LESSON 4.1

Factors *and* Prime Factorization

Vocabulary

prime number, p. 173
composite number, p. 173
prime factorization, p. 173
factor tree, p. 173
monomial, p. 174

BEFORE	*Now*	WHY?
You found the product of two or more numbers.	You'll write the prime factorization of a number.	So you can count ways to display a firefly collection, as in Ex. 56.

Yearbook You are working on your school yearbook. Each page will have 24 student photos. The photos will be arranged in a rectangular display with the same number of photos in each row. How many ways can you arrange the photos so that there are no more than 10 photos in any row or column?

You can use factors to determine the number of possible displays. In this chapter, finding the factors of a given whole number means finding whole numbers that divide the given number without a remainder. For example, two factors of 50 are 5 and 10.

Example 1 *Writing Factors*

For the yearbook described above, each possible display will consist of 24 photos. Because there will be the same number of photos in each row, the number of photos in each row will be a factor of 24.

1 Write 24 as a product of two whole numbers in all possible ways.

$1 \cdot 24 \qquad 2 \cdot 12 \qquad 3 \cdot 8 \qquad 4 \cdot 6$

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

2 Use the factors to find all the rectangular displays with no more than 10 photos in any row or column.

3 rows of 8 photos — 6 rows of 4 photos

8 rows of 3 photos — 4 rows of 6 photos

Answer There are 4 possible displays.

Checkpoint

Write all the factors of the number.

1. 30 **2.** 31 **3.** 45 **4.** 87

Prime and Composite Numbers A **prime number** is a whole number that is greater than 1 and has exactly two whole number factors, 1 and itself. A **composite number** is a whole number that is greater than 1 and has more than two whole number factors. The number 1 is neither prime nor composite.

Study *Strategy*

The table lists all the factors of each number. However, to determine whether a number is composite, you need to find only one factor other than the number itself and 1.

Examples of Prime and Composite Numbers

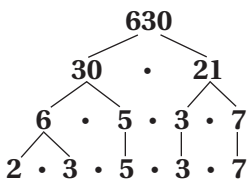
Number	Factors	Prime or composite?
24	1, 2, 3, 4, 6, 8, 12, 24	Composite
41	1, 41	Prime
51	1, 3, 17, 51	Composite
89	1, 89	Prime
121	1, 11, 121	Composite

Prime Factorization When you write a number as a product of prime numbers, you are writing its **prime factorization**. You can use a diagram called a **factor tree** to write the prime factorization of a number.

Example 2 ***Writing a Prime Factorization***

Write the prime factorization of 630.

One possible factor tree:



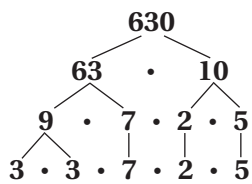
Write original number.

Write 630 as 30 • 21.

Write 30 as 6 • 5. Write 21 as 3 • 7.

Write 6 as 2 • 3.

Another possible factor tree:



Write original number.

Write 630 as 63 • 10.

Write 63 as 9 • 7. Write 10 as 2 • 5.

Write 9 as 3 • 3.

Both trees give the same result: $630 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 = 2 \cdot 3^2 \cdot 5 \cdot 7$.

Answer The prime factorization of 630 is $2 \cdot 3^2 \cdot 5 \cdot 7$.

Note *Worthy*

It is helpful to include diagrams in your notes. Include a different factor tree for the number 630 in your notes.

Checkpoint

Tell whether the number is *prime* or *composite*. If it is composite, write its prime factorization.

5. 32 **6.** 56 **7.** 59 **8.** 83

9. 101 **10.** 175 **11.** 180 **12.** 420

Factoring Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables raised to whole number powers.

Monomials	Not monomials
$7x$	$7 + x$
$25mn^2$	$25m - n^2$
$24y^3z^2$	$24 + y^3 + z^2$

To *factor* a monomial, write the monomial as a product of prime numbers and variables with exponents of 1.

Example 3 *Factoring a Monomial*

Factor the monomial $28xy^3$.

$28xy^3 = 2 \cdot 2 \cdot 7 \cdot x \cdot y^3$ Write 28 as $2 \cdot 2 \cdot 7$.

$= 2 \cdot 2 \cdot 7 \cdot x \cdot y \cdot y \cdot y$ Write y^3 as $y \cdot y \cdot y$.

Checkpoint

Factor the monomial.

13. $6ab$ **14.** $15n^3$ **15.** $3x^3y^2$ **16.** $36s^4t$

4.1 Exercises

More Practice, p. 806

INTERNET eWorkbook Plus CLASSZONE.COM

Guided Practice

Vocabulary Check

1. Describe how to write the prime factorization of a number.

2. Explain why 34 is a composite number.

Skill Check

Write all the factors of the number.

3. 16 **4.** 32 **5.** 29 **6.** 55

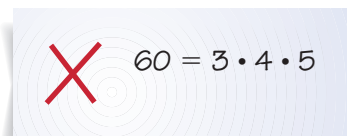
Tell whether the number is *prime* or *composite*.

7. 9 **8.** 15 **9.** 17 **10.** 23

Write the prime factorization of the number.

