

# Theoretical Ground Reflection Model for UWB Communication Systems

Sathaporn Promwong<sup>1,2</sup>, Jun-ichi Takada<sup>1</sup>, Pichaya Supanakoon<sup>2</sup> and Prakit Tangtisanon<sup>2</sup> <sup>1</sup>Graduate School of Science and Engineering, Tokyo Institute of Technology

O-okayama Minami 6 Bldg., 2-12-1, O-okayama, Meguro-ku, 152-8550, Tokyo, Japan.

E-mail: ken@ap.ide.titech.ac.jp

<sup>2</sup>Department of Information Engineering, Faculty of Engineering,

King Mongkut's Institute of Technology Ladkrabang, Chalongkrung Road, Ladkrabang, Bangkok 10520, Thailand. E-mail:kspichay@kmitl.ac.th

Abstract-In this paper, the theoretical ground reflection model for UWB communication systems is proposed. The passband rectangular pulse with specific center frequency and bandwidth is used as the transmitted signal. Complex form Friis' transmission formula is applied for the UWB ground reflection channel. The received signal is evaluated. The UWB path loss is defined as the ratio between the maximum amplitude of the transmitted and received signal waveforms. The closed form path loss expression is derived. For the data processing, the path loss and rms delay spread of ground reflection model are considered for the 1.00, 0.10 and 0.01 m antenna heights. The double linear regression for the path loss is modeled by using the first Fresnel zone and MMSE break points. The rms delay spread as the function of the antenna height and the path loss is investigated. The characteristics of the path loss and rms delay spread are discussed in the paper.

#### I. INTRODUCTION

Recently, ultra wideband (UWB) radio technology has become an important topic for microwave communication. UWB is different from other radio wave (RF) technology. Instead of using a narrow carrier frequency, UWB transmits pulses of power in the range of the ultra wide frequency spectrum. The Federal Communication Commission (FCC) [1], in US specifies that UWB has a frequency spectrum ranging from 3.1 to 10.6 GHz. The FCC defined UWB signal as those which have a fractional bandwidth greater than 0.20 or a bandwidth greater than 500 MHz measured at -10 dB points. The power density of UWB signal is considered to be noise for other communication systems because its power spectrum is below the noise level. The UWB receiver collects the power of the received signal to rebuild the pulse. Therefore, UWB radio technology can exist with other RF technology without interference. UWB radio technology is an ideal candidate that can be utilized for commercial, short-range, low power, low cost indoor communication systems such as wireless personal area networks (WPAN) [2]-[4].

Friis' transmission formula [5] is widely used to calculate the free space path loss for narrow band systems. Complex form Friis' transmission formula and the use of matched filter are developed for UWB systems [6]-[7]. In the closed form expressions, the UWB path loss and matched filter gain for free space channel are derived [8]. In this paper, the theoretical ground reflection model for UWB communication systems is proposed. The passband rectangular pulse with specific center frequency and bandwidth is used as the transmitted signal. Complex form Friis' transmission formula is applied for the UWB ground reflection channel. The received signal is evaluate. The UWB path loss is defined as the ratio between the maximum amplitude of the transmitted and received signal waveforms. The closed form path loss expression is derived. For the data processing, the path loss and rms delay spread of ground reflection model are considered for the 1.00, 0.10 and 0.01 m antenna heights. The double linear regression for the path loss is modeled by the first Fresnel zone and MMSE break points. The rms delay spread as a function of the antenna height and the path loss is investigated. The characteristics of the path loss and rms delay spread are discussed in the paper.

# II. THEORETICAL GROUND REFLECTION MODEL

The ground reflection (2-ray) model considers of the direct and ground reflection paths. The ground reflection model is shown in Fig. 1. The distances of the direct, d' and ground reflection d'' paths are function of the transmitter-receiver (T-R) separation distance d, transmitter (Tx) and receiver (Rx) antenna heights,  $h_t$  and  $h_r$ , and can be written as [9]

$$d' = \sqrt{(h_{\rm t} - h_{\rm r})^2 + d^2},$$
 (1)

$$d'' = \sqrt{(h_{\rm t} + h_{\rm r})^2 + d^2}.$$
 (2)

The frequency transfer function of the direct  $H_d$  and ground reflection  $H_r$  paths are considered in the same way as the free space [8] and can be expressed as

j

$$H_{\rm d}(f, d, h_{\rm t}, h_{\rm r}) = \frac{1}{4\pi |f| t'} \exp(-j2\pi f t'),$$
 (3)

$$H_{\rm r}(f, d, h_{\rm t}, h_{\rm r}) = \Gamma \frac{1}{4\pi |f| t''} \exp(-j2\pi f t''), \quad (4)$$



Fig. 1. Ground reflection model.

where  $\Gamma$  is the ground reflection coefficient, t' = d'/c and t'' = d''/c are the delayed times of the direct and ground reflection paths, respectively, and c is the velocity of the light.

