cointegration and by providing confidence intervals measuring the uncertainty surrounding the point estimates of the impulses responses. (ii) We add to the empirical literature by presenting results for a large sample of OECD countries for many of which there is no VAR evidence so far<sup>64</sup> and by proposing a new identification scheme, extending the approach proposed by Blanchard and Perotti (2002).

<sup>&</sup>lt;sup>63</sup> Some of the studies listed in Table 3.1 do not perform a policy analysis. In these cases, the last column of the table has an "n.a." (not available) entry.

<sup>&</sup>lt;sup>64</sup> Note that the two studies that come closest to ours in scope, Mittnik and Neumann (2001) and Pereira (2001b), both use public investment as model variable whereas we use public capital.

# **3** Econometric Methodology

This section presents the vector autoregressive (VAR) methodology used in the empirical application later in this paper.<sup>65</sup> A VAR model is a *k*-equation, *k*-variable linear model in which each variable is in turn explained by its own lagged values, past values of the remaining *k*-1 variables and possibly deterministic terms such as constants and linear time trends. Estimation of unrestricted VAR models is straightforward and is briefly sketched in Section 3.1. However, complications arise if some or all of the variables included in a VAR model are non-stationary. In this case, the appropriate estimation approach depends on whether the variables are cointegrated or not.<sup>66</sup> If the variables are cointegrated the VAR model is said to have an error-correction representation. As estimation of cointegrated VAR models is more involved than estimation of unrestricted models, Section 3.2 describes a popular estimation approach in some detail: the Johansen (1988, 1991) maximum likelihood approach. As the unrestricted and cointegrated VAR model used at the estimation stage are reduced-form models, they cannot directly be used for structural inference and policy analysis. An identification problem has to be solved such that the VAR model can be given a structural interpretation. The identification problem is discussed in Section 3.3.

#### **3.1** The Unrestricted VAR Model

A *p*-th order vector autoregressive model, denoted VAR(p), can be expressed as<sup>67</sup>

$$X_{t} = A_{1}X_{t-1} + A_{2}X_{t-2} + \ldots + A_{p}X_{t-p} + \Phi D_{t} + \varepsilon_{t}, \qquad (3.1)$$

where  $X_t = [x_{1t}, ..., x_{kt}]'$  is a set of variables collected in a  $(k \times 1)$  vector,  $A_j$  denotes a  $k \times k$ matrix of autoregressive coefficients for j = 1, 2, ..., p, and  $\Phi$  denotes a  $k \times d$  matrix of coefficients on deterministic terms collected in the  $d \times 1$  vector  $D_1$ . The vector  $\varepsilon_t = [\varepsilon_{1t}, ..., \varepsilon_{kt}]'$  is a k-dimensional white noise process, i.e.,  $E[\varepsilon_t] = 0$ ,  $E[\varepsilon_t \varepsilon'_t] = \Omega$ , and  $E[\varepsilon_t \varepsilon'_s] = 0$  for  $s \neq t$ , with  $\Omega$  a  $(k \times k)$  symmetric positive definite matrix.

<sup>&</sup>lt;sup>65</sup> See Lütkepohl (2001) and Stock and Watson (2001) for surveys on the VAR methodology.

<sup>&</sup>lt;sup>66</sup> See Hendry and Juselius (2000, 2001) for an introduction to the concept of cointegration, Johansen (1995) and Juselius (2003) for an extensive treatment of cointegrated VAR models.

Estimation of the unrestricted VAR model is particularly easy. Conditioning on the first p observations (denoted  $X_{-p+1}, X_{-p+2}, ..., X_0$ ) and basing estimation on the sample  $X_1, X_2, ..., X_T$ , the k equations of the VAR can be estimated separately by ordinary least squares (OLS). Since the set of regressors is identical across equations, the OLS estimator is identical to the generalized least squares (GLS) estimator of the seemingly unrelated regressions model. Moreover, under the assumption that the  $\varepsilon_t$  are Gaussian white noise, it can be shown that the simple OLS estimator is identical to the full information maximum likelihood (FIML) estimator (see, e.g., Hamilton (1994: 293-296)). Finally, under general conditions, the OLS estimator of  $A = [A_1, ..., A_p]$  is consistent and asymptotically normally distributed. Remarkably, this result not only holds in the case of stationary variables, but also in the case in which some variables are integrated and possibly cointegrated (Sims et al. (1990)).

Based on this result many researchers have ignored nonstationarity issues and estimated unrestricted VAR models in levels. This approach is characteristic, e.g., of the literature on the empirical effects of monetary policy shocks surveyed in Christiano et al. (1999). A drawback of this approach is that, while the autoregressive coefficients in equation (3.1) are estimated consistently, this may not be true for other quantities derived from these estimates. In particular, Phillips (1998) showed that impulse responses and forecast error variance decompositions based on the estimation of unrestricted VAR models are inconsistent at long horizons in the presence of non-stationary variables. In contrast, vector error correction models (VECMs) produce consistent estimates of impulse responses and of forecast error variance decompositions if the number of cointegration relations is estimated consistently. As impulse response analysis is one of the main tools for policy analysis based on VAR models, a careful investigation of the cointegration properties of the VAR system is warranted. The next section presents Johansen's (1988, 1991) maximum likelihood approach for the estimation of cointegrated VAR processes.

 $<sup>^{67}</sup>$  This section builds on the assumption of a known lag order *p*. In the empirical application, the optimal lag order is explicitly tested for.

#### **3.2** The Cointegrated VAR Model

The starting point of the analysis is that any VAR(p) model (3.1) can always be written in equivalent form

$$\Delta X_{t} = \Pi X_{t-1} + \Gamma_{1} \Delta X_{t-1} + \Gamma_{2} \Delta X_{t-2} + \dots + \Gamma_{p-1} \Delta X_{t-p+1} + \Phi D_{t} + \varepsilon_{t}, \qquad (3.2)$$

where  $\Pi \equiv -I + \sum_{i=1}^{p} A_i$  and  $\Gamma_j \equiv -\sum_{i=j+1}^{p} A_i$  (j = 1, 2, ..., p-1) denote  $(k \times k)$  matrices of coefficients, respectively. If no restrictions are imposed on  $\Pi$ , then the *k* equations of system (3.2) can be estimated by simple OLS. In this case, the estimation results will be identical to those obtained from the OLS estimation of the unrestricted VAR(*p*) model, taking into account the relationship between  $\Pi$ ,  $\Gamma \equiv [\Gamma_1, ..., \Gamma_{p-1}]$  and  $A \equiv [A_1, ..., A_p]$ . As we will see, this is a special case, however, arising when none of the variables collected in the vector  $X_t$  is non-stationary.

In the following, we assume that each of the series in  $X_t$  taken individually is integrated of order one (I(1)). Under this assumption the vector  $X_t$  is said to be cointegrated if for some nonzero  $(k \times 1)$  vector  $a_1$  the linear combination  $a'_1X_t$  is stationary (see, e.g., Hamilton (1994: 574)). Moreover, in a system with more than two variables, there may be r < klinearly independent vectors  $a_1, ..., a_r$  such that  $\beta' X_t$  is a stationary  $(r \times 1)$  vector, where  $\beta'$  is the transpose of the  $(k \times r)$  matrix  $\beta \equiv [a_1, ..., a_r]$ . In this case, there are exactly rcointegrating relations among the series collected in  $X_t$ . Furthermore, if the process can be described as a p-th order VAR in levels as in (3.1), then there exists a  $(k \times r)$  matrix  $\alpha$  such that  $\Pi = \alpha\beta'$  and there further exist  $(k \times k)$  matrices  $\Gamma_1, ..., \Gamma_{p-1}$  such that

$$\Delta X_t = \alpha \beta' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \dots + \Gamma_{p-1} \Delta X_{t-p+1} + \Phi D_t + \varepsilon_t.$$
(3.3)

This follows from Granger's representation theorem, stating that cointegrated series can be represented by error correction models (see Engle and Granger (1987: 255-256)). The system (3.3) differs from the system (3.2) in that it imposes a reduced-rank restriction on  $\Pi$  for r < k.

The foregoing discussion allows us to distinguish three interesting cases: (i) If r = 0, then rank $(\Pi) = 0$  and the variables collected in  $X_t$  are not cointegrated. In this case, there are k

independent stochastic trends in the system and it is appropriate to estimate the VAR model in first differences, dropping  $X_{t-1}$  as regressor in equation (3.2). (ii) At the other extreme, if r = k, then rank( $\Pi$ ) = k and each variable in  $X_t$  taken individually must be stationary. Or, in other words, the number of stochastic trends, given by k-r, is equal to zero. As mentioned above, in this case, the system can be estimated by applying OLS either to the unrestricted VAR in levels (equation 3.1) or to its equivalent representation given by (3.2). (iii) In the intermediate case, 0 < r < k, the variables in  $X_t$  are driven by 0 < k-r < kcommon stochastic trends and rank( $\Pi$ ) = r < k. In this case, estimating the system given by (3.3) by OLS is not appropriate since cross-equation restrictions have to be imposed on the matrix  $\Pi$ . Instead, the maximum likelihood approach developed by Johansen (1988, 1991) can be applied in order to estimate the space spanned by the cointegrating vectors collected in  $\beta$ . An additional asset of Johansen's approach is that it enables us to test for the number of cointegrating relations, which in many applications is unknown a priori.<sup>68</sup> Thus, it is a unifying framework that allows us to investigate which of the three cases is the relevant one in empirical applications.

Before turning to the details of Johansen's estimation approach it is worthwhile to discuss the role of the deterministic terms collected in the vector  $D_t$ . The specification of the deterministic terms in equation (3.3) plays an important role in the analysis because the asymptotic distributions of the test statistics used for the determination of the number of cointegrating vectors depends on the assumptions made on these terms (see Johansen (1994)). In the following, the deterministic term in equation (3.3) is assumed to be expressible as  $\Phi D_t \equiv \mu_0 + \mu_1 t$ , i.e., equation (3.3) potentially includes k constants (for  $\mu_0 \neq 0$ ) as well as k linear trends (for  $\mu_1 \neq 0$ ), where  $\mu_0$  and  $\mu_1$  are  $(k \times 1)$  vectors, respectively. The coefficient on these two deterministic terms can be further decomposed such that

$$\mu_0 \equiv \alpha \rho_0 + \alpha_\perp \gamma_0 ,$$

$$\mu_1 \equiv \alpha \rho_1 + \alpha_\perp \gamma_1 ,$$
(3.4)

<sup>&</sup>lt;sup>68</sup> See Hubrich et al. (2001) for a review of systems cointegration tests, including Johansen's likelihood ratio tests.

where  $\alpha_{\perp}$  is a  $(k \times (k - r))$  matrix orthogonal to  $\alpha$ , i.e.  $\alpha' \alpha_{\perp} = 0$ . The definitions (3.4) imply that the constant and the trend coefficients can each be decomposed into one part belonging to the cointegrating space  $(\rho_i, i = 0, 1)$  and another part that is orthogonal to the cointegrating space  $(\gamma_i, i = 0, 1)$ . Without any restrictions on the coefficients, the model given by (3.3), assuming that  $\Phi D_t \equiv \mu_0 + \mu_1 t$ , is consistent with linear trends in the differenced process  $\Delta X_t$  and, hence, quadratic trends in the process  $X_t$ . Johansen (1994) distinguishes five alternative models, corresponding to alternative sets of restrictions on the deterministic terms. In the following, we concentrate on the model which seems to be the most relevant for our problem: the constant is left unrestricted  $(\rho_0, \gamma_0 \neq 0)$  and the trend is restricted to the cointegrating space  $(\rho_1 \neq 0, \gamma_1 = 0)$ .<sup>69</sup> This specification eliminates the potential for quadratic trends in  $X_t$ , while allowing for linear trends in  $X_t$  and for trend-stationary cointegrating relations. The latter may be justified on the grounds that the cointegrating space might contain a production function as one cointegrating vector (see ,e.g., Sturm and De Haan (1995)). Based on this specification of the deterministic terms, the vector error correction model (3.3) may be written as

$$\Delta X_t = \alpha \widetilde{\beta}' \widetilde{X}_{t-1} + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{p-1} \Delta X_{t-p+1} + \mu_0 + \varepsilon_t, \qquad (3.5)$$

where  $\tilde{\beta}' \equiv [\beta', \rho_1]$  is an  $(r \times (k+1))$  matrix and  $\tilde{X}_{t-1} \equiv [X'_{t-1}, t]'$  is a  $((k+1)\times 1)$  vector. Having chosen the relevant model, we can now describe Johansen's (1988, 1991) estimation approach. The following presentation of Johansen's algorithm draws on Hamilton (1994: 636-637) who presents it for a model without trend  $(\mu_1 = 0)$ . Under the assumption that the error terms  $\varepsilon_t$  are Gaussian white noise, it can be shown that the estimates calculated with this algorithm are identical to the maximum likelihood estimates (Johansen (1988)). Following Hamilton (1994: 636-637), Johansen's algorithm can be divided into three steps:

In the first step, a number of auxiliary regressions is carried out in order to concentrate out  $\Gamma_1, ..., \Gamma_{p-1}$  and  $\mu_0$ . These parameters are eliminated by OLS regression of  $\Delta X_t$  and  $\tilde{X}_{t-1}$  on

<sup>&</sup>lt;sup>69</sup> The resulting model corresponds to model  $H^*(r)$  in Johansen (1994), case IV (without exogenous variables) in Pesaran et al. (2000) and case 2\* in Osterwald-Lenum (1992). Franses (2001) as well as Pesaran and Smith (1998: 483) argue that the case analyzed here is one of two cases particularly relevant in practice, the other one being that of a restricted constant ( $\rho_0 \neq 0, \gamma_0 = 0$ ) and no trend ( $\rho_1, \gamma_1 = 0$ ).

