CSE 2320 Name ____________________

Test 1

Spring 2014 Last 4 Digits of Student ID # ____________

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Suppose $f(x)$ is a monotonically increasing function. Which of the following approximates the summation?

   A. $\int_{m-1}^{n-1} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m+1}^{n+1} f(x)dx$
   B. $\int_{m-1}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n+1} f(x)dx$
   C. $\int_{m}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n} f(x)dx$
   D. $\int_{m}^{n+1} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m-1}^{n} f(x)dx$

2. Which of the following is solved heuristically by a greedy method?
   A. Fractional knapsack
   B. Finding the shortest paths from a designated source vertex in a sparse graph.
   C. Minimum spanning tree
   D. 0/1 knapsack

3. Which of the following is not true regarding dynamic programming?
   A. It is a form of exhaustive search
   B. It is a form of divide-and-conquer
   C. A cost function must be defined
   D. The backtrace may be based on recomputing the cost function

4. What is the definition of H_n?

   A. $\Theta\left(\sqrt{n}\right)$ B. $\sum_{k=1}^{n} k$ C. $\ln n$ D. $\sum_{k=1}^{n} \frac{1}{k}$

5. Which of the following functions is not in $\Omega(n)$?

   A. $\lg n$ B. $n \lg n$ C. $n^2 \lg n$ D. n^3

6. Bottom-up heap construction is based on applying `fixDown` in the following fashion:
   A. In ascending slot number order, for each slot that is a parent.
   B. In descending slot number order, for each slot that is a parent.
   C. $\frac{n}{2}$ times, each time from subscript 1.
   D. n - 1 times, each time from subscript 1.

7. Suppose that a binary search is to be performed on a table with 100 elements. The maximum number of elements that could be examined (probes) is:

   A. 4 B. 5 C. 6 D. 7

8. The number of calls to `PQdelmin` to build a Huffman code tree for n symbols is:

   A. $\Theta(\log n)$ B. n - 1 C. n D. $2n$ - 2

9. The recursion tree for mergesort has which property?
   A. each level has the same contribution
   B. it leads to a definite geometric sum
   C. it leads to a harmonic sum
   D. it leads to an indefinite geometric sum

10. What is the value of $\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k$?

    A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 2

11. What is indicated when `find(i)==find(j)` while maintaining disjoint subsets?
    A. `i` and `j` are leaders for the same subset
    B. `i` is the ancestor of `j` in one of the trees
    C. `i` and `j` are in the same subset
    D. `i` and `j` are leaders for different subsets

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in \mathrm{O}(n^2)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in \mathrm{O}(\log n)$ B. $T(n) \in \mathrm{O}(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

13. Which of the following is not true regarding a minheap with 1000 elements?
    A. Subscript 1 will store the maximum priority.
    B. The parent for the node with subscript 500 is stored at subscript 250.
    C. The left child for the node with subscript 200 is stored at subscript 400.
    D. The right child for the node with subscript 405 is stored at subscript 811.
14. The time to run the code below is in:

```
for (i=n; i>=0; i--)
  for (j=0; j<n; j+=2)
    sum+=i+j;
```

    A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

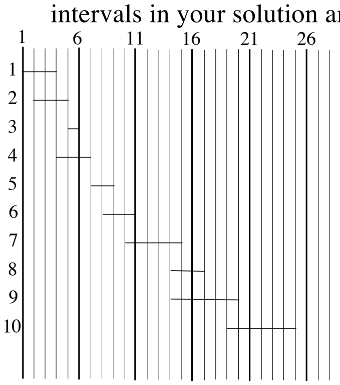
15. The time to run the code below is in:

```
sum=1;
for (i=1; i<n; i=3*i)
  sum++;
```

    A. $\Theta(n^2)$ B. $\Theta(\log n)$ C. $\Theta(\sqrt{n})$ D. $\Theta(n)$

Short Answer:
1. Explain what it means for a sort to be stable. (5 points)

Long Answer
1. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points

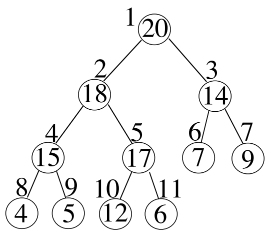


	v_i	p_i	$m(i)$
1	1	0	
2	5	0	
3	3	2	
4	8	1	
5	1	4	
6	2	4	
7	3	5	
8	1	6	
9	4	6	
10	1	8	