Then, the frequency transfer function of the ground reflection model  $H_{2ray}$  is the combination of the frequency transfer functions of the direct and ground reflection paths,

$$H_{2ray}(f, d, h_{t}, h_{r}) = H_{d}(f, d, h_{t}, h_{r}) + H_{r}(f, d, h_{t}, h_{r}),$$
  
$$= \frac{1}{4\pi |f| t_{2ray}} \exp(-j\theta_{2ray}),$$
(5)

where

$$t_{2ray} = \frac{1}{\sqrt{\frac{1}{t'^2} + \frac{2\Gamma}{t't''}\cos(2\pi f\Delta t) + \frac{\Gamma^2}{t''^2}}}, \theta_{2ray} = \tan^{-1} \left[ \frac{\frac{1}{t'}\sin(2\pi ft') + \frac{\Gamma}{t''}\sin(2\pi ft'')}{\frac{1}{t'}\cos(2\pi ft') + \frac{\Gamma}{t''}\cos(2\pi ft'')} \right], \Delta t = t'' - t'.$$

The UWB transmitted signal is a passband rectangular pulse. The expressions of this pulse in time domain  $v_{\rm t}$  and its spectral density function  $V_{\rm t}$  are

$$v_{\rm t}(t) = \frac{1}{f_{\rm b}} \begin{bmatrix} f_{\rm max} {\rm sinc}(2f_{\rm max}t) \\ -f_{\rm min} {\rm sinc}(2f_{\rm min}t) \end{bmatrix}, \qquad (6)$$

$$V_{\rm t}(f) = \begin{cases} \frac{1}{2f_{\rm b}} & ||f| - f_{\rm c}| \le \frac{f_{\rm b}}{2} \\ 0 & ||f| - f_{\rm c}| > \frac{f_{\rm b}}{2} \end{cases},$$
(7)

where t is the time, f is the frequency,  $f_c$  is the center frequency,  $f_b$  is the spectral bandwidth,  $f_{\min} = f_c - f_b/2$ is the minimum frequency,  $f_{\max} = f_c + f_b/2$  is the maximum frequency, and  $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$ . This signal can be approximated to be the impulse function with constant spectral density ranges from  $f_{\min}$  to  $f_{\max}$  Hz, and the area is

$$\int_{-\infty}^{\infty} v_{\rm t}(t) dt = 1.$$

The spectral density function  $V_{r,2ray}$  of the received signal can be found from

$$V_{\rm r,2ray}(f,d,h_{\rm t},h_{\rm r}) \tag{8}$$

$$= V_{t}(f) \cdot H_{2ray}(f, d, h_{t}, h_{r}),$$
  
= 
$$\begin{cases} \frac{1}{8\pi f_{b}|f|t_{2ray}} \exp(-j\theta_{2ray}) & ||f| - f_{c}| \leq \frac{f_{b}}{2} \\ 0 & ||f| - f_{c}| > \frac{f_{b}}{2} \end{cases}$$

The received signal waveform from the direct  $v_{\rm rd}$  and ground reflection  $v_{\rm rr}$  paths can be found from

$$v_{\rm rd}(t, d, h_{\rm t}, h_{\rm r}) \tag{9}$$

$$= \begin{cases} \frac{1}{4\pi f_{\rm b}t'}\ln\left(\frac{f_{\rm max}}{f_{\rm min}}\right) & t=t'\\ \frac{1}{4\pi f_{\rm b}t'}\begin{bmatrix} {\rm C}_{\rm i}(2\pi f_{\rm max}|t-t'|)\\ -{\rm C}_{\rm i}(2\pi f_{\rm min}|t-t'|) \end{bmatrix} & t\neq t' \end{cases},$$

$$v_{\rm rr}(t, d, h_{\rm t}, h_{\rm r})$$
(10)  
= 
$$\begin{cases} \frac{\Gamma}{4\pi f_{\rm b} t''} \ln\left(\frac{f_{\rm max}}{f_{\rm min}}\right) & t = t'' \\ \frac{\Gamma}{4\pi f_{\rm b} t''} \begin{bmatrix} C_{\rm i}(2\pi f_{\rm max}|t - t''|) \\ -C_{\rm i}(2\pi f_{\rm min}|t - t''|) \end{bmatrix} & t \neq t'' \end{cases}$$