 $\Delta X_{t-1}, \dots, \Delta X_{t-p+1}$  and a constant term. Denoting OLS estimates by a hat, the first set of *k* regressions can be expressed as

$$\Delta X_t = \hat{\Psi}_1 \Delta X_{t-1} + \dots + \hat{\Psi}_{p-1} \Delta X_{t-p+1} + \hat{\zeta}_1 + \hat{u}_t, \qquad (3.6)$$

where  $\hat{\Psi}_i$  (i = 1, ..., p-1) is a ( $k \times k$ ) matrix of OLS coefficient estimates,  $\hat{\zeta}_1$  is a ( $k \times 1$ ) vector of coefficient estimates of the constant and  $\hat{u}_t$  denotes the ( $k \times 1$ ) vector of OLS residuals. Note that equation (3.6) is just a vector autoregressive model for  $X_t$  in first differences.<sup>70</sup>

The second set of (k + 1) OLS regressions can be expressed as

$$\widetilde{X}_{t-1} = \hat{B}_1 \Delta X_{t-1} + \dots + \hat{B}_{p-1} \Delta X_{t-p+1} + \hat{\zeta}_2 + \hat{w}_t, \qquad (3.7)$$

where  $\hat{B}_i(i = 1, ..., p-1)$  is a  $((k+1) \times k)$  matrix of OLS coefficient estimates,  $\hat{\zeta}_2$  is a  $((k+1) \times 1)$  vector of coefficient estimates of the constant and  $\hat{w}_t$  denotes the  $((k+1) \times 1)$  vector of OLS residuals.

In the second step, the canonical correlations between  $\hat{u}_t$  and  $\hat{w}_t$  are calculated.<sup>71</sup> For this purpose, first, the sample variance-covariance matrices of  $\hat{u}_t$  and  $\hat{w}_t$  are calculated as

$$\hat{\Sigma}_{ww} = \frac{1}{T} \sum_{i=1}^{T} \hat{w}_{i} \hat{w}_{i}^{'}, \qquad \hat{\Sigma}_{uu} = \frac{1}{T} \sum_{i=1}^{T} \hat{u}_{i} \hat{u}_{i}^{'}, \qquad (3.8)$$

$$\hat{\Sigma}_{uw} = \frac{1}{T} \sum_{i=1}^{T} \hat{u}_{i} \hat{w}_{i}^{'}, \qquad \hat{\Sigma}_{wu} = (\hat{\Sigma}_{wu})^{'}.$$

Then, the eigenvalues  $\hat{\lambda}_i$  of the  $((k+1)\times(k+1))$  matrix  $(\hat{\Sigma}_{ww})^{-1}\hat{\Sigma}_{wu}(\hat{\Sigma}_{uu})^{-1}\hat{\Sigma}_{uw}$  are calculated, with the eigenvalues ordered  $1 > \hat{\lambda}_1 > ... > \hat{\lambda}_k > \hat{\lambda}_{k+1} > 0$ . The matrix holding the

 $<sup>^{70}</sup>$  If for some reason it were known that the cointegrating rank was zero, then (3.6) would be the appropriate model and the estimation problem would already have been solved. However, in empirical applications the cointegrating rank is usually unknown. Johansen's algorithm, at a later stage, allows to test for the number of cointegrating relations.

<sup>&</sup>lt;sup>71</sup> See Hamilton (1994: 630-635) for an exposition of canonical correlation analysis.

eigenvectors associated with these eigenvalues is denoted by  $\hat{V} = [\hat{v}_1, ..., \hat{v}_{k+1}]$ , where the eigenvectors are normalized such that  $\hat{V} \cdot \hat{\Sigma}_{ww} \hat{V} = I$ . The eigenvalues  $\hat{\lambda}_i$  can be interpreted as squared canonical correlations between  $\Delta X$  and  $\tilde{X}_{t-1}$ , conditional on  $\Delta X_{t-1}, ..., \Delta X_{t-p+1}$  and the constants. Thus, the magnitude of  $\hat{\lambda}_i$  can be thought of as measuring the "stationarity" of the corresponding  $\hat{v}'_i \tilde{X}_{t-1}$ . Intuitively, the larger  $\hat{\lambda}_i$  the more confident we can be that  $\hat{v}'_i \tilde{X}_{t-1}$  is stationary (a cointegrating relation). In contrast, a small value for  $\hat{\lambda}_i$  is an indication that  $\hat{v}'_i \tilde{X}_{t-1}$  is only weakly correlated with  $\Delta X_1$  and, thus, probably non-stationary. This reasoning suggests that the number of cointegrating relations is equivalent to the number of eigenvalues  $\hat{\lambda}_i$  that are significantly different from zero. A formal test for the number of cointegrating relations can be based on the following likelihood ratio test statistic often referred to as trace test statistic (see Hamilton (1994: 645)):

$$2(L_{A}^{*}-L_{0}^{*})=-T\sum_{i=r+1}^{k}\log(1-\hat{\lambda}_{i}), \qquad (3.9)$$

where  $L_0^*$  is the maximum value the log likelihood function can attain under the null hypothesis of *r* cointegrating vectors and where  $L_A^*$  is the maximum value the log likelihood function can attain under the alternative hypothesis that there are as many cointegrating vectors as there are variables in  $X_t$ . The null hypothesis of the test, thus, is that the k-rsmallest eigenvalues are equal to zero. If this hypothesis can be accepted, then the process  $X_t$ is driven by  $g \equiv k - r$  stochastic trends. Since under the null hypothesis stochastic trends are present, the asymptotic distribution of the trace test statistic is non-standard. Johansen (1994: 215-216) shows that the asymptotic distribution of the test statistic depends on the number of stochastic trends and on the assumptions on the deterministic terms. Critical values taking this into account have been tabulated by MacKinnon et al. (1999), among others. In practice, the cointegrating rank can be determined by a nested sequence of hypotheses (see Johansen (2000: 364)), starting with the hypothesis that r = 0 and, if this hypothesis is rejected, testing r = 1, and so on, continuing until the null hypothesis cannot be rejected anymore. Note that we can relate the outcome of this test sequence to the three model cases discussed above: (i) if the null hypothesis r = 0 cannot be rejected, then the appropriate model is a VAR for  $\Delta X_t$  as given by (3.6), (ii) if 0 < r < k, then the VECM given by (3.5) under the restriction rank( $\pi$ ) = r is the appropriate model, and, (iii) if the last null hypothesis of the test sequence, r = k - 1, is rejected, then the variables in  $X_t$  are (trend-)stationary and in this case the appropriate model is the unrestricted VAR model for  $X_t$  in levels (3.1).

In the third and final step of Johansen's approach, the maximum likelihood estimates of the model parameters are calculated. Based on the choice of cointegrating rank r, the maximum likelihood estimate of  $\tilde{\beta}$  is given by the  $((k+1) \times r)$  matrix

$$\hat{\vec{\beta}} = [\hat{v}_1, \hat{v}_2, \dots, \hat{v}_r].$$
(3.10)

where  $[\hat{v}_1, ..., \hat{v}_r]$  are the eigenvectors associated with the *r* largest eigenvalues of the matrix  $(\hat{\Sigma}_{ww})^{-1}\hat{\Sigma}_{wu}(\hat{\Sigma}_{uu})^{-1}\hat{\Sigma}_{uw}$ .

Then, the maximum likelihood estimate of the  $(k \times r)$  matrix of adjustment coefficients  $\alpha$  is given by

$$\hat{\alpha} = \hat{\Sigma}_{uw} \hat{\tilde{\beta}} , \qquad (3.11)$$

and the maximum likelihood estimate of the  $(k \times (k+1))$  matrix  $\widetilde{\Pi} = \alpha \widetilde{\beta}'$  obtains as

$$\hat{\widetilde{\Pi}} = \hat{\alpha}\hat{\widetilde{\beta}}'.$$
(3.12)

Finally, the maximum likelihood estimates of the  $(k \times k)$  matrices  $\Gamma_i$  and the  $(k \times 1)$  vector of constants can be calculated as

$$\hat{\Gamma}_i = \hat{\Psi}_i - \widetilde{\Pi}\hat{B}_i, \quad \text{for } i = 1, ..., p - 1, \quad (3.13)$$

$$\hat{\mu}_0 = \hat{\zeta}_1 - \hat{\Pi} \hat{\zeta}_2.$$
(3.14)

Under general conditions the estimators of  $\Pi$ ,  $\Gamma_i$  and  $\mu_0$  are consistent and asymptotically normally distributed (see, e.g., Lütkepohl and Breitung (1997: 307)). Note, however, that the

same is not true for the estimators of  $\alpha$  and  $\tilde{\beta}$ . Without identifying restrictions only the cointegration space is estimated consistently, but not the cointegration parameters  $\tilde{\beta}$ .<sup>72</sup> A necessary condition for the parameters  $\tilde{\beta}$  to be identified is that at least r-1 restrictions be imposed on the parameters of each cointegrating vector. Without such restrictions it is not possible to give a structural interpretation to the cointegration parameters. Yet, even if such restrictions are imposed, it is in general not possible to infer the long-run effects of shocks hitting the system from the cointegrating vectors alone (see Lütkepohl and Reimers (1992: 69)). Instead, such effects can be obtained from an impulse response analysis as described in the next section. For such an analysis it is not necessary that the cointegrating vectors be identified. Thus, the structural analysis of the cointegrated VAR model can be based on the  $\tilde{\Pi}$  matrix that, as was noted above, is estimated consistently in Johansen's approach even if no identifying restrictions are imposed on the matrix  $\tilde{\beta}$ . Thus, in the empirical application, we will only impose the appropriate rank restriction on  $\tilde{\Pi}$ , but not identify the individual cointegrating relations.

#### **3.3** The Structural VAR Model

The previous two sub-sections have described how the VAR model can be estimated for alternative assumptions on the cointegrating rank. As these models are reduced-form models, little can be learned about the underlying economic structure unless identifying restrictions are imposed. This sub-section shows how to give VAR models a structural interpretation and, in particular, shows how to derive impulse response functions from the reduced-form parameter estimates. Impulse responses give an insight into the reaction of key macroeconomic variables to an unexpected change in one variable (here, e.g., public capital).

The subsequent analysis is based on the following reduced-form model:

$$X_{t} = A_{1}X_{t-1} + A_{2}X_{t-2} + \dots + A_{p}X_{t-p} + \mu_{0} + \mu_{1}t + \varepsilon_{t}.$$
(3.15)

<sup>&</sup>lt;sup>72</sup> This identification problem is sometimes called the long-run identification problem because it concerns the long-run structure (the cointegrating relations). It is distinct from the short-run identification problem discussed in the next section. See Juselius (2003) for an insightful treatment of both identification problems.

Equation (3.15) is equivalent to the unrestricted VAR model (3.1), except that  $\mu_0 + \mu_1 t$  has been substituted for  $\Phi D_t$ . Yet, this model can serve in the structural analysis irrespective of whether the variables in  $X_t$  are non-stationary or not.<sup>73</sup> Pre-multiplying equation (3.15) by the  $(k \times k)$  matrix  $A_0$  gives the structural form

$$A_0 X_t = A_1^* X_{t-1} + A_2^* X_{t-2} + \dots + A_p^* X_{t-p} + A_0 \mu_0 + A_0 \mu_1 t + Be_t, \qquad (3.16)$$

where  $A_i^* \equiv A_0 A_i$  for i = 1, ..., p, and  $Be_t = A_0 \varepsilon_t$  describes the relation between the structural disturbances  $e_t$  and the reduced-form disturbances  $\varepsilon_t$ . In the following, it is assumed that the structural disturbances  $e_t$  are white noise and uncorrelated with each other, i.e. the variance-covariance matrix of the structural disturbances, denoted *D*, is diagonal. The matrix  $A_0$  describes the contemporaneous relation among the variables collected in the vector  $X_t$ .<sup>74</sup> In the literature, this representation of the structural form is often called the *AB* model (see Amisano and Giannini (1997)).