2. Use the substitution method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n$ is in $\Theta(n \log n)$. 10 points
3. Use the recursion-tree method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n$ is in $\Theta(n \log n)$. 10 points
4. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
   p[0]=5       p[1]=4       p[2]=3       p[3]=4       p[4]=5       p[5]=6
          1            2            3            4            5
1      0     0    60     1   120     2   195     2   ???     ?
2   -------        0     0    48     2   120     2   222     2
3   -------    -------         0     0    60     3   150     4
4   -------    -------    -------          0     0   120     4
5   -------    -------    -------    -------           0     0
```

5. Show the maxheap after performing `PQinsert` for a priority of 22, followed by a second `PQinsert` for a priority of 21. 10 points



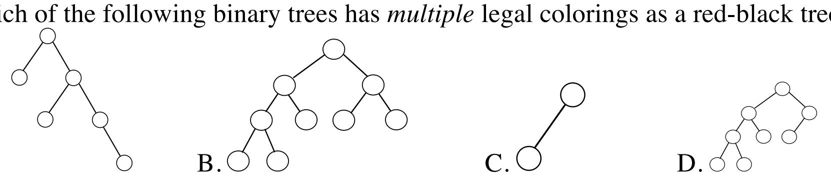
CSE 2320 Name ______________________________

Test 2

Spring 2014

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which version of rat-in-a-maze finds the shortest path?
   A. queue B. recursive C. red-black tree D. stack
2. How should the successor of a leaf in an unbalanced binary search tree be found?
   A. Examine the ancestors B. Go right, then proceed to the left
   C. Inorder traversal D. Preorder traversal
3. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:
   A. return stack[SP++] B. stack[SP++] = X C. stack[--SP] = X D. stack[++SP] = X
4. What is the worst-case time to perform MAXIMUM(L) for a circular, sorted, doubly-linked list with n nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
5. For which of the following sorts does the decision tree model not apply?
   A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT
6. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(h)$
7. Based on dictionary search performance alone, the best justification for ordering a linked list is:
   A. Many more misses than hits are expected
   B. Sentinels are more effective in speeding up search
   C. Many more hits than misses are expected
   D. Less storage will be needed
8. The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:
   A. to assure that the final solution is free of duplicate values
   B. to determine the longest possible increasing subsequence terminated by a particular input value
   C. to search a table that will contain only the LIS elements at termination
   D. to sort the original input
9. An unsorted integer array with 1500 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?
   A. If 1470 is returned from PARTITION, we are done.
   B. If 1481 is returned from PARTITION, we must continue.
   C. If 1468 is returned from PARTITION, we must continue.
   D. If 1469 is returned from PARTITION, we must continue.
10. Which of the following is a longest common subsequence for 0 1 2 0 1 2 and 0 0 1 2 1 2?
    A. 0 0 1 1 B. 0 1 2 1 2 C. 0 0 1 2 D. 0 1 2 0
11. Which of the following binary trees has *multiple* legal colorings as a red-black tree?



A. B. C. D.

12. The expected number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?
    A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$
13. Which of the following will not be true regarding the decision tree for MERGESORT for sorting n input values?
    A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
    B. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.

C. The height of the tree is $\Omega(n \log n)$.

D. There will be n! leaves.

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
if (x==null)
  return;
traverse(x.left);
traverse(x.right);
// process x here
}
```

A. inorder B. postorder C. preorder D. search for key `x`

15. Recently, we considered an abstraction supporting the operations *allocate*, *allocateAny*, and *freeup* which was implemented in constant time. Which of the following was not a feature of the implementation?