The received signal  $v_{r,2ray}$  is the combination between the signals that come from the direct and ground reflection paths. Then, it can be written as

$$v_{\rm r,2ray}(t,d,h_{\rm t},h_{\rm r}) \tag{11}$$

$$= v_{rd}(t, d, h_{t}, h_{r}) + v_{rr}(t, d, h_{t}, h_{r}),$$

$$= \begin{cases} \frac{1}{4\pi f_{b}} \begin{bmatrix} \frac{1}{t'} \ln\left(\frac{f_{max}}{f_{min}}\right) \\ +\frac{\Gamma}{t''} C_{i}(2\pi f_{max}\Delta t) \\ -\frac{\Gamma}{t''} C_{i}(2\pi f_{max}\Delta t) \end{bmatrix} & t = t' \\ \frac{1}{4\pi f_{b}} \begin{bmatrix} \frac{\Gamma}{t''} \ln\left(\frac{f_{max}}{f_{min}}\right) \\ +\frac{1}{t'} C_{i}(2\pi f_{max}\Delta t) \\ -\frac{1}{t'} C_{i}(2\pi f_{max}\Delta t) \end{bmatrix} & t = t'' \\ \frac{1}{4\pi f_{b}} \begin{bmatrix} \frac{1}{t'} C_{i}(2\pi f_{max}|t - t'|) \\ -\frac{1}{t'} C_{i}(2\pi f_{max}|t - t'|) \\ +\frac{1}{t''} C_{i}(2\pi f_{max}|t - t''|) \\ -\frac{\Gamma}{t''} C_{i}(2\pi f_{min}|t - t''|) \end{bmatrix} & t \neq t', t'' \end{cases}$$

This received signal equation can be defined as the impulse response of the ground reflection model.

Let vs define the UWB path loss of ground reflection model  $PL_{2ray}$  as the ratio between the maximum amplitude of the transmitted and received signal waveforms. Therefore, the UWB ground reflection path loss in dB can be derived as

$$PL_{2ray}(d, h_{t}, h_{r})[dB]$$

$$= 20 \log \left\{ \frac{4\pi f_{b}}{\left[\frac{1}{t'} \ln \left(\frac{f_{max}}{f_{min}}\right) + \frac{\Gamma}{t''} C_{i}(2\pi f_{max}\Delta t) \\ -\frac{\Gamma}{t''} C_{i}(2\pi f_{min}\Delta t) \right]} \right\}.$$
(12)



Fig. 2. Transmitted signal waveform.

## III. DATA PROCESSING

In this paper, the ground reflection coefficient is set to be  $\Gamma = -1$ . The T-R separation distance is considered from 0.01 to 10 m. The transmitted signal is set in the full UWB spectrum bandwidth. The center frequency is  $f_c = 6.85$  GHz. The frequency bandwidth is  $f_b = 7.5$  GHz. The transmitted signal waveform is shown in Fig. 2.

In order to consider the path loss and delay spread of ground reflection model, three cases of Tx and Rx antenna heights are studied using the break point. The break point is considered as the T-R separation distance for which the ground begins to obstruct the first Fresnel zone. If the propagation path is before the break point, meaning that the particular obstacle does not impinge on the first Fresnel zone volume, then the signal attenuation with distance is the same as free space propagation. For the propagation path after the break point, the path loss becomes greater than that for free space propagation. The distance,  $d_f$ , at which the first Fresnel zone becomes obstructed, is given by [10]-[11]

$$d_{\rm f} = \frac{1}{\lambda} \sqrt{(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2) \left(\frac{\lambda}{2}\right)^2 + \left(\frac{\lambda}{2}\right)^4}, \quad (13)$$

where  $\Sigma = h_t + h_r$  and  $\Delta = h_t + h_r$ .

For the first case, the Tx and Rx antenna heights are set to be  $h_t = h_r = 1.00$  m. In this case, the considered T-R separation distance is before the first Fresnel zone break point region. In the second case, Tx and Rx antenna heights are set to be  $h_t = h_r = 0.10$  m. In this case, the first Fresnel zone break point is at the T-R separation distance of about 0.9 m. In the last case, the Tx and Rx antenna height are set to be  $h_t = h_r = 0.01$  m. In this case the T-R separation distance is after the first Fresnel zone break point region.