Without restrictions on the parameters  $A_{0,} A_i^*$  and B, model (3.16) is not identified. Estimation of the reduced-form model (3.15) yields parameter estimates for  $A_i, \mu_0, \mu_1$  and for k(k+1)/2 distinct elements of the variance-covariance matrix  $\Omega$  of the reduced-form disturbances. Yet, these k(k+1)/2 elements of  $\Omega$  do not allow to uniquely determine the  $2k^2$  free parameters in matrices  $A_0, B$  and D of the structural form. The relationship between  $\Omega$  on the one hand and  $A_{0,} B$  and D on the other hand can be formalized as follows:

$$\Omega = E[\varepsilon_t \varepsilon_t'] = A_0^{-1} B E[e_t e_t'] B'(A_0^{-1}) = A_0^{-1} B D B'(A_0^{-1})', \qquad (3.17)$$

 $A_1 = I_k + \Psi_1$ ,  $A_i = \Psi_i - \Psi_{i-1}$  for i = 2, ..., p-1,  $A_p = -\Psi_{p-1}$ .

 $A_1 = \Pi + I_k + \Gamma_1, \quad A_i = \Gamma_i - \Gamma_{i-1} \text{ for } i = 2, \dots, p-1, \quad A_p = -\Gamma_{p-1}.$ 

<sup>&</sup>lt;sup>73</sup> While in the estimation of the VAR parameters it is crucial to distinguish the three cases analyzed in the previous section, the analysis can proceed based on the representation (3.15) once the estimation stage has been completed. Note that if the cointegrating rank r is equal to 0, then the following relations can be used to map the parameters  $\Psi_i$  from equation (3.6) to the  $A_i$  parameter matrices in (3.15):

If the cointegrating rank is 0 < r < k, the following relations can be used to map the parameters  $\Pi$  and  $\Gamma_i$  from the VECM (3.5) to the  $A_i$  matrices:

<sup>&</sup>lt;sup>74</sup> Note that the equations are normalized such that  $A_0$  is a matrix with ones along its principal diagonal.

where *E* is the expectations operator. Given appropriate restrictions on  $A_0$ , *B*, *D*, the freely varying parameters of these matrices can be estimated by full-information maximum likelihood or by the generalized method of moments (see, e.g., Breitung (2000: 57-61)). The existence of a unique maximum of the likelihood function necessitates both an order condition and a rank condition to be satisfied. The order condition is that  $A_0$ , *B* and *D* have no more than k(k+1)/2 unknown parameters.<sup>75</sup> Accordingly, at least  $k^2 + k(k-1)/2$  restrictions have to be placed on the parameters of  $A_0$ , *B* and *D* in order for the order condition to be satisfied. In the empirical literature, a large number of alternative identification procedures have been applied. Two of these will be used in the empirical application: (i) the recursive approach originally proposed by Sims (1980) that restricts *B* to a *k*-dimensional identity matrix and  $A_0$  to a lower triangular matrix, and, (ii) the approach of Blanchard and Perotti (2002) discussed in Section 5.2 that places restrictions on the  $A_0$  and *B* matrices substantially differing from those of the recursive approach.

The solution to the identification problem given by the recursive VAR approach implies that equation (3.17) can be rewritten as

$$\Omega = A_0^{-1} D \left( A_0^{-1} \right)' = A_0^{-1} D^{1/2} D^{1/2} \left( A_0^{-1} \right)' = PP', \qquad (3.18)$$

where  $P = A_0^{-1}D^{1/2}$  and  $A_0$  is lower triangular. This, in turn, implies that *P* is a lower triangular matrix with the standard deviations of the structural disturbances on its principal diagonal. Moreover, it can be shown that *P* is the (unique) Cholesky factor of the symmetric positive definite matrix  $\Omega$  (Hamilton (1994: 91-92)). Thus, in this identification approach it is particularly easy to recover the estimates of the structural parameters. Note, however, that while *P* is unique for a given ordering of the variables in  $X_t$ , there are *k*! possible orderings in total. Hence, it is important to check how sensitive the dynamic properties of the model are to alternative orderings of the variables.

Based on these considerations, it is possible to distinguish two types of impulse responses (see, e.g., Lütkepohl and Reimers (1992: 55)): (i) those impulse responses that give the dynamic effects of innovations in the reduced-form disturbances  $\varepsilon_t$ , and, (ii) the impulse

<sup>&</sup>lt;sup>75</sup> As the rank condition requires a lengthy derivation, we refer the interested reader to its detailed exposition in Amisano and Giannini (1997: 48-57). The structural VAR models analyzed in Sections 3.3 and 3.4 all satisfy the rank condition.

responses that give the dynamic effects of innovations in the structural-form disturbances  $e_t$ . In empirical applications, in general, only the second set of impulse responses is of interest because it allows to study the effects of shocks to one variable in isolation since the variancecovariance matrix of the structural disturbances is diagonal. In contrast, the reduced-form residuals are in general correlated ( $\Omega$  is not diagonal) so that little can be learned from the study of the effects of a change in a single element of  $\varepsilon_t$  if historically changes in this element have coincided with changes in other elements of  $\varepsilon_t$ .

Still, as the structural disturbances can be interpreted as linear combinations of the reducedform disturbances, it is useful to calculate the impulse responses giving the effects of innovations in  $\varepsilon_t$  in an intermediate step. These quantities are given by (see Lütkepohl (1991: 18)):

$$\Xi_n = \sum_{j=1}^n \Xi_{n-j} A_j, \qquad n = 1, 2, \dots,$$
(3.19)

where  $\Xi_0 = I_k$  and  $A_j = 0$  for j > p. The row *i*, column *k* element of  $\Xi_n$  gives the response of variable  $X_i$  to a one-unit increase in the *k*th variable, *n* periods ago. Given these quantities it is easy to obtain the impulse responses to innovations in the structural disturbances  $e_t$ . For the general structural model, they are given by

$$\Theta_n = \Xi_n A_0^{-1} B D^{1/2}, \qquad n = 1, 2, \dots,$$
(3.20)

and for the recursive VAR they obtain as

$$\Theta_n = \Xi_n P, \qquad n = 1, 2, \dots$$
(3.21)

The elements of  $\Theta_n$  have an interpretation analogous to that of the elements of  $\Xi_n$ , except that the size of the impulses is one standard deviation here. Impulses of size one unit could easily be obtained by post-multiplying (3.20) and (3.21) with  $D^{-1/2}$ .

As the impulse responses are random variables it is useful to provide confidence intervals in order to measure the uncertainty surrounding the estimated impulse responses. Confidence intervals can be constructed based either on analytical derivatives (see Lütkepohl (1990) for stationary VAR models and Lütkepohl and Reimers (1992) for cointegrated VAR models), on Monte Carlo simulation methods (see, e.g., Sims and Zha (1999)) or on the bootstrap methodology (see, e.g., Runkle (1987)). In the empirical application, we report confidence intervals based on the bootstrap methodology.<sup>76</sup> The simple bootstrap algorithm can be summarized as follows:

- 1. Estimate the parameters of the model (3.15) by the appropriate method.<sup>77</sup>
- 2. Generate bootstrap residuals  $\varepsilon_1^*, ..., \varepsilon_T^*$  by randomly drawing with replacement from the set of estimated residuals  $\hat{\varepsilon}_1, ..., \hat{\varepsilon}_T$ .<sup>78</sup>
- 3. Condition on the pre-sample values  $(X_{t-p+1}^*,...,X_0^*) = (X_{t-p+1},...,X_0)$  and construct bootstrap time series  $X_t^*$  recursively using equation (3.15),

$$X_t^* = \hat{A}_1 X_{t-1}^* + \dots + \hat{A}_p X_{t-p}^* + \hat{\mu}_0 + \hat{\mu}_1 t + \varepsilon_t^*, \qquad t = 1, \dots, T.$$

- 4. Re-estimate the parameters  $A_1, ..., A_p, \mu_0$  and  $\mu_1$  from the generated data and calculate the impulse response functions  $\hat{\Theta}_n^*, n = 1, 2, ...$
- 5. Repeat steps 2–4 a large number of times (in the empirical application: 1000) and calculate the  $\alpha$  and  $1-\alpha$  percentile interval endpoints of the distribution of the individual elements of  $\hat{\Theta}_n^*$ , n = 1, 2, ... In the empirical application, we set  $\alpha = 0.16$  and accordingly report 68% confidence intervals.<sup>79</sup>

<sup>&</sup>lt;sup>76</sup> See Hall (1994) and Horowitz (2001) for extensive treatments of the bootstrap methodology in general.

<sup>&</sup>lt;sup>77</sup> Kilian (1998) proposes a bias-corrected boostrap method for stationary VAR models based on an adjustment of the estimated autoregressive parameters  $\hat{A}$ . As it is unclear whether this bias correction improves the accuracy of estimated impulse responses also in the case of non-stationary VAR models, we do not pursue this route here.

<sup>&</sup>lt;sup>78</sup> A major difference between the bootstrap approach and the Monte Carlo simulation method is that the former builds on random draws from the set of estimated residuals while the latter in general builds on random draws from a normal distribution. The bootstrap approach, instead, allows for non-normality of the residuals.

<sup>&</sup>lt;sup>79</sup> In the empirical VAR literature, typically either 68% or 95% confidence intervals are reported. Sims (1987: 443) argues against the use of 95% confidence intervals in VAR studies on the grounds that "there is no scientific justification for testing hypotheses at the 5% significance level in every application". He suggests to treat the statistical significance of impulse responses derived from VAR coefficient estimates differently from that of coefficient estimates in standard econometric models. It is inherent in VAR models that most of the parameter estimates are insignificantly different from zero when tested at the 5% level, and this translates into relatively large confidence intervals for impulse responses. Still, estimates from unconstrained VAR models are widely thought to provide a useful data summary. Against this background, Sims and Zha (1999: 1118) recommend the use of 68% confidence intervals for estimated impulse responses. In the empirical application,

# 4 **Empirical Results**

This section presents empirical evidence on the dynamic effects of public capital for 22 OECD countries based on VAR models. Section 4.1 deals with model selection and determination of cointegration rank. Section 4.2 presents the results of an impulse response analysis based on a set of benchmark identifying assumptions. The results of a sensitivity analysis employing alternative identifying assumptions are presented in Section 5.

#### 4.1 Model Specification and Estimation

#### Data

The countries considered in this paper are the same as those considered in Kamps (2004).<sup>80</sup> With a few exceptions, the sample periods cover the years 1960-2001. For each country, we specify a four-variable VAR model including the public net capital stock,  $K^G$ , the private net capital stock,  $K^P$ , the number of employed persons, N, and real GDP, Y.<sup>81</sup> Expressing all variables in natural logarithms multiplied by 100 and denoting the transformed variables by lower-case letters, the vector of endogenous variables  $X_t$  can be expressed as  $X_t \equiv [k_t^G, k_t^P, n_t, y_t]'$ .<sup>82</sup>

#### VAR Order Selection

The exposition of the VAR methodology in Section 3 was based on the implicit assumption of a known lag order *p*. In empirical applications, however, the lag order is typically unknown. In the econometric literature, a number of selection criteria have been proposed that can be used to determine the optimal lag order. A review of popular selection criteria can be found in Lütkepohl (1991: Chapters 4.3 and 11.4.1). The starting point is to estimate VAR(*m*) models with orders m = 0, ..., M conditional on *M* pre-sample observations ( $X_{M-1}, ..., X_0$ ) and then

we follow this advice, yet we refrain from drawing strong conclusions about the statistical significance of the estimated impulse responses.

<sup>&</sup>lt;sup>80</sup> Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States

<sup>&</sup>lt;sup>81</sup> Real GDP and employment are drawn from the OECD Analytical Database, Version June 2002. The public and private capital stocks are taken from Kamps (2004). The dataset is available on request.

 $<sup>^{82}</sup>$  Multiplying the variables in logarithms by 100 facilitates the interpretation of the estimated impulse responses. In this case, the impulse responses give the percentage change in the level of the respective variable.

to choose an estimator of the order p that minimizes some selection criterion. The general structure of the selection criteria applied here can be expressed as follows:

$$Cr(m) = \log |\hat{\Omega}(m)| + \frac{1}{T} \varphi(m),$$
  $m = 0, 1, ..., M,$  (3.22)

where *M* is the maximum lag length considered. The first addend in equation (3.22) is the log determinant of the residual covariance matrix, which in general is decreasing in *m*, while the second addend is increasing in *m*. The selection criteria, thus, strike a balance between goodness of fit and parsimonious specification of the model. The selection criteria considered here differ in the specification of the term  $\varphi(m)$ : (i) for the Akaike (1974) information criterion (AIC)  $\varphi(m) = 2mk^2$ , (ii) for the Schwarz (1978) information criterion (SC)  $\varphi(m) = mk^2 \log(T)$ , and (iii) for the Hannan and Quinn (1979) information criterion (HQ)  $\varphi(m) = 2mk^2 \log(\log(T))$ . For each criterion, the optimal lag length  $\hat{p}$  is found by minimizing (3.22):

$$Cr(\hat{p}) = \min\{Cr(m)|m=0,1,...,M\}.$$
 (3.23)

Asymptotically, the AIC overestimates the lag order with positive probability, whereas the two other criteria estimate the order consistently if the VAR process has a finite order and the maximum order M is larger than the true order p (see Lütkepohl (1991: 130-132)). These results not only hold for stationary variables, but also in the case of I(1) variables. Thus, VAR order selection can be based on the unrestricted VAR model (3.1) discussed in Section 3.1. Given the small sample size in our empirical application, we choose not to discard the AIC on the grounds of asymptotic results.