A. a header B. a recycling list C. a circular, doubly-linked list D. arrays

Long Answer

1. Give the unbalanced binary search tree that results when the keys 50, 80, 70, 60, 90, 100, 80, 120 are inserted, in the given order, into an initially empty tree. (5 points)
2. Twenty million positive integers in the range 0 . . . 99,999,999 are to be sorted by LSD radix sort. Compare the performance for using radix 0 . . . 9999 and radix 0 . . . 9. Show your work. (10 points)
3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

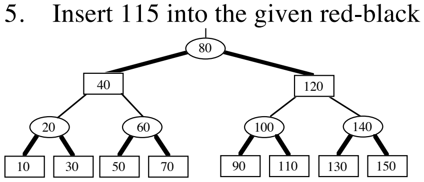
8 2 5 3 6 1 9 0 7 4

Version: 1 2/Sedgewick

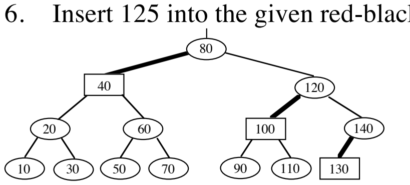
4. Use dynamic programming to solve the following instance of the strictly longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. (No points for solving by inspection.) (10 points)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	10	15	20	25	7	10	15	22	25	5	10	22	26	27

5. Insert 115 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



6. Insert 125 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



CSE 2320 Name ________________________________

Test 3

Spring 2014

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Suppose a maximum bipartite matching with k edges is found using Edmonds-Karp. Which of the following does not hold?

A. The capacity of the minimum cut is k. B. There will be $k + 1$ breadth-first searches.
C. All residual network capacities are zero or one. D. Every augmenting path uses three edges.

2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:

A. Back B. Cross C. Forward D. Tree

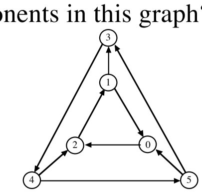
3. During a breadth-first search, the status of a gray vertex is:

A. It has been completely processed. B. It is in the FIFO queue.
C. It is in the priority queue. D. It is undiscovered.

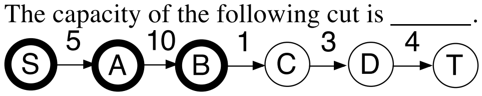
4. The fastest method for finding the diameter of a tree is to:

A. Use the Floyd-Warshall algorithm. B. Use the Ford-Fulkerson algorithm.
C. Use breadth-first search. D. Use Dijkstra's algorithm.

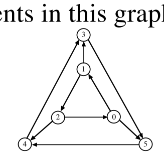
5. The capacity of any cut is:
   A. An upper bound on the maximum flow.
   B. The same as the capacity of all other cuts.
   C. The same as the maximum attainable flow.
   D. A lower bound on the maximum flow.
6. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:
   A. finish(X) < finish(Y)
   B. finish(X) > finish(Y)
   C. finish(X) = finish(Y)
   D. could be either A. or B.
7. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:
   A. They are equal.
   B. The amount of flow does not exceed the net flow.
   C. The net flow does not exceed the amount of flow.
   D. There is no relationship.
8. What is the number of strongly connected components in this graph?



   A. 1 B. 2 C. 3 D. 4
9. The capacity of the following cut is ______. (S vertices are bold.)

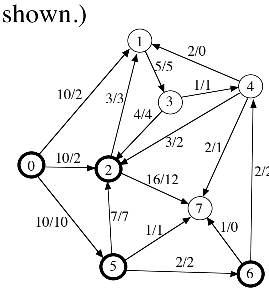


   A. 1 B. 10 C. 16 D. 23
10. Which edge is chosen in a phase of Kruskal's algorithm?
    A. An edge of maximum-weight in a cycle (to be excluded)
    B. An edge that is on a shortest path from the source
    C. The unprocessed edge (x, y) of smallest weight such that `find(x)!=find(y)`
    D. The unprocessed edge (x, y) of smallest weight such that `find(x)==find(y)`
11. The Edmonds-Karp variant is important because:
    A. It solves the bipartite matching problem.
    B. It solves the network flow problem in polynomial time.
    C. It solves the network flow problem using critical edges.
    D. It solves the network flow problem without using augmenting paths.
12. Which of the following cannot occur when additional edges are included in a directed graph?
    A. The number of strong components may decrease.
    B. The number of strong components may increase.
    C. The graph acquires a cycle.
    D. The number of strong components may remain the same.
13. The number of potential probe sequences when using linear probing with a table with m entries is:
    A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$
14. What is the number of strongly connected components in this graph?



