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An Ethnomathematics Exercise for Analyzing a *Khipu* Sample from Pachacamac (Perú)

Ejercicio de Etnomatemática para el análisis de una muestra de *quipu* de Pachacamac (Perú)

Alberto Saez-Rodríguez[1]

Abstract

A *khipu* sample studied by Gary Urton embodies an unusual division into quarters. Urton's research findings allow us to visualize the information in the pairing quadrants, which are determined by the distribution of S- and Z-knots, to provide overall information that is helpful for identifying the celestial coordinates of the brightest stars in the Pleiades cluster. In the present study, the linear regression attempts to model the relationship between two variables (which are determined by the distribution of the S- and Z-knots). The scatter plot illustrates the results of our simple linear regression: suggesting a map of the Pleiades represented by seven points on the Cartesian coordinate plane.

Keywords: Inca khipu, Linear regression analysis, Celestial coordinate system, Pleiades.

Resumen

Una muestra de *quipu* estudiada por Gary Urton comporta una división por cuadrantes poco común. Utilizando dicho hallazgo de Urton, podemos visualizar la información contenida en los pares de cuadrantes, la cual está determinada por una distribución de nudos con orientaciones opuestas de 'S' y de 'Z', brindándonos, así, toda la información necesaria para identificar las coordenadas celestes de las estrellas más brillantes del cúmulo de las Pléyades. En el presente estudio se usa el análisis de la Regresión Lineal con el fin de construir un modelo que permita predecir el comportamiento de la variable dependiente *y* (valores de los nudos con orientación de 'Z') en función de la variable independiente *x* (valores de los nudos con orientación de 'S'). Dicho modelo se logró usando un diagrama de dispersión, lo que nos condujo a un mapa con la posición de las siete estrellas más brillantes de las Pléyades, como un modelo empírico de la relación que mantienen las variables en estudio.

Palabras clave: Quipu, Análisis de regresión lineal, Sistema de coordenadas celeste, Pléyades.

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Introduction

D'Ambrosio (1997) used the term 'ethnomathematics' mostly associated with mathematics practiced in "cultures without written expression". The Incas had developed a method of recording numerical information which did not require writing. It involved knots in strings called *khipu*. According to Orey (2000, p. 250), the application of "ethnomathematical techniques and the tools of mathematical modelling allows us to see a different reality and give us insight into science done in a different way".

Khipu Recording in the Inca Empire

The *khipu* is the most complex non-alphabetic recording system known from the ancient world, but its techniques of information registry have eluded scholars for centuries. The broader impacts of the study of the *khipu* can be compared with those of other ancient writing/recording systems (for example, Sumerian cuneiform and Mayan hieroglyphs). Deciphering *khipus* is an exceedingly difficult task because we lack the equivalent of a Rosetta Stone for *khipus*, and this study presents one of several possible solutions to this puzzle. In this study, I explore this ancient and potentially powerful system of coding information from the pre-Columbian South America. The numerical data recorded in *khipus* were calculated by means of decimal and 'fortiethal' *yupanas* (In Quechua *yupar* means "to count"), using the Fibonacci arithmetic system: $F_n = F_{n-1} + F_{n-2}$, and powers of 10, 20 and 40 as place values for the different fields in the instrument.

While numerous Spanish chronicles in Peru left accounts of the *khipu* that inform us on certain features and operations of these devices, none of these accounts is extensive or detailed enough to put us on solid ground in our attempts today to understand exactly how the Inca made and consulted these knotted and dyed records. Urton (2003, p. 53) notes that "Our own challenge today is to seek by every means possible to try to understand and appreciate the full range and potential of record keeping that the Inca realized in their use of this device."

The former Inca record keepers, known as *khipukamayuq* (knot makers/keepers), supplied Inca rulers with a colossal variety and quantity of information pertaining to censuses, accounting, tributes, ritual and calendrical organization, genealogies, astronomical observations (Zuidema, 1982), and other such matters. In general terms, khipus are composed of a primary cord to which a variable number of pendant strings are attached (see Figure 10). Referring to the direction of slant of the main axis of each knot, i.e., "S" or "Z" (see Figure 1), Urton (2003) argued that the people who made these knotted-string devices knew what they were doing and fabricated these complicated objects for meaningful reasons, not because they were right-handed or left-handed.