The first three columns of Table 2 give the optimal lag order selected by the three criteria for each of the 22 OECD countries considered. The results reveal an interesting relation among the criteria that holds for sample sizes  $T \ge 16$  (see Lütkepohl (1991: 133)):  $\hat{p}(SC) \le \hat{p}(HQ) \le \hat{p}(AIC)$ . Whereas the AIC selects a lag order of 4 for most countries, the HQ and SC criteria select a lag order of 2 in most cases. Given the small sample size, we are interested in a parsimonious specification of the model. Thus, we choose the lag order selected by the SC criterion in general. Yet, we also perform specification tests that check whether for the lag length selected by the SC criterion the residuals are free from first-order

Country	VAR order minimizing			Chosen VAP	Specification tests (p-values) <sup>e</sup>		
	AIC <sup>a</sup>	SCb	HQc	order <sup>d</sup>	Autocorre- lation <sup>f</sup>	Hetero- scedasticity <sup>g</sup>	Normality <sup>h</sup>
Australia	2	2	2	2	0.604	0.336	0.073
Austria	4	1	2	2	0.587	0.209	0.185
Belgium	2	2	2	2	0.654	0.083	0.188
Canada	4	2	2	2	0.745	0.902	0.059
Denmark	4	3	3	3	0.110	0.344	0.001*
Finland	4	1	1	2	0.680	0.118	0.354
France	4	2	4	2	0.515	0.291	0.154
Germany	4	2	2	2	0.577	0.275	0.211
Greece	3	2	2	3	0.657	0.672	0.031*
Iceland	4	2	4	2	0.276	0.496	0.063
Ireland	4	2	2	2	0.524	0.042*	0.144
Italy	4	2	2	2	0.400	0.025*	0.445
Japan	4	2	2	3	0.182	0.188	0.003*
Netherlands	4	1	4	2	0.168	0.061	0.022*
New Zealand	4	1	3	2	0.054	0.142	0.062
Norway	4	2	2	2	0.370	0.757	0.136
Portugal	4	2	2	2	0.355	0.292	0.322
Spain	4	2	4	4	0.118	0.343	0.000*
Sweden	4	2	4	2	0.101	0.157	0.032*
Switzerland	4	2	2	2	0.238	0.139	0.077
United Kingdom	2	2	2	2	0.562	0.145	0.078
United States	4	2	4	2	0.054	0.546	0.115

Table 2: Specification of VAR orders

*Notes*: The maximum order considered is equal to 4. The underlying VAR model contains constants and linear time trends. In the case of Germany, the VAR model also contains a dummy variable (set to 1 in 1991 and 0 otherwise) as well as its lagged value. In the case of Denmark, the VAR model also contains a dummy variable (set to 1 in 1973, -1 in 1974 and 0 otherwise) as well as its lagged value.

<sup>a</sup>Akaike information criterion (Akaike (1974)). – <sup>b</sup>Schwarz information criterion (Schwarz (1978)). – <sup>c</sup>Hannan-Quinn information criterion (Hannan and Quinn (1979)). – <sup>d</sup>The VAR order is chosen on the basis of the information criteria and on the basis of specification tests. – <sup>e</sup>The specifications tests are based on the residuals from the estimation of an unrestricted VAR (*p*), where *p* is the integer reported in the column "Chosen VAR order". \* denotes statistical significance at the 5 percent level. – <sup>f</sup>Multivariate autocorrelation LM test (Johansen (1995: 22)). Under the null hypothesis of no serial correlation of order *h* (here: *h*=1) the test statistic is asymptotically distributed  $\chi^2$  with 16 degrees of freedom. – <sup>g</sup>Multivariate extension of White's (1980) heteroscedasticity test (Doornik (1996)). Under the null hypothesis of homoscedastic residuals the test statistic is asymptotically distributed  $\chi^2$  with 10(8*p*+2) degrees of freedom, where *p* is the chosen VAR order. – <sup>h</sup>Multivariate residual normality test (Lütkepohl (1991: 155–158)). Under the null hypothesis of normally distributed residuals the test statistic is asymptotically distributed  $\chi^2$  with 8 degrees of freedom.

autocorrelation, homoscedastic and normally distributed. Since the trace test for cointegration is robust to deviations from the normality assumption (see Cheung and Lai (1993: 324)) and since the asymptotic properties of the VAR parameter estimators do not depend on the normality assumption (see Lütkepohl (1991: 359)), we do not dismiss the specification chosen by the SC criterion if the normality test indicates that the residuals are non-normal. However, if the autocorrelation test indicates that the residuals are autocorrelated, we increase the lag

order compared to the one selected by the SC criterion until the autocorrelation test does not reject the null hypothesis anymore.<sup>83</sup> The last three columns of Table 2 show the results of the three specification tests for the chosen lag order for each of the 22 OECD countries considered. The results show that at the 5% significance level there are no signs of residual autocorrelation and in general no signs of heteroscedastic residuals.<sup>84</sup> The following steps of the empirical analysis are, thus, based on the lag orders displayed in the middle column of Table 2.

#### Determination of Cointegration Rank

Neoclassical growth theory suggests that along the balanced growth path (steady state) the socalled great ratios are constant, i.e., variables such as output, capital, consumption and investment grow at the same constant rate. King et al. (1991) first investigated the cointegration implications of neoclassical growth theory. They showed that the constancy of the great ratios implies that if the individual variables are non-stationary they must be driven by a single common stochastic trend. Translated to our problem this implies that the public capital to output ratio and the private capital to output ratio are potential cointegrating relations. In addition, a third potential cointegrating relation might be given by a production function of the type considered, e.g., by Aschauer (1989). Yet, this critically hinges on the nature of technology. If technology is modeled as a trend-stationary process (see, e.g., Sturm and De Haan (1995)), then the production function could be a cointegrating relation.<sup>85</sup> However, if technology is a non-stationary process (see, e.g., Crowder and Himarios (1997)) then the production function will not describe a stationary relation between the variables collected in the vector  $\tilde{X}_t = [k_t^G, k_t^P, n_t, y_t, t]^r$ . To sum up, based on economic theory we expect to find at most three cointegrating relations.

We test for the number of cointegrating relations using Johansen's (1988, 1991) trace test.

 $<sup>^{83}</sup>$  In the case of Denmark, a dummy variable (set to 1 in 1973, -1 in 1974 and 0 otherwise) was included because without the dummy variable the null hypothesis of no serial correlation had to be rejected at the 5% significance level for all lag orders between 0 and 4. In the case of Germany, a dummy variable (set to 1 in 1991 and 0 otherwise) was included in order to account for the level shift in the variables due to German Reunification.

<sup>&</sup>lt;sup>84</sup> Exceptions are Ireland and Italy for which the heteroscedasticity test statistic is significant at the 5% level. In both cases, increasing the lag length to 4, as suggested by the AIC, worsened the performance of the model with respect to residual autocorrelation. As autocorrelation is more detrimental than heteroscedasticity, we choose the shorter lag length in both cases.

<sup>&</sup>lt;sup>85</sup> This, of course, raises the question of where the stochastic trends in the data come from. Technology is widely viewed to be the prime candidate for a stochastic trend.

Country	VAR order		Cointegration			
		H <sub>0</sub> : $r = 0$	H <sub>0</sub> : $r = 1$	H <sub>0</sub> : $r = 2$	H <sub>0</sub> : $r = 3$	rank <sup>a</sup>
Australia	2	97.41	57.73	32.46	11.34	3
Austria	2	89.55	50.24	25.82	6.02	2
Belgium	2	63.88	34.59	17.24	5.72	1
Canada	2	81.59	52.03	27.48	12.73	3°
Denmark	3	104.79	58.79	30.06	8.49	3
Finland	2	70.25	38.76	17.37	8.43	1
France	2	80.34	45.24	21.89	10.56	2
Germany	2	72.53	38.35	16.13	0.96	1
Greece	3	107.93	46.95	23.53	7.58	2
Iceland	2	73.03	43.10	19.13	6.44	2
Ireland	2	79.85	47.98	23.15	10.02	2
Italy	2	98.94	57.27	31.98	12.56	3°
Japan	3	100.85	46.34	19.66	8.82	2
Netherlands	2	69.85	40.66	20.41	8.58	1
New Zealand	2	58.06	34.98	15.08	5.56	0
Norway	2	84.45	49.09	27.67	10.09	3
Portugal	2	58.10	38.03	22.70	10.57	0
Spain	4	121.30	65.80	31.82	9.21	3
Sweden	2	86.35	54.62	30.96	10.87	3
Switzerland	2	71.84	40.01	20.66	5.55	1
United Kingdom	2	89.08	56.76	27.10	8.64	3
United States	2	120.30	59.18	26.71	11.89	3
Critical values <sup>b</sup>		63.87	42.92	25.86	12.52	

Table 3: Johansen (1988, 1991) cointegration test

*Notes:* The underlying VAR model contains unrestricted intercepts and restricted trend coefficients and is of order p, where p is the integer reported in the column "VAR order". In the case of Germany, the VAR model also contains a dummy variable (set to 1 in 1991 and 0 otherwise) as well as its lagged value. In the case of Denmark, the VAR model also contains a dummy variable (set to 1 in 1973, -1 in 1974 and 0 otherwise) as well as its lagged value.

<sup>a</sup>The test decision is based on the asymptotic critical values reported in the bottom row of the table. – <sup>b</sup>The asymptotic critical values for a 5% significance level for Johansen's log-likelihood based trace statistic are taken from MacKinnon et al. (1999), Table V. – <sup>c</sup>In the cases of Canada and Italy, the test results suggest that the model variables are stationary (r=4). However, recursively calculated eigenvalues and trace statistics (see Hansen and Juselius (1995: 50-63) for details) suggest that for both countries the fourth eigenvalue is not significantly different from zero. Against this background, we choose r=3 for both countries.

The test statistics are computed using equation (3.9), and are then compared with the appropriate critical values tabulated by MacKinnon et al. (1999).<sup>86</sup> The testing sequence can

<sup>&</sup>lt;sup>86</sup> The MacKinnon et al. (1999) critical values are also used in the case of Denmark and Germany. The empirical models for these two countries include dummy variables. It is well known that dummy variables may affect the asymptotic distribution of the trace test statistic. This is particularly true for step dummies that give rise to broken linear trends in the levels of the variables. The dummy variables considered here, instead, are asymptotically negligible.

be expressed as follows (Lütkepohl (2001)):

$$H_0(r_0)$$
:  $rank(\Pi) = r_0$  versus  $H_1(r_0)$ :  $rank(\Pi) = k$ ,  $r_0 = 0, 1, ..., 3$ . (3.24)

The testing sequence starts with the null hypothesis that the cointegration rank is zero. If this hypothesis cannot be rejected, then the testing sequence terminates and a VAR model in first differences is the appropriate model. At the other extreme, if all null hypotheses have to be rejected, then the variables can be regarded as (trend-)stationary in levels.

Table 3 displays the test results for each of the 22 countries considered here. The results show that for a large majority of countries the number of cointegrating relations is either two or three. For the remaining countries, the cointegration rank is lower; for two countries, New Zealand and Portugal, it is even zero. As a consequence, for these two countries we estimate a VAR model for the variables in first differences. For the other countries, we estimate a VECM imposing the appropriate rank restriction.

#### 4.2 Impulse Response Analysis

This section analyzes the dynamic properties of the estimated VAR models for the 22 OECD countries considered in this study with the help of impulse response functions. As was discussed in Section 3.3, there is a need to identify VAR models in order to be able to give the impulse response functions a structural interpretation. In the empirical literature on the effects of monetary policy shocks, a large number of identification schemes have been proposed. While in principle all of these identification schemes could also be applied to study the effects of fiscal policy shocks, some of these schemes do not seem to be useful for our setting. For example, it does not seem to be advisable to impose restrictions on the long-run effects of fiscal policy shocks. In other settings, long-run restrictions can be justified by neutrality propositions derived from economy theory: e.g., Blanchard and Quah (1989) impose the restriction that shocks to aggregate demand do not affect output in the long run, and Shapiro and Watson (1988) impose restrictions such that shocks to nominal variables such as the money supply or prices have no effect on real variables in the long run. In our setting,

economic theory does not suggest any neutrality propositions.<sup>87</sup> As shown, e.g., by Baxter and King (1993), there is no reason to expect that a permanent change in government expenditure has a zero long-run effect on output and other real variables. Therefore, in this study we instead concentrate on identification schemes that involve short-run restrictions on impulse responses.