# A. Path Loss Analysis

The UWB path loss for the 1.00 m antenna heights along T-R separation distance from 0.1 to 1 m compared with narrow



Fig. 3. UWB path loss for the 1.00 m antenna heights along T-R separation distance from 0.1 to 1 m compared with narrow band path loss.



Fig. 4. UWB path loss for the 0.10 m antenna heights along T-R separation distance from 0.1 to 1 m compared with narrow band path loss.

band path loss is shown in Fig. 3. From the figure we can see that, in the region before the break point, the UWB path loss of ground reflection model is flatter than the narrow band path loss.

Figure 4 shows the UWB path loss for the 0.10 m antenna heights along T-R separation distance from 0.1 to 1 m compared with narrow band path loss. We can clearly see that the UWB path loss of ground reflection model has two regions divided by the break point same as the narrow band path loss. But in the region before the break point, the UWB path loss has smallen fading than the narrow band path loss. After the break point, the UWB path loss is higher than the narrow band path loss.

The UWB path loss for the 0.01 m antenna heights along T-R separation distance from 0.1 to 1 m compared with narrow band path loss is shown in Fig. 5. We can see that, in the



Fig. 5. UWB path loss for the 0.01 m antenna heights along T-R separation distance from 0.1 to 1 m compared with narrow band path loss.

region after the break point, the UWB path loss of ground reflection model is higher than the narrow band path loss.

The frequency used model [12]-[13] indicates that the mean path loss increases exponentially with distance. Absolute mean path loss in decibels is defined as the path loss in decibels from the transmitter to the reference distance  $d_0$  plus the additional path loss in decibiels,

$$\overline{PL}(d)[dB] = PL_{\rm f}(d_0)[dB] + 10n \log\left(\frac{d}{d_0}\right). \tag{14}$$

We assume the path loss  $PL_f(d_0)$  is due to the free-space propagation. The free space UWB path loss is defined by

$$PL_{\rm f}(d_0)[dB] = 20 \log \left[ \frac{4\pi f_{\rm b} d_0}{c \ln \left(\frac{f_{\rm max}}{f_{\rm min}}\right)} \right].$$
 (15)

The path loss results are modeled using two different forms of a double linear regression to compute values of the path loss exponents  $n_1$  and  $n_2$  for the two regions and the standard deviation  $\sigma$  in decibels about the best fit mean power law model. In the first form of the double linear regression model, the break point between the two linear regions is fixed at the first Fresnel zone clearance distance. The first double linear regression model for path loss  $PL_1$  and T-R separation distance d is

$$PL_{1}(d)[dB]$$
(16)  
= 
$$\begin{cases} PL_{f}(d_{f})[dB] + 10n_{1}\log\left(\frac{d}{d_{f}}\right) & 0.1 < d < d_{f} \\ PL_{f}(d_{f})[dB] + 10n_{2}\log\left(\frac{d}{d_{f}}\right) & d_{f} < d < 10 \end{cases}$$
.

In the second form of the double linear regression model, the break point between the two linear regions is not fixed, but is instead an unknown parameter for determining the MMSE best curve fit. The alternate double linear regression model for the path loss  $PL_2$  is

$$PL_2(d)[dB] \tag{17}$$

TABLE I PATH LOSS EXPONENTS, STANDARD DEVIATIONS, FIRST FRESNEL ZONE BREAK POINT BEST CURVE FITS FOR 1.00, 0.10 and 0.01 m antenna HEIGHTS.

-	Antenna height (m)	$n_1$	$n_2$	$\sigma$ (dB)	$d_{\rm f}$ (m)
-	1.00	2.00	-	0.13	91.39
	0.10	2.12	3.87	3.45	0.90
	0.01	-	4.37	9.37	0.00

TABLE II PATH LOSS EXPONENTS, STANDARD DEVIATIONS, MMSE BREAK POINT BEST CURVE FITS FOR 1.00, 0.10 AND 0.01 M ANTENNA HEIGHTS.

Antenna height (m)	$n_1^*$	$n_2^*$	$\sigma^*$ (dB)	$d_b$ (m)
1.00	2.00	-	0.13	-
0.10	2.15	5.79	1.52	1.76
0.01	-	5.99	0.05	0.02

$$= \begin{cases} PL_{\rm f}(d_{\rm b})[dB] + 10n_1 \log\left(\frac{d}{d_{\rm b}}\right) & 0.1 < d < d_{\rm b} \\ PL_{\rm f}(d_{\rm b})[dB] + 10n_2 \log\left(\frac{d}{d_{\rm b}}\right) & d_{\rm b} < d < 10 \end{cases}$$

Table 1 lists the first Fresnel zone break point best curve fit parameters for 1.00, 0.10 and 0.01 m antenna heights. From the table, its can be seen that the path loss exponent  $n_1$  before the break point is about 2. After the break point, the path loss exponent  $n_2$  is about 4.