    A. 1 B. 2 C. 3 D. 4
15. The worst-case time for Prim's algorithm implemented with a minheap is:
    A. θ(V + E) B. θ(E lg V) C. θ(V lg V) D. θ(V lg E)

Problems 16, 17, and 18 refer to the following network. 0 is the source. 7 is the sink. Each edge is labeled with capacity/flow. (Additional edges for the residual network are not shown.)



16. The net flow across the given cut (S vertices are bold) is:
    A. 14 B. 16 C. 18 D. 20
17. The capacity of the indicated cut is:
    A. 31 B. 32 C. 33 D. 34
18. Suppose the flow is increased as much as possible using the augmenting path $0 \rightarrow 1 \rightarrow 2 \rightarrow 7$. Which is the critical edge?
    A. $0 \rightarrow 1$ B. $1 \rightarrow 2$ C. $2 \rightarrow 7$ D. Insufficient information
19. An adjacency matrix is the most useful representation for which problem?
    A. Breadth-first search B. Finding strongly-connected components
    C. Maximum network flow D. Warshall's algorithm
20. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:
    A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. α D. 2α

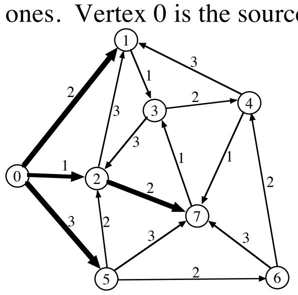
Long Answer

1. Consider the following hash table whose keys were stored by double hashing using
h_1(key) = key % 17 and h_2(key) = 1 + (key % 16).

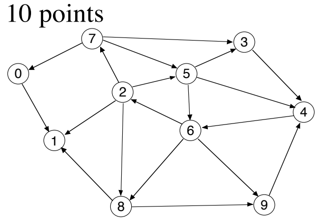
```
0   -1
1   800
2   -1
3   -1
4   701
5   -1
6   601
7   -1
8   501
9   -1
10  401
11  -1
12  301
13  -1
14  201
15  -1
16  101
```

a. Suppose 1002 is to be inserted (using double hashing). Which slot will be used? (5 points)
b. Suppose 1003 is to be inserted (using double hashing) *after* 1002 has been stored. Which slot will be used? (5 points)

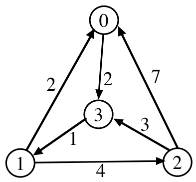
2. What are the entries in the heap (for Dijkstra's algorithm) ***before*** and ***after*** moving the next vertex and edge into the shortest path tree? DO NOT COMPLETE THE ENTIRE TREE!!! Edges already in the shortest path tree are the thick ones. Vertex 0 is the source. 10 points.



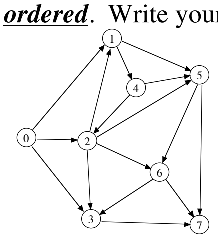
3. Demonstrate, for the graph below, the algorithm that uses ***two*** depth-first searches to determine the strongly-connected components. 10 points



4. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

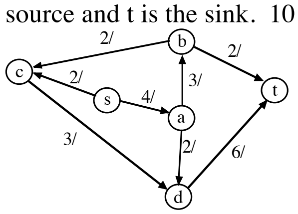


5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ***ordered***. Write your answer in the tables below. 10 points



Vertex	Start	Finish	Edge	Type	Edge	Type
0	1	____	0 1	____	2 6	____
1	____	____	0 2	____	3 7	____
2	____	____	0 3	____	4 2	____
3	____	____	1 4	____	4 5	____
4	____	____	1 5	____	5 6	____
5	____	____	2 1	____	5 7	____
6	____	____	2 3	____	6 3	____
7	____	____	2 5	____	6 7	____

6. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. s is the source and t is the sink. 10 points.



Minimum Cut:

S vertices: s

T vertices: t

Augmenting Paths and Contribution to Flow:

Extra Credit: Fill in the KMP failure links. 10 points

	1	pattern	2
0		a	
1		b	
2		c	
3		d	
4		a	
5		b	
6		c	
7		a	
8		b	
9		c	