As was discussed in Section 3.3, structural VAR models can be identified by imposing restrictions on the  $A_0$ , B and D matrices, remembering that  $A_0\varepsilon_t = Be_t$  and  $D = E(e_te_t)$ . In this section, we identify the VAR models for the individual countries by assuming that the relation between the reduced-form disturbances  $\varepsilon_t$  and the structural disturbances  $e_t$  takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^{k^G} \\ \varepsilon_t^{k^P} \\ \varepsilon_t^{N} \\ \varepsilon_t^{N} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t^{k^G} \\ e_t^{k^P} \\ e_t^{N} \\ e_t^{N} \end{bmatrix},$$
(3.25)

There are six unknown parameters in equation (3.25) as well as four unknown parameters in the diagonal covariance matrix of the structural disturbances, D. Since there are ten distinct elements in the covariance matrix of the reduced-form residuals,  $\hat{\Omega}$ , the model is just identified. This set of identifying assumptions is an example for the recursive approach originally proposed by Sims (1980). It has been widely applied in related literature (see Section 2) and is, thus, a natural starting point for our analysis. As was mentioned earlier, the ordering of variables in the recursive approach may be important for the results. The robustness of the results presented in the following to alternative orderings of the variables is explored in Section 5.1. In Section 5.2, we extend an alternative identification scheme originally proposed by Blanchard and Perotti (2002) that departs from the recursiveness assumption.

The particular ordering of variables resulting from the benchmark identification scheme has the following implications: (i) Public capital does not react contemporaneously to shocks to

<sup>&</sup>lt;sup>87</sup> In fiscal policy analysis in general, an exception is the so-called Ricardian Equivalence proposition (Barro (1974)). This proposition states that for a given path of government expenditure it is irrelevant whether expenditure is financed by lump-sum taxes or government debt. Thus, e.g., a deficit-financed tax cut has no effects on output. Yet, this result not only holds in the long run, but in all periods starting with the period when the shock occurs. There is, thus, no need to impose a long-run restriction on the impulse responses in this case.

the other variables in the system, (ii) private capital does not react contemporaneously to shocks to employment and real GDP, but is affected contemporaneously by shocks to public capital, (iii) employment does not react contemporaneously to shocks to real GDP, but is affected contemporaneously by shocks to both private and public capital, and, (iv) real GDP is affected contemporaneously by shocks to all other variables in the system. Note that after the initial period the variables in the system are allowed to interact freely, i.e., for example, shocks to real GDP can affect public capital in all periods after the one in which the shock occurs.

The assumptions on the contemporaneous relations between the variables can be justified as follows: Movements in government spending, unlike movements in taxes, are largely unrelated to the business cycle. In particular, government spending on capital items involves large decision and implementation lags. Therefore, it seems sensible to assume that public capital is not affected contemporaneously by shocks originating in the private sector. In a similar vein, private capital is largely unrelated to the business cycle.<sup>88</sup> Employment, while being strongly pro-cyclical, in general lags the business cycle.<sup>89</sup> Thus, it seems appropriate to assume that employment is unaffected contemporaneously by output shocks. Ordering output last can be justified by, e.g., a production function which shows that the three inputs affect output contemporaneously.

#### The Dynamic Effects of Public Capital

Figure 1 shows the effects of a shock to public capital for the 22 OECD countries considered here for a horizon of 25 years. Each subplot in the figure displays a point estimate of the impulse responses as well as a 68% confidence interval computed with the bootstrap procedure described in Section 3.3. The shocks to public capital have size one standard deviation for each country. While this precludes a quantitative comparison of the effects across countries, shocks of such size have the attractive feature that they can be viewed as representative for typical shocks that occurred during the sample period in the individual countries. A quantitative comparison of the long-run effects across countries is given at the

<sup>&</sup>lt;sup>88</sup> See, e.g., King and Rebelo (1999: 938) for evidence for the United States. In contrast, capital utilization is highly correlated with the cycle. Apparently, in the short run firms vary the degree of capital utilization in response to shocks affecting the demand for their products, rather than adjusting the productive capacity itself. <sup>89</sup> See, e.g., Stock and Watson (1999: 41) for evidence for the United States. Note, however, that their results are computed for quarterly data. It is, thus, unclear whether the finding that employment lags output also applies on an annual basis. We follow the literature and order employment before output in the structural VAR model.



Figure 1: Impulse responses of GDP to a shock to public capital

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.



#### Figure 1 (continued): Impulse responses of GDP to a shock to public capital

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

end of this section.

The subplots in Figure 1 show that in general the output effect of a shock to public capital is positive. For most countries, the output response is positive at all plotted horizons up to the endpoint of 25 years. The figure also reveals that the impulse responses are estimated quite imprecisely, as witnessed by large confidence intervals for some countries. Judged by the 68% confidence intervals, the output responses are statistically significant in about half of all cases. Apart from the general pattern described above, two other interesting patterns can be observed. First, there are a few countries for which the output response is negative at all plotted horizons (Ireland, Japan, Portugal). Second, there are some countries for which the short-run output response is negative, while the medium-run response is positive (Canada, Iceland, Norway, Spain, United Kingdom).

Given these result patterns, it is interesting to investigate whether they can be traced back to the responses of the other three variables. If a neoclassical production function was a valid description of the relation between the four endogenous variables, then the impulse responses of public capital, private capital and employment taken together should enable us to explain the observed patterns of output responses. Figure 2 plots the impulse responses of public capital to a one standard deviation shock to public capital. The subplots of this figure reveal that the point estimates of the responses of public capital are positive for all countries. For the majority of countries, the point estimates are positive for all plotted horizons, and, judged by the 68% confidence intervals, the responses are statistically significant in most of these cases. Thus, the responses of public capital are consistent with the general pattern observed for the output responses. As regards the two other patterns of the output responses, in most cases they can not be explained by the pattern of the public capital responses alone. In particular, negative output responses as observed for some countries are not easily reconciled with positive public capital responses unless public capital is conceived to have a negative marginal productivity. Among those countries with a negative output response, this is only conceivable in the case of Japan. As shown in Kamps (2004), Japan exhibits by far the largest public capital to output ratio among the OECD countries in our sample. It is, thus, conceivable that the public capital to output ratio in Japan is beyond its optimal level so that additional public capital has a negative effect on output. While this might be an explanation for the negative output response in the case of Japan, it is implausible for the other countries exhibiting negative output responses. In particular, Portugal exhibits the lowest public capital



# Figure 2: Impulse responses of public capital to a shock to public capital

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in public capital in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.



Figure 2 (continued): Impulse responses of public capital to a shock to public capital

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in public capital in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

stock per head among the OECD countries in our sample (see Table 3 in Kamps (2004)). Against this background, it is hardly imaginable that the marginal productivity of public capital is negative in Portugal.

Another possible explanation is that public capital crowds out private capital and employment. The impulse responses of private capital to a shock to public capital – not plotted here<sup>90</sup> – show that in the vast majority of countries private capital and public capital are complements in the medium run.<sup>91</sup> Interestingly, however, in almost half of the countries private capital and public capital are substitutes in the short run. Among these countries are Canada and Spain, two of those countries for which a negative short-run but a positive medium-run output response was observed. Thus, for these two countries the responses of private capital may explain the pattern of the output responses. The general equilibrium analysis performed by Baxter and King (1993) suggests the following explanation for the sign switch of the private capital responses: There are two opposing forces determining the response of private capital. One of these forces is the resource cost associated with financing an additional unit of public capital. This cost reduces the resources available to the private sector and all other things being equal induces a fall in private investment. The other force is the positive effect of an increase in public capital on the marginal productivity of private capital, all other things being equal inducing a rise in private investment. If public capital accumulates gradually, then the first force will dominate the second in the short run, whereas in the medium to long run the second force will dominate.

As regards the impulse responses of employment to a shock to public capital – not plotted here<sup>92</sup> –, they do not show a general pattern. For roughly one third of the countries the responses are negative – implying that employment and public capital are substitutes –, while for the other countries the responses are either positive – implying that employment and public capital are complements – or not significantly different from zero, judged by the 68% confidence interval.<sup>93</sup> The lack of clear-cut results for employment is deplorable also from a

 $<sup>^{90}</sup>$  The figure holding the impulses responses of private capital to shocks to public capital is available upon request.

<sup>&</sup>lt;sup>91</sup> This is true for Portugal, implying that crowding out of private capital does not seem to be the reason for the negative output response observed for this country.

 $<sup>^{92}</sup>$  The figure holding the impulses responses of employment to shocks to public capital is available upon request.

<sup>&</sup>lt;sup>93</sup> The employment responses of Portugal are not significantly different from zero, implying that the employment responses – like the public and private capital responses – cannot rationalize the negative output response observed for this country. Note that Portugal is one of only two countries for which both the Engle-Granger test and the Johansen test fail to reject the null hypothesis of no cointegration. Possibly, the empirical model is mis-

theoretical perspective because the responses of employment can be very useful in order to test competing theoretical models. For example, traditional Keynesian models predict that employment will rise in response to an increase in government spending, which is a testable hypothesis. Issues are more complicated when it comes to neoclassical models such as the general equilibrium model such as the one considered by Baxter and King (1993). The policy experiments performed with this model suggest a possible explanation for the inconclusive evidence on the employment response. An increase in public capital exerts two opposing wealth effects and – depending on the way additional public capital is financed – possibly also a substitution effect. For example, if public capital is financed by non-distortionary taxes and is only mildly productive, then employment will rise in response to a shock to public capital. However, if public capital is financed by distortionary taxes and is only mildly productive, then employment will fall in response to such a shock. The empirical model is silent on these issues because it does not include any government revenue variables. The reason is, of course, that including all variables in the VAR model that are interesting in this respect (non-distortionary taxes, distortionary taxes, government debt and government consumption) would quickly exhaust the available degrees of freedom. We will come back to the financing decision in Section 5.2. where we discuss the Blanchard and Perotti (2002) identification scheme.

Table 4 displays summary information about the long-run effects of public capital for the 22 OECD countries in our sample. The table displays long-run elasticities of private capital, employment and real GDP with respect to public capital, respectively. These long-run elasticities are special in that they capture the dynamic feedback between the four variables in the system.<sup>94</sup> Therefore, they can be viewed as the empirical counterpart of the general equilibrium effects typically considered in theoretical models. The long-run elasticities considered here are conceptually different from the elasticities of a production function. Whereas for a production function, e.g., the elasticity of output with respect to public capital gives the percentage change in output per exogenous one-percent change in public capital holding fixed the private inputs and excluding feedback effects, e.g., from output to public

specified even though the specification tests reported in Table 3.2 suggest otherwise. Alternatively, data quality might be an issue in the case of Portugal. For example, whereas the real GDP series for Portugal contained in the OECD Analytical Database starts in 1960, the Portuguese statistics office INE and Eurostat currently publish GDP data according to ESA 1995 starting only in 1987 and 1995, respectively. As we do not know the underlying cause for the puzzling results of the impulse response analysis, we choose to treat Portugal as an outlier in the following.

Country	Long-run elasticity of with respect to public capital <sup>a</sup>				
Country	Private capital	Employment	Real GDP		
Australia	0.33*	-0.24	0.29*		
Austria	0.22*	0.12	0.07		
Belgium	-0.18	0.06	0.15		
Canada	1.54	0.85	1.25*		
Denmark	0.63**	0.03	0.41**		
Finland	0.68*	0.50	0.72*		
France	1.44*	-0.48	0.84*		
Germany	0.22	-0.12	0.53*		
Greece	1.32*	-0.32*	1.77**		
Iceland	0.64*	0.31	0.78*		
Ireland	0.58*	-0.36	0.01		
Italy	1.25*	-0.38*	1.73		
Japan	-11.14	-5.81	-8.58		
Netherlands	0.24	-0.24	0.52		
New Zealand	0.15	0.06	0.11		
Norway	1.46*	0.11	0.41*		
Portugal	0.30	-0.33	-0.77*		
Spain	1.13	-0.27	1.09*		
Sweden	0.84*	0.55*	0.55*		
Switzerland	0.38**	-0.05	0.41*		
United Kingdom	0.40**	-0.27*	0.08		
United States	-0.71*	-0.48	0.33		

Table 4: Long-run effects of public capital

*Notes*: \* (\*\*) denotes that the 68% (95%) confidence interval does not include zero. The confidence intervals for the individual countries are computed using the bootstrap procedure described in Section 3.3.

<sup>a</sup>The long-run elasticities give the long-run percentage change in private capital, employment and real GDP per one-percent long-run change in public capital. They are obtained by dividing the long-run response of private capital, employment and real GDP to a shock to public capital, respectively, by the long-run response of public capital to a shock to public capital. In the computations, we set the response horizon n=500 which ensures that for all countries the impulse responses have converged to their long-run levels.

capital, the long-run elasticities with respect to public capital reported here account for the dynamic interaction between the variables in the system.<sup>95</sup>

The results reported in Table 4 show that for most countries the long-run elasticity of output with respect to public capital is positive, giving support to the hypothesis that public capital is

<sup>&</sup>lt;sup>94</sup> See, e.g., Pereira (2001b) for another study using this concept.