11. 10 **12.** 18 **13.** 25 **14.** 39

15. Error Analysis Describe and correct the error in writing the prime factorization of 60.



Practice and Problem Solving

Homework *Help*

Example	Exercises
1	16–23, 46, 56
2	24–45
3	48–55

Online Resources
CLASSZONE.COM
- More Examples
- eTutorial Plus

Write all the factors of the number.

16. 8 **17.** 53 **18.** 12 **19.** 33

20. 36 **21.** 60 **22.** 71 **23.** 144

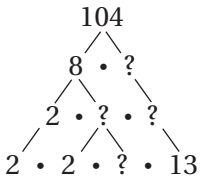
Tell whether the number is *prime* or *composite*.

24. 7 **25.** 16 **26.** 21 **27.** 19

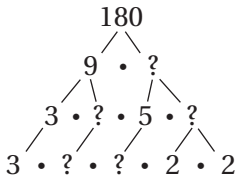
28. 121 **29.** 51 **30.** 84 **31.** 141

Copy and complete the factor tree. Then write the prime factorization of the number.

32.



33.



Write the prime factorization of the number.

34. 26 **35.** 58 **36.** 63 **37.** 85

38. 120 **39.** 160 **40.** 154 **41.** 195

42. 202 **43.** 210 **44.** 217 **45.** 225

46. Coin Collecting The U.S. Mint began issuing state quarters in 1999. There will be one state quarter for each of the 50 states. You are collecting the state quarters and want to design a rectangular display with the same number of quarters in each row. How many ways can you arrange your display?

47. *Writing* Give an expression that is a monomial and tell why it is an example of a monomial. Then give an expression that is *not* a monomial and tell why it is not an example of a monomial.

Factor the monomial.

48. $11cd$ **49.** $19m^3$ **50.** $3f^6$ **51.** $21ab$

52. $5xy^2$ **53.** $35rs^5$ **54.** $2y^4z^3$ **55.** $40m^2n$

56. Fireflies There are 69 species of flashing fireflies, also known as lightning bugs, in the United States. A museum is designing a rectangular display of these 69 species with the same number of fireflies in each row. How many displays are possible?

57. Critical Thinking Explain why all two-digit whole numbers with 5 as the ones' digit are composite.

Study *Strategy*

In Exercises 58–61, to list all the factors of a number, first list all the prime factors. Then list all products of two prime factors. Then list all products of three prime factors, and so on.

Use the prime factorization of the number to list all of its factors.

58. 240 **59.** 335 **60.** 500 **61.** 201

List all the factors of the monomial.

62. $6ab^2$ **63.** $52w$ **64.** $2r^3s$ **65.** $7xyz$

In the **Real World**

Silicate Minerals About $\frac{9}{10}$ of Earth's crust is made up of silicate minerals. The mass of the crust is about 30 quintillion tons, which is 30 followed by 18 zeros. What is the approximate mass of the silicate minerals in the crust?

66. Extended Problem Solving A geologist has collected 102 different types of silicate minerals. The geologist has taken a photograph of each mineral and wants to make a display of the photographs.

a. Calculate How many rectangular arrangements of the photographs are possible?

b. The geologist wants no more than 15 photographs in any row or column. How many rectangular arrangements satisfying this requirement are possible?

c. Analyze The geologist decreases the number of photographs in the display to 96. How many rectangular arrangements, with no more than 15 photographs in any row or column, are now possible?

67. Conjecture The square of an integer is called a *perfect square*. Write the prime factorizations, with exponents, for these perfect squares: 4, 9, 16, 25, 36, and 64. Make a conjecture about the exponents in the prime factorization of a perfect square.

68. Perfect Numbers A *perfect number* is a number that is the sum of all its factors except for itself. The smallest perfect number is 6, because $6 = 1 + 2 + 3$. The next perfect number is between 20 and 30. Find the next perfect number.

69. Critical Thinking If 18 is a factor of a number, what other numbers must also be factors of that number? Give examples to support your answer.

70. Challenge What is the least whole number that has exactly 7 factors, including 1 and itself? Explain your answer.

Mixed Review

Algebra Basics **Solve the equation. Check your solution.** *(Lessons 2.5, 2.6)*

71. $a + 24 = 16$ **72.** $33 + b = 58$ **73.** $c - 14 = 18$ **74.** $d - 10 = 10$

75. $6r = 48$ **76.** $-10s = 50$ **77.** $\frac{t}{9} = -7$ **78.** $\frac{u}{-2} = -14$

Write the verbal sentence as an equation. Then solve the equation. *(Lesson 3.3)*

79. Fifteen plus a number is equal to 21 minus the number.

80. Two times the sum of 3 and a number is equal to 5 plus the number.

81. Eight plus a number is equal to -3 times the number.

Standardized Test Practice

82. Multiple Choice For which value of x is the value of the expression $7x + 1$ a prime number?

A. 0 **B.** 1 **C.** 3 **D.** 4

83. Multiple Choice Which expression is the prime factorization of 252?

F. $2^2 \cdot 3^2 \cdot 7^2$ **G.** $2^2 \cdot 3^2 \cdot 7$ **H.** $2 \cdot 3^2 \cdot 7$ **I.** $2 \cdot 3^2 \cdot 7^2$

84. Short Response The area of a rectangle is 54 square inches. The length and width are whole numbers of inches. Find all possible dimensions of the rectangle. Which dimensions result in the rectangle having the greatest perimeter?