The MMSE best fit curve is considered. The MMSE break point best curve fit parameters for 1.00, 0.10 and 0.01 m antenna heights are listed in the Table 2. From the table, it can be seen that the path loss exponent  $n_1^*$  before the break point is about 2, the same as the first Fresnel zone break point model. But after the break point, the path loss exponent  $n_2^*$  is about 6.

The results form these tables indicate that the first Fresnel zone break point model can be used to characterize the path loss for the region before the break point. There is much error in the path loss exponent for the region after the break point, with values of  $\sigma$  up to 9. Then, the MMSE break point best curve fits is necessary for the region after the break point. From these results, letting  $n_1 = 2$  and  $n_2 = 6$  is the reasonable assumption for the region before and after the break point. There is a significant trend toward increasing path loss exponent as antenna height decreases.

## B. RMS Delay Spread Analysis

where

=

One method of characterizing UWB ground reflection model is by calculating its rms delay spread  $\sigma_{\tau}$ . The rms delay spread is the square root of the second central moment of the power delay profile and is calculated as [8]

$$\sigma_{\tau} = \sqrt{\overline{\tau^2} - (\overline{\tau})^2},\tag{18}$$

$$\begin{array}{rcl} \overline{\tau} & = & \displaystyle \frac{\sum_{\mathbf{k}} a_{\mathbf{k}}^2 \tau_{\mathbf{k}}}{\sum_{\mathbf{k}} a_{\mathbf{k}}^2}, \\ \\ \overline{\tau^2} & = & \displaystyle \frac{\sum_{\mathbf{k}} a_{\mathbf{k}}^2 \tau_{\mathbf{k}}^2}{\sum_{\mathbf{k}} a_{\mathbf{k}}^2} \end{array}$$

TABLE III MAXIMUM, MEAN AND STANDARD DEVIATIONS OF THE RMS DELAY SPREAD  $\sigma_{\tau}$  of 1.00, 0.10 and 0.01 antenna heigths.

Antenna height	Maximum	Mean	Std. deviation
(m)	(ps)	(ps)	(ps)
1.00	1,562.71	935.00	404.12
0.10	166.23	55.00	52.03
0.01	1.81	0.39	0.45



Fig. 6. RMS delay spread versus path loss for 1.00, 0.10 and 0.01 antenna heights.

The rms delay spread of the UWB ground reflection model is considered by using the expression of the received signal  $v_{r,2ray}$ .

For this study, the rms delay spread is analyzed as the function of antenna heights and path loss. Table 3 shows the maximum, mean and standard deviations of the rms delay spread values as a function of the three antenna heights. From the table it is clear that the maximum, mean, and standard deviations of the rms delay spread increases as a function of the antenna height. The lower antenna height provides the lower rms delay spread, but it causes the larger path loss for a given distance. Then, the tradeoff between path loss and rms delay spread for a given antenna height may be found.

Figure 6 shows rms the delay spread versus path loss for 1.00, 0.10 and 0.01 m antenna heights. For the 0.10 and 0.01 m antenna heights, there is clearly a trend toward decreasing rms delay spread as path loss increases. The rms delay spread exponentially decreases for the path loss after the break point region. For the 1.00 m antenna height, the trend is toward increasing rms delay spread as path loss increases to about 50 dB. As the path loss increases, the rms delay spread increases because the signal from the ground reflection path is very low compared with that from the direct path. After that the rms delay spread decreases as the path loss increases.

## IV. CONCLUSION

In this paper, the theoretical ground reflection model for UWB communication system is proposed. From the results,

we can see that the first Fresnel zone break point model can be used to characterize the path loss for the region before the break point. For the region after the break point, the MMSE break point best curve fit is necessary. The path loss exponent  $n_1 = 2$  and  $n_2 = 6$  is the reasonable assumption for the region before and after the break point respectively. There is a significant trend toward increasing path loss exponent as antenna height decreases. The maximum, mean, and standard deviations of the rms delay spread increases as a function of the antenna height. The lower antenna height provides the lower rms delay spread, but it causes the larger path loss for a given distance. Then, the tradeoff between path loss and rms delay spread for a given antenna height may be found. The rms delay spread exponentially decreases for the path loss after the break point region.

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