<sup>&</sup>lt;sup>95</sup> Baxter and King (1993: 330)), e.g., make a similar distinction in their quantitative analysis of a dynamic general equilibrium model.

productive.<sup>96</sup> Judged by the 68% confidence intervals, this long-run elasticity is statistically significant in the majority of countries. In most of these countries, the long-run elasticity is smaller than 1, i.e., a one-percent long-run increase in public capital is associated with a less than proportional increase in output. The long-run elasticity of private capital with respect to public capital is positive for most countries, indicating that private capital and public capital are complements in the long run. As is the case for output, this elasticity – again judged by the 68% confidence intervals – is statistically significant and smaller than 1 in the majority of countries. As already noted in the interpretation of the impulse responses, the results for employment are less conclusive. In all countries except four, the long-run elasticity of employment with respect to public capital is not statistically significant. In those countries in which it is significant, this elasticity is either positive or negative. Taken together, the results for employment seem to suggest that public capital and employment are neither complements nor substitutes in the long run, but rather that they are unrelated in the long run. To sum up, an increase in public capital in OECD countries on average can be expected to lead to an increase in output in the long run, but there is little evidence that it is the appropriate policy measure if the aim is to increase employment in the long run.

#### Is There Evidence for Reverse Causation?

In Kamps (2004), estimation of a production function was based on the assumption that public capital, private capital and employment are exogenous with respect to output. This implied that feedback from output to the inputs was excluded by assumption. The VAR model, instead, allows for such feedback by treating all variables as endogenous. Whether it is important to do so in an empirical application, can be clarified with the help of a causality analysis. While it is possible to formally test for causality in the sense of Granger (1969), the impulse response analysis can also be regarded as a type of causality analysis (Lütkepohl 2001)).<sup>97</sup> In our context, it is most interesting to investigate whether there is feedback from

<sup>&</sup>lt;sup>96</sup> There are two exceptions to this general finding: Japan and Portugal. As was mentioned above, the estimate for Portugal is difficult to rationalize, therefore we treat it as outlier. As regards Japan, the long-run elasticity taken on its own seems to suggest a very strong negative output effect of public capital. However, none of the three elasticities reported for Japan is statistically significant judged by the 68% confidence interval. Moreover, the long-run response of public capital to a shock to public capital is almost zero. While the long-run responses of the other three variables are also close to zero, they are larger in absolute value than the long-run response of public capital. This translates into misleadingly large long-run elasticities of private capital, employment and output, respectively. It is more likely that the true long-run elasticities are zero in the case of Japan.

<sup>&</sup>lt;sup>97</sup> The methodology for testing Granger causality in higher dimensional systems is developed in Dufour and Renault (1998). These authors show that impulse responses do not summarize all information about causal links in higher dimensional systems (Dufour and Renault (1998: 1113)). As a consequence, even if the impulse

output to public capital, i.e., whether the impulse responses of public capital to an output shock are significantly different from zero at some point in the response horizon.

Figure 3 depicts the impulse responses of public capital to a shock to real GDP. Note that our identifying assumptions restrict the impact response of public capital to be zero. The impulse responses show that in the vast majority of countries public capital increases after a positive output shock. In most cases, these responses are statistically significant, judged by the 68% confidence intervals. These results suggest that it is indeed important to treat public capital as endogenous variable in empirical investigations.<sup>98</sup> The general result that public capital positively reacts to output shocks has a straightforward interpretation: An unanticipated increase in output will in general entail an increase in government revenue so that the resources available for public investment increase. Likewise, an unanticipated decline in output will lead to a deterioration of public finances. The historical record suggests that in general governments in OECD countries in the 1970s and 1980s tended to react to high budget deficits – that arose at a time when trend growth in output declined – by cutting public investment (see De Haan et al. (1996: 71)).

# 5 Sensitivity Analysis: Alternative Identification Assumptions

The benchmark results on the dynamic effects of public capital presented in the previous section relied on specific identification assumptions. While these are standard in the related literature, the question arises as to how sensitive the results are to variations in these assumptions. In the present context, two types of sensitivity analyses seem worthwhile. First, as was mentioned in Section 3.3, the results of the impulse response analysis may be sensitive to the ordering of variables in the recursive VAR model. It is, thus, important to check whether the benchmark results presented in the previous section are robust to alternative orderings of the variables. A sensitivity analysis along these lines is performed in Section 5.1. Second, the empirical literature on the effects of fiscal policy has proposed a number of

response coefficients giving the effects of variable j on variable k are zero at all horizons, there may still be causality running from k to j. Applied to our context, this means that impulse responses not significantly different from zero are not sufficient to rule out causality. Yet, if the impulse responses are significantly different from zero, then this is a clear indication of causality. As is shown below, this is the case for the vast majority of countries in our sample.

<sup>&</sup>lt;sup>98</sup> The same is true for private capital and employment which both also show responses to an output shock significantly different from zero. Detailed results are available upon request.

identification schemes that depart from the recursiveness assumption. In the following, we briefly discuss three approaches to identification that have been applied in fiscal policy analysis recently and that can be viewed as the main alternatives to the recursive



Figure 3.3: Impulse responses of public capital to a shock to GDP

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in public capital in response to a one standard deviation shock to GDP for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.



Figure 3.3 (continued): Impulse responses of public capital to a shock to GDP

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in public capital in response to a one standard deviation shock to GDP for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

approach (see, e.g. Perotti (2002: 8-10)):

- 1. The first approach is the so-called fiscal dummy variable approach introduced by Ramey and Shapiro (1998) and further developed in Eichenbaum (1998), Edelberg et al. (1999) and Burnside et al. (2003). These studies analyze the effects of large increases in military spending in the United States. The underlying idea is that the spending increases associated with the Korean war, the Vietnam war and the Reagan military buildup can be viewed as essentially unrelated to the state of the economy. Accordingly, these studies proceed by estimating a reduced-form VAR model that includes a dummy variable set to 1 during these episodes and to 0 otherwise. This dummy variable is treated as exogenous variable. It is then possible to calculate impulse responses to a shock to the dummy variable based on the reduced-form VAR model only. Thus, the major advantage of this approach is that there is no need for identification of a structural form. However, a number of assumptions have to be made so that these episodes can be treated as truly exogenous events (see Perotti (2001: 28)). Even if these assumptions are satisfied, this approach cannot be easily extended to other OECD countries for lack of candidate fiscal episodes in the period since World War II. Moreover, it is unclear whether the results obtained with this approach can be viewed as representative for categories of government spending other than military spending – which is implicitly assumed in most studies belonging to this literature. Given these qualifications, we conclude that this approach cannot be applied in our context.
- 2. The second approach identifies fiscal policy shocks via sign restrictions on the impulse responses. This approach was introduced by Uhlig (2001) to study the effects of monetary policy shocks and was applied to fiscal policy analysis by Mountford and Uhlig (2002). Unlike the recursive VAR approach and the third approach discussed below, the sign-restrictions approach does not impose linear restrictions on the contemporaneous relations between reduced-form and structural disturbances. Rather, Mountford and Uhlig (2002) impose restrictions directly on the impulse responses: For example, a "business cycle" shock is identified by the requirement that the impulse responses of government revenue and of GDP are positive for the four quarters following the shock. This turns out to be their crucial identifying assumption, having implications also for the identification of fiscal policy shocks (see Mountford and Uhlig (2002: 8)). It states that whenever government revenue and output move in

the same direction, this must be due to a change in the business cycle. Accordingly, this assumption rules out that an increase (fall) in government revenue can generate an increase (fall) in output. While this assumption accords well with both standard Keynesian and neoclassical theory, it rules out such phenomena as "expansionary fiscal contractions" that have received a lot of attention in the recent literature on the effects of fiscal policy.<sup>99-100</sup> Yet, what is more important in our context, is that given our system it does not seem to be possible to identify the effects of shocks to public capital with the help of sign restrictions. For example, we expect a positive co-movement of public capital and output in response both to a shock to public capital (the "fiscal" shock) and to an output shock (the "business cycle" shock). Also, it does not seem to be advisable to impose sign restrictions on the impulse responses of employment and private capital as it is unknown a priori, e.g., whether employment and private capital on the one hand and public capital on the other hand are complements or substitutes. Against this background, we choose not to apply this approach in our context.

3. A third approach, due to Blanchard and Perotti (2002), relies on institutional information about tax and transfer systems and about the timing of tax collections in order to identify the automatic response of taxes and government spending to economic activity. This identification scheme relies on a two-step procedure: In a first step, the institutional information is used to estimate cyclically adjusted taxes and government expenditures. In a second step, estimates of fiscal policy shocks are obtained. Blanchard and Perotti (2002) applied this approach to estimate the effects of tax and government spending shocks for the United States. Other studies using this approach are Höppner (2003: Chapter 3.3) for Germany, Kuttner and Posen (2002) for Japan and Perotti (2002) for five OECD countries (Australia, Canada, the United

<sup>&</sup>lt;sup>99</sup> See, e.g., Giavazzi et al. (2000). These authors find empirical support for the hypothesis that the effects of taxes and government spending are non-linear in OECD countries. For example, the sign of the effects of a change in taxes on national savings depends on the size and persistence of the fiscal impulse. Note, however, that there is conflicting evidence suggesting that the finding of non-linear effects of fiscal policy is not robust (see, e.g., van Aarle and Garretsen (2003) and Kamps (2001)).

<sup>&</sup>lt;sup>100</sup> Aside from this assumption, the sign-restrictions approach has a number of advantages over alternative approaches. For example, the number of identified shocks need not be equal to the number of variables in the VAR model. In his analysis of the effects of monetary policy shocks, e.g., Uhlig (2001) identifies a single shock in a system with six variables. Also, the sign-restrictions approach, unlike the other approaches, allows to capture the effects of anticipated policy changes because there is no underlying assumption that (what the other approaches would estimate to be) a fiscal policy shock has to occur before the response of variables such as output and private consumption.

Kingdom, the United States and West Germany). All these studies have in common that they use aggregate government purchases as the spending variable. Yet, it is straightforward to extend Blanchard and Perotti's identification scheme to allow for a decomposition of government spending into public investment and public consumption. Section 5.2 presents results for this identification scheme for the United States and a comparison with results obtained from a recursive VAR model.

#### 5.1 Alternative Orderings of Variables in the Recursive VAR Model

As was mentioned in Section 3.3, the results of the impulse response analysis may be sensitive to the ordering of variables in the recursive VAR approach. In the benchmark VAR model analyzed in Section 4, the variables were ordered as follows:  $X_t \equiv [k_t^G, k_t^P, n_t, y_t]'$ . All in all, there are 4!= 24 possible orderings of the variables. As an analysis of all possible orderings would be extremely arduous in the present context, we choose to present results here for one alternative ordering that places public capital last in the list of variables:  $X_t \equiv [k_t^P, n_t, y_t, k_t^G]'$ . This implies that public capital is affected contemporaneously by shocks to all other variables, but that the other variables are unaffected contemporaneously by shocks to public capital. This can be regarded as an extreme departure from the benchmark case in which public capital was unaffected contemporaneously by shocks to private capital, employment and output. While the benchmark ordering of variables seems more plausible given the decision and implementation lags involved in fiscal policy, it would be reassuring if the results obtained for this alternative ordering were similar to those obtained in the benchmark case.

Figure 4 displays the impulse responses of GDP to a shock to public capital for this alternative ordering of variables. The figure shows that, with a few exceptions, the results are qualitatively very similar to those obtained for the benchmark ordering of variables (see Figure 1). The main exceptions are Finland and New Zealand for which the impulse responses switch signs. Quantitatively, the impulse responses are in general somewhat smaller in absolute value than in the benchmark case. All in all, Figure 4 suggests that the output effects of public capital – which are the focus of interest – are not very sensitive to alternative orderings of the model variables.



Figure 4: Impulse responses of GDP to a shock to public capital

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: private capital, employment, GDP, public capital.



#### Figure 4 (continued): Impulse responses of GDP to a shock to public capital

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: private capital, employment, GDP, public capital.

# 5.2 Blanchard and Perotti's (2002) Identification Scheme: Evidence for the United States

This section presents results for an identification scheme due to Blanchard and Perotti (2002). Their identification scheme relies on detailed institutional information on tax and transfer systems. As this information is not readily available for most of the countries in our sample, we confine the analysis to the United States. Moreover, this identification scheme necessitates the use of quarterly data in order for the fiscal shocks to be identifiable. As capital stock estimates are not available at quarterly frequency, we need to use investment data instead. Thus, the results of this analysis are not directly comparable to those in Section 4. However, given the VAR model considered by Blanchard and Perotti (2002), it is possible to analyze whether the effects of spending shocks obtained for their identification scheme are similar to those obtained for a recursive scheme. If the results were similar, then this would be reassuring with respect to the benchmark analysis reported in Section 4.

Blanchard and Perotti (2002) estimate a three-dimensional VAR model for the United States for the sample period 1960:1 to 1997:4. The variables included in their VAR model are the logarithms of GDP, net taxes and government purchases of goods and services, all expressed in real per capita terms.<sup>101</sup> In this section, we depart from their setup by decomposing government purchases into public investment and public consumption.<sup>102</sup> The vector of variables can then be expressed as  $X_t = [i_t^G, c_t^G, \tau_t, y_t]$ , where  $i_t^G$  denotes public investment,  $c_t^G$  denotes public consumption,  $\tau_t$  denotes net taxes and  $y_t$  denotes GDP, all expressed in natural logarithms and real per capita terms. Estimation of our VAR model is based on the same sample period and the same lag order (4) as in Blanchard and Perotti (2002). The Johansen cointegration test suggests the presence of one cointegrating vector.<sup>103</sup> Accordingly, we estimate a vector error correction model imposing the reduced-rank restriction.<sup>104</sup>

<sup>&</sup>lt;sup>101</sup> Net taxes are defined as the sum of personal tax and non-tax receipts, corporate profits tax receipts, indirect business tax and non-tax accruals and contributions for social insurance, less net transfer payments to persons and net interest paid by government (see Blanchard and Perotti (2002: 1336)). I thank Roberto Perotti for kindly providing me with the data used in their study.

<sup>&</sup>lt;sup>102</sup> The public investment and public consumption series are taken from the Bureau of Economic Analysis' National Income and Product Accounts.

<sup>&</sup>lt;sup>103</sup> Detailed results are available upon request.

<sup>&</sup>lt;sup>104</sup> Blanchard and Perotti (2002) present results for two assumptions on the VAR process: (i) all variables are trend-stationary (VAR in levels), and, (ii) all variables are integrated of order one but not cointegrated (VAR in

Identification proceeds as follows: Adapting Blanchard and Perotti's (2002: 1333) starting point to our context, the relationship between the reduced-form disturbances  $\varepsilon_t$  and the structural disturbances  $e_t$  can be written as<sup>105</sup>

$$\varepsilon_t^{i^G} = b_{11}\varepsilon_t^{y} + b_{21}e_t^{\tau} + d_1e_t^{c^G} + e_t^{i^G}, \qquad (3.26)$$

$$\varepsilon_t^{c^G} = b_{12}\varepsilon_t^{y} + b_{22}e_t^{\tau} + d_2e_t^{i^G} + e_t^{c^G}, \qquad (3.27)$$

$$\varepsilon_t^{\tau} = a_1 \varepsilon_t^{y} + a_{21} e_t^{i^G} + a_{22} e_t^{c^G} + e_t^{\tau}, \qquad (3.28)$$

$$\varepsilon_t^{y} = c_1 \varepsilon_t^{\tau} + c_{21} \varepsilon_t^{i^G} + c_{22} \varepsilon_t^{c^G} + e_t^{y}.$$
(3.29)

Equation (3.26) states that unanticipated changes in government investment within a quarter can be due to (i) the response to unanticipated changes in GDP, captured by  $b_{11}\varepsilon_t^{\gamma}$ , (ii) the response to structural shocks to taxes, captured by  $b_{21}e_t^{\tau}$ , (iii) the response to structural shocks to government consumption, captured by  $d_1e_t^{c^G}$ , or, (iv) the response to structural shocks to government investment, captured by  $e_t^{i^G}$ . A similar interpretation can be given to Equations (3.27) and (3.28). The fourth equation states that unanticipated changes in GDP can be due to unanticipated changes in taxes or in either of the two government spending components or to other unanticipated shocks,  $e_t^{\gamma}$ .

The above system of equations contains twelve unknown parameters. Adding the four unknown variances of the structural disturbances, the structural model has sixteen unknown parameters altogether. As the covariance matrix of the reduced-form disturbances has only ten distinct elements, identification of the structural model requires at least six restrictions. The solution to the identification problem can be summarized as follows: (i) The parameters  $b_{11}$ ,  $b_{12}$  and  $a_1$  measure the cyclical sensitivity of government investment, government consumption and net taxes, respectively. Blanchard and Perotti (2002: 1334-1335) estimate these parameters in preliminary regressions. They find that net taxes are strongly pro-cyclical, the average value of their estimate of  $a_1$  being equal to 2.16 over the sample period. In

first differences). Given the results of Phillips (1998) on the consistency of impulse response estimates, we choose to explicitly account for cointegration.

<sup>&</sup>lt;sup>105</sup> The parameter notation follows Blanchard and Perotti (2002), adjusted for the decomposition of government purchases into its two components.

contrast, their results show that government purchases are a-cyclical, justifying to set  $b_{11} = b_{12} = 0$ . (ii) The second part of the identification problem concerns the relationship between taxes and government spending. Imagine we observe that the government increases taxes and spending at the same time. The question then is whether taxes respond to spending (i.e.,  $b_{21}, b_{22} = 0$ ,  $a_{21}, a_{22} \neq 0$ ) or vice versa. There is no easy answer to this question. Yet, as it turns out, the ordering of taxes and spending makes little difference to the impulse responses. In their analysis of the effects of spending shocks, Blanchard and Perotti (2002: 1345) assume that spending is ordered first. We follow this approach and set  $b_{21} = b_{22} = 0$ . (iii) The third part of the identification problem concerns the relationship between government consumption and government investment. There is no reason a priori to assume that one or the other spending component should be ordered first. Rather, one can argue that decisions on government consumption and government investment are contemporaneously independent from each other. In fact, setting both  $d_1$  and  $d_2$  equal to zero provides us with an overidentifying restriction whose validity can be explicitly tested for.

All in all, we have set seven restrictions implying that the structural model is overidentified. A likelihood ratio test shows that the validity of the overidentifying restriction cannot be rejected.<sup>106</sup> Recalling that structural models in general can be expressed in the form  $A_0\varepsilon_t = Be_t$ , we can write our system of equations as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2.16 \\ -c_{21} & -c_{22} & -c_{1} & 1 \end{bmatrix} \begin{vmatrix} \varepsilon_{t}^{i^{G}} \\ \varepsilon_{t}^{\tau} \\ \varepsilon_{t}^{y} \\ \varepsilon_{t}^{y} \end{vmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a_{21} & a_{22} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{vmatrix} \varepsilon_{t}^{i^{G}} \\ \varepsilon_{t}^{\sigma} \\ \varepsilon_{t}^{\gamma} \\ \varepsilon_{t}^{\gamma} \end{vmatrix},$$
(3.30)

where the identifying restrictions have been imposed. Comparing this system of equations with the one given by (3.25) that is characteristic for the recursive approach, reveals the following differences between the two approaches to identification: Whereas in the recursive approach all elements of  $A_0$  above the principal diagonal are restricted to be zero, there is one exception in Blanchard and Perotti's identification approach. This exception turns out to be

<sup>&</sup>lt;sup>106</sup> The test statistic is distributed  $\chi^2$  with one degree of freedom under the null hypothesis. The *p*-value of the test statistic is 0.97.

crucial when impulse responses to tax shocks are considered. The reason is that empirically the reduced-form residuals of taxes and output tend to be positively correlated. If taxes are ordered before output, the recursive approach attributes this correlation to the effect of taxes on output. In other words, results from this identification approach would suggest that an unanticipated increase in taxes leads to an increase in output. This is clearly a counterfactual prediction. Blanchard and Perotti's approach avoids this pitfall by attributing the positive correlation between taxes and output to automatic stabilizers, captured by the parameter  $a_1$ (estimated to be equal to 2.16 for the United States). This cyclical adjustment of taxes has the important implication that the estimate of the parameter  $c_1$ , capturing the effect of taxes on output, is negative. Other differences between the recursive approach and Blanchard and Perotti's approach are: (i) whereas in the recursive approach all parameters of  $A_0$  below the principal diagonal are freely varying, in the alternative approach this is the case only for the three parameters measuring the effect of (cyclically adjusted) government spending and taxes on output, and, (ii) whereas in the recursive approach B is restricted to an identity matrix, in the alternative approach there are two freely varying parameters in B capturing the effect of structural spending shocks on taxes. However, these two differences turn out to have little effect on the results.

This becomes clear from inspection of Figure 5, which displays the impulse responses of government investment, (net) taxes and output to a one-percent shock to government investment. The left-hand column of Figure 5 displays impulse responses for Blanchard and Perotti's identification approach, while the right-hand column displays impulse responses for the recursive approach. As can be seen the results for the two identification approaches are virtually identical.<sup>107</sup> In both cases, government investment, taxes and output exhibit a positive response to a shock to government investment. In the long run, output and net taxes increase by around 0.1 percent. Given that over the sample period the average share of government investment in output was roughly 4 percent in the United States, the point estimates suggest that an extra dollar of government investment leads to an output increase of roughly 2.5 dollars.<sup>108</sup> Moreover, an increase in government investment by one dollar

<sup>107</sup> Note, however, that this is only true for the two spending shocks. For the tax shock, not shown here, the results differ substantially, as suggested by the discussion above.

<sup>&</sup>lt;sup>108</sup> Yet, the output response is statistically significant only in the first year following the shock.



#### Figure 5: Impulse responses of GDP to a shock to public investment for the United States

*Notes*: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in the respective variable in response to a one-percent shock to public investment for a horizon of 50 quarters. The dotted lines represent 68% bootstrap error bands. The left-hand column displays impulse responses obtained for an identification scheme corresponding to the one used by Blanchard and Perotti (2002). The right-hand column displays impulses responses obtained for a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public investment, public consumption, net taxes, GDP.

generates around 0.5 dollars of additional net taxes. Thus, the point estimates suggest that roughly one half of additional spending is financed by taxes, implying that parts of the increase in government investment are deficit financed.<sup>109</sup> Note, however, that the response of net taxes is estimated quite imprecisely as indicated by the relatively large confidence interval. As a consequence, we can neither reject the hypothesis that additional government investment is entirely tax financed nor the hypothesis that it is entirely deficit financed. All in all, the results shown in Figure 5 confirm the results for the United States reported in Section 4.2: While there is some evidence for positive output effects of public investment (public capital) for the United States, the confidence intervals include zero for large parts of the response horizon so that a zero output effect of additional public capital cannot be excluded for this particular country. Finally, the results of this section are reassuring in that for the analysis of the effects of government spending shocks the recursive approach to identification indeed seems to be appropriate.

# 6 Conclusion

This paper has provided new evidence on the dynamic effects of public capital in OECD countries based on the VAR methodology. In contrast to the production function approach routinely applied in the literature, this methodology treats all variables as endogenous and, thus, allows for feedback effects from output to the three input variables. Moreover, application of the Johansen (1988,1991) method has shown that it is important to account for the possibility of multiple cointegrating vectors among the model variables. The main results of the analysis can be summarized as follows:

- 1. For the majority of countries in our sample, shocks to public capital tend to have significant positive output effects.
- 2. In contrast to the results documented in the literaure for the production function approach, there is little evidence for "supernormal" returns to public capital. The results presented in this paper suggest that one reason for the extremely high returns to public capital obtained for some countries for the production function approach might be that the latter approach ignores feedback effects from output to public capital.

<sup>109</sup> The response of government consumption – not plotted in Figure 5 – to a shock to government investment is positive over the response horizon.

3. For the vast majority of countries in our sample, public capital and private capital are long-run complements. As regards the short-run relation between these variables, two groups of countries can be distinguished: (i) one group for which public capital and private capital are short-run substitutes, i.e., private capital declines in response to a shock to public capital, and, (ii) a second group for which public capital and private capital are short-run complements.

For the vast majority of countries, the response of employment to a shock to public capital is statistically insignificant. In other words, there is little evidence for the hypothesis that employment can be fostered by additional public capital.

# 7 References

- Aarle, B. van, and H. Garretsen (2003). Keynesian, Non-Keynesian or No Effects of Fiscal Policy Changes? The EMU Case. *Journal of Macroeconomics* 25 (2): 213-240.
- Akaike, H. (1974). A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control* 19 (6): 716-723.
- Amisano, G., and C. Giannini (1997). *Topics in Structural VAR Econometrics*. 2<sup>nd</sup> Edition. Berlin: Springer.
- Aschauer, D.A. (1989). Is Public Expenditure Productive? *Journal of Monetary Economics* 23 (2): 177-200.
- Barro, R.J. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy* 82 (6): 1095-1117.
- Batina, R.G. (1998). On the Long Run Effects of Public Capital and Disaggregated Public Capital on Aggregate Output. *International Tax and Public Finance* 5 (3): 263-281.
- Baxter, M., and R.G. King (1993). Fiscal Policy in General Equilibrium. *American Economic Review* 83 (3): 315-333.
- Blanchard, O.J., and R. Perotti (2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. *Quarterly Journal of Economics* 117 (4): 1329-1368.
- Blanchard, O.J., and D. Quah (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances. *American Economic Review* 79 (4): 655-673.
- Breitung, J. (2000). *Structural Inference in Cointegrated Vector Autoregressive Models*. XploRe e-book. Online source (access on July 13, 2003): http://www.quantlet.com/ mdstat/scripts/bre/pdf/brepdf.pdf.
- Burnside, C., M. Eichenbaum, and J.D.M. Fisher (2003). Fiscal Shocks and Their Consequences. *Journal of Economic Theory* (forthcoming). Also available as: NBER Working Paper 9772. National Bureau of Economic Research, Cambridge, MA.
- Cheung, Y.-W., and K.S. Lai (1993). Finite-Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration. *Oxford Bulletin of Economics and Statistics* 55 (3): 313-328.
- Christiano, L.J., M. Eichenbaum, and C.L. Evans (1999). Monetary Policy Shocks: What Have We Learned and to What End? In J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*. Vol. 1A. Amsterdam: Elsevier.
- Crowder, W.J., and D. Himarios (1997). Balanced Growth and Public Capital: An Empirical Analysis. *Applied Economics* 29 (8): 1045-1053.
- Cullison, W.E. (1993). Public Investment and Economic Growth. *Federal Reserve Bank of Richmond Economic Quarterly* 79 (4): 19-33.
- De Haan, J., J.-E. Sturm, and B.J. Sikken (1996). Government Capital Formation: Explaining the Decline. *Weltwirtschaftliches Archiv* 132 (1): 55-74.
- Doornik, J.A. (1996). Testing Vector Error Autocorrelation and Heteroscedasticity. Nuffield College, Oxford. Online Source (Access on November 1, 2003): http://www.nuff.ox.ac.uk/Users/Doornik/papers/vectest.pdf.

- Dufour, J.-M., and E. Renault (1998). Short Run and Long Run Causality in Time Series: Theory. *Econometrica* 66 (5): 1099-1125.
- Edelberg, W., M. Eichenbaum, and J.D.M. Fisher (1999). Understanding the Effects of a Shock to Government Purchases. *Review of Economic Dynamics* 2 (1): 166-206.
- Eichenbaum, M. (1998). Comment on "V.A. Ramey and M.D. Shapiro, Costly Capital Reallocation and the Effects of Government Spending". *Carnegie-Rochester Conference Series on Public Policy* 48 (June): 195-209.
- Engle, R.F., and C.W.J. Granger (1987). Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica* 55 (2): 251-276.
- Everaert, G. (2003). Balanced Growth and Public Capital: An Empirical Analysis with I(2) Trends in Capital Stock Data. *Economic Modelling* 20 (4): 741-763.
- Favero, C. (2001). Applied Macroeconometrics. Oxford: Oxford University Press.
- Flores de Frutos, R., M. Gracia-Diez, and T. Perez-Amaral (1998). Public Capital Stock and Economic Growth: An Analysis of the Spanish Economy. *Applied Economics* 30 (8): 985-994.
- Franses, P.H. (2001). How to Deal with Intercept and Trend in Practical Cointegration Analysis? *Applied Economics* 33 (5): 577-579.
- Giavazzi, F., T. Jappelli, and M. Pagano (2000). Searching for Non-linear Effects of Fiscal Policy: Evidence from Industrial and Developing Countries. *European Economic Review* 44 (7): 1259-1289.
- Granger, C.W.J. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. *Econometrica* 37 (3): 424-438.
- Hall, P. (1994). Methodology and Theory for the Bootstrap. In R.F. Engle and D.L. McFadden (eds.), *Handbook of Econometrics*. Vol. 4. Amsterdam: Elsevier.
- Hamilton, J.D. (1994). Time Series Analysis. Princeton, NJ: Princeton University Press.
- Hannan, E.J., and B.G. Quinn (1979). Determination of the Order of an Autoregression. *Journal of the Royal Statistical Society, Series* B, 41: 190-195.
- Hansen, H., and K. Juselius (1995). *CATS in RATS: Cointegration Analysis of Time Series*. Evanston, IL: Estima.
- Hendry, D.F., and K. Juselius (2000). Explaining Cointegration Analysis: Part I. *Energy* Journal 21 (1): 1-42.
- Hendry, D.F., and K. Juselius (2001). Explaining Cointegration Analysis: Part II. *Energy Journal* 22 (1): 75-120.
- Höppner, F. (2003). Business Cycle Effects of Fiscal Policy Empirical Evidence from Germany. Berlin: Dissertation.de.
- Horowitz, J.L. (2001). The Bootstrap. In J.J. Heckman and E. Leamer (eds.), *Handbook of Econometrics*. Vol. 5. Amsterdam: Elsevier.
- Hubrich, K., H. Lütkepohl, and P. Saikkonen (2001). A Review of Systems Cointegration Tests. *Econometrics Reviews* 20 (3): 247-318.
- Johansen, S. (1988). Statistical Analysis of Cointegration Vectors. *Journal of Economic Dynamics and Control* 12 (2-3): 231-254.

- Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica* 59 (6): 1551-1580.
- Johansen, S. (1994). The Role of the Constant and Linear Terms in Cointegration Analysis of Nonstationary Variables. *Econometric Reviews* 13 (2): 205-229.
- Johansen, S. (1995). Likelihood-Based Inference in Cointegrated Vector Autoregressive Models. Oxford: Oxford University Press.
- Johansen, S. (2000). Modelling of Cointegration in the Vector Autoregressive Model. *Economic Modelling* 17 (3): 359-373.
- Juselius, K. (2003). *The Cointegrated VAR Model: Econometric Methodology and Macroeconomic Applications*. Institute of Economics, University of Copenhagen. Online source (access on September 25, 2003): http://www.econ.ku.dk/okokj/
- Kamps, C. (2001). Fiscal Consolidation in Europe: Pre- and Post-Maastricht. Kiel Working Paper 1028. Institute for World Economics, Kiel.
- Kamps, C. (2004). New Estimates of Government Net Capital Stocks for 22 OECD Countries 1960-2001. IMF Working Paper 04/67. International Monetary Fund, Washington, DC.
- Kilian, L. (1998). Small-Sample Confidence Intervals for Impulse Response Functions. *Review of Economics and Statistics* 80 (2): 218-230.
- King, R.G., and S.T. Rebelo (1999). Resuscitating Real Business Cycles. In J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*. Volume 1B. Amsterdam: Elsevier.
- Kuttner, K.N., and A.S. Posen (2002). Fiscal Policy Effectiveness in Japan. *Journal of the Japanese and International Economies* 16 (4): 536-558.
- Ligthart, J.E. (2002). Public Capital and Output Growth in Portugal: An Empirical Analysis. *European Review of Economics and Finance* 1 (2): 3-30.
- Lütkepohl, H. (1990). Asymptotic Distributions of Impulse Response Functions and Forecast Error Variance Decompositions of Vector Autoregressive Models. *Review of Economics and Statistics* 72 (1): 116-125.
- Lütkepohl, H. (1991). Introduction to Multiple Time Series Analysis. Berlin: Springer.
- Lütkepohl, H. (2001). Vector Autoregressions. In B.H. Baltagi (ed.), A Companion to Theoretical Econometrics. Oxford: Blackwell.
- Lütkepohl, H., and J. Breitung (1997). Impulse Response Analysis of Vector Autoregressive Processes. In C. Heij, H. Schumacher, B. Hanzon and K. Praagman (eds.), *System Dynamics in Economic and Financial Models*. Chichester: Wiley.
- Lütkepohl, H., and H.-E. Reimers (1992). Impulse Response Analysis of Cointegrated Systems. *Journal of Economic Dynamics and Control* 16 (1): 53-78.
- King, R.G., C.I. Plosser, J.H. Stock, and M.W. Watson (1991). Stochastic Trends and Economic Fluctuations. *American Economic Review* 81 (4): 819-840.
- MacKinnon, J.G., Haug, A.A., and L. Michelis (1999). Numerical Distribution Functions of Likelihood Ratio Tests for Cointegration. *Journal of Applied Econometrics* 14 (5): 563-577.

- Mamatzakis, E.C. (1999). Testing for Long Run Relationship Between Infrastructure and Private Capital Productivity: A Time Series Analysis for the Greek Industry. *Applied Economics Letters* 6 (4): 243-246.
- McMillin, W.D., and D.J. Smyth (1994). A Multivariate Time Series Analysis of the United States Aggregate Production Function. *Empirical Economics* 19 (4): 659-673.
- Mittnik, S., and T. Neumann (2001). Dynamic Effects of Public Investment: Vector Autoregressive Evidence from Six Industrialized Countries. *Empirical Economics* 26 (2): 429-446.
- Mountford, A., and H. Uhlig (2002). What Are the Effects of Fiscal Policy Shocks? CEPR Discussion Paper 3338. Center for Economic Policy Research, London.
- Osterwald-Lenum, M. (1992). A Note with the Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics. *Oxford Bulletin of Economics and Statistics* 54 (3): 461-471.
- Otto, G.D., and G.M. Voss (1996). Public Capital and Private Production in Australia. *Southern Economic Journal* 62 (3): 723-738.
- Pereira, A.M. (2000). Is All Public Capital Created Equal? *Review of Economics and Statistics* 82 (3): 513-518.
- Pereira, A.M. (2001a). On the Effects of Public Investment on Private Investment: What Crowds in What? *Public Finance Review* 29 (1): 3-25.
- Pereira, A.M. (2001b). Public Investment and Private Sector Performance International Evidence. *Public Finance & Management* 1 (2): 261-277.
- Pereira, A.M., and J.M. Andraz (2003). On the Impact of Public Investment on the Performance of U.S. Industries. *Public Finance Review* 31 (1): 66-90.
- Pereira, A.M., and R. Flores de Frutos (1999). Public Capital Accumulation and Private Sector Performance. *Journal of Urban Economics* 46 (2): 300-322.
- Pereira, A.M., and O. Roca Sagales (1999). Public Capital Formation and Regional Development in Spain. *Review of Development Economics* 3 (3): 281-294.
- Pereira, A.M., and O. Roca Sagales (2001). Infrastructures and Private Sector Performance in Spain. *Journal of Policy Modeling* 23 (4): 371-384.
- Pereira, A.M., and O. Roca Sagales (2003). Spillover Effects of Public Capital Formation: Evidence from the Spanish Regions. *Journal of Urban Economics* 53 (2): 238-256.
- Perotti, R. (2001). What Do We Know About the Effects of Fiscal Policy? In M. Bordignon and D. Da Empoli (eds.), *Politica fiscale, flessibilità dei mercati e crescita*. Milano: Angeli.
- Perotti, R. (2002). Estimating the Effects of Fiscal Policy in OECD Countries. ECB Working Paper 168. European Central Bank, Frankfurt am Main.
- Pesaran, M.H., Y. Shin, and R.P. Smith (2000). Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables. *Journal of Econometrics* 97 (2): 293-343.
- Pesaran, M.H., and R.P. Smith (1998). Structural Analysis of Cointegrating VARs. *Journal of Economic Surveys* 12 (5): 471-505.

- Phillips, P.C.B. (1998). Impulse Response and Forecast Error Variance Asymptotics in Nonstationary VARs. *Journal of Econometrics* 83 (1-2): 21-56.
- Ramey, V.A. and M.D. Shapiro (1998). Costly Capital Reallocation and the Effects of Government Spending. *Carnegie-Rochester Conference Series on Public Policy* 48 (June): 145-194.
- Runkle, D.E. (1987). Vector Autoregressions and Reality. *Journal of Business and Economic Statistics* 5 (4): 437-442.
- Schwarz, G. (1978). Estimating the Dimension of a Model. Annals of Statistics 6 (2): 461-464.
- Shapiro, M.D., and M.W. Watson (1988). Sources of Business Cycle Fluctuations. In S. Fischer (ed.), *NBER Macroeconomics Annual*. Cambridge, MA: MIT Press.
- Sims, C.A. (1980). Macroeconomics and Reality. *Econometrica* 48 (1): 1-48.
- Sims, C.A. (1987). Comment on "D.E. Runkle, Vector Autoregressions and Reality". *Journal* of Business and Economic Statistics 5 (4): 443-449.
- Sims, C.A., J.H. Stock, and M.W. Watson (1990). Inference in Linear Time Series Models with some Unit Roots. *Econometrica* 58 (1): 113-144.
- Sims, C.A., and T. Zha (1999). Error Bands for Impulse Responses. *Econometrica* 67 (5): 1113-1155.
- Stock, J.H., and M.W. Watson (1999). Business Cycle Fluctuations in US Macroeconomic Time Series. In J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*. Vol. 1A. Amsterdam: Elsevier.
- Stock, J.H., and M.W. Watson (2001). Vector Autoregressions. *Journal of Economic Perspectives* 15 (4): 101-115.
- Sturm, J.-E., and J. de Haan (1995). Is Public Expenditure Really Productive? New Evidence for the USA and the Netherlands. *Economic Modelling* 12 (1): 60-72.
- Sturm, J.-E., J. de Haan, and G.H. Kuper (1998a). Modelling Government Investment and Economic Growth on a Macro Level: A Review. In S. Brakman, H. van Ees, and S.K. Kuipers (eds.), *Market Behaviour and Macroeconomic Modelling*. London: Macmillan Press.
- Sturm, J.-E., J. Jacobs, and P. Groote (1999). Output Effects of Infrastructure Investment in the Netherlands 1853-1913. *Journal of Macroeconomics* 21 (2): 355-380.
- Runkle, D.E. (1987). Vector Autoregressions and Reality. *Journal of Business & Economic Statistics* 5 (4): 437-442.
- Uhlig, H. (2001). What Are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure. Institute of Economic Policy, Humboldt University, Berlin. Online source (access on July 15, 2003): http://www.wiwi.huberlin.de/wpol/papers/neutral15.pdf.
- Voss, G.M. (2002). Public and Private Investment in the United States and Canada. *Economic Modelling* 19 (4): 641-664.
- White, H. (1980). A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity. *Econometrica* 48 (4): 817-838.