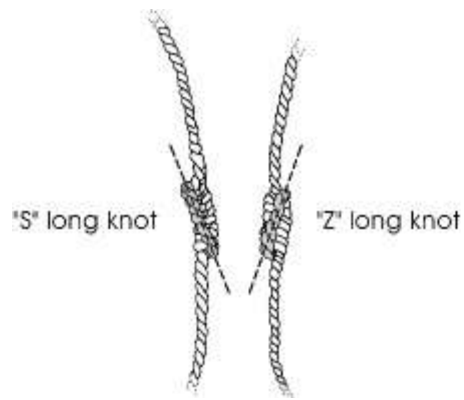

Figure 1. S- and Z-tied long knots. (Source: G. Urton, 2003).

Topographic references as analogues for positions of the stars on the celestial sphere

It is important to remember that the two-dimensional Cartesian coordinate system, which was developed independently in 1637 by René Descartes and Pierre de Fermat, is commonly defined by two axes that are aligned at right angles to each other to form the *x*, *y*-plane (Newton c. 1760; Bell, 1945). Maps began as two dimensional drawings. According to Delambre (1817), Hipparchos (c. 190 BC – c. 120 BC) may have used a globe reading values off coordinate grids drawn on it.

Although the ancient Sumerians were the first to record the names of constellations on clay tablets 5,000 years ago, the earliest known star catalogues were compiled by the ancient Babylonians of Mesopotamia in the late 2nd millennium BC (ca. 1531 BC to ca. 1155 BC). The oldest Chinese graphical representation of the sky is a lacquer box dated to 430 BC, although this depiction does not show individual stars. Figure 2 shows the oldest Peruvian cosmological chart, created by Pachacuti Yamqui (1968 [c.1613]). However, Claudius Ptolemaei (1843 and 1845) first suggested precise methods for fixing the position of geographic features on its surface using a coordinate system with parallels of latitude and meridians of longitude.

The celestial coordinate system, which serves modern astronomy so well, is firmly grounded in the faulty world-view of the ancients. They believed the Earth was motionless and at the center of creation. The sky, they thought, was exactly what it looks like: a hollow hemisphere arching over the Earth like a great dome. When the ancient Incas looked up at the night sky, they described it simply as it was seen.

Ancient Incan Astronomy

Early astronomers used many instruments to study the heavens. There is some evidence to suggest that Incan astronomers used several tools to chart the position of objects in the sky and to predict the movements of the sun, moon, and certain stars (see Figures 4 and 6). All of these tools were basically tools for measuring or calculating the positions of objects in the sky. Within the limits of naked eye astronomy, they used a variety of basic observational techniques. Later, they used very simple instruments to define, monitor, and predict the motion of celestial objects (Krupp, 2003). What tools did Incan astronomers use? During the Spanish conquest of the Incas, the Spanish melted down all of the Incan gold artefacts they could find. It is impossible to imagine how many artefacts the Spanish conquerors might have destroyed. Once the invaders found a new temple, they looted it, collected all of the gold and silver they found, melted it, made coins from it, and shipped it to Spain.

Hiltunen (2003) used astronomical phenomena described in Montesinos' chronicle (1920, originally published in 1644) as historical evidence. His approach attempted to link astronomical phenomena with historical events that were described in Montesinos' chronicle. Montesinos (1920, originally published in 1644) speaks of the appearance of two comets during the reign of Yupanqui; one had the form of a lion, and the other had the form of a serpent. According to Hiltunen (2003), when historical narratives contain well-documented and contextually credible descriptions of eclipses or comets, they can be a useful method to synchronize oral traditions within an absolute chronology. In Montesinos' chronicle (1920 [1644]), several such references exist, but these are poorly documented and contextually dubious. Fortunately, good manuals exist that can be used to check occurrences of eclipses, comets, and even supernovae, thereby yielding an extensive historical perspective. Hiltunen (2003) first tested this idea with Incan historical records and found that references to extraordinary astronomical phenomena that occurred shortly before the arrival of the Spaniards can be correlated quite well (although these cases may, at least partly, be later interpolations intended to impute a prognosticated drama to these events). Montesinos also gives one such reference for Incan times, which may be relevant. In this reference, Cobo (1990, pp. 27–29) and Garcilaso (1966, pp. 118–119) wrote that when a solar eclipse occurred, the Incas would consult their diviners, who usually determined that a great prince was about to die and the Sun had thus gone into mourning. Lunar eclipses were thought to occur because a puma or snake was eating the moon. The Memoirs of Garcilaso mention one comet that appeared at the time of the death of the Inca Huascas and another that was visible for some time afterward (while Atahualpa was a prisoner of Pizarro).

Combining statements of Bernabé Cobo ([1635] 1956, II, pp. 160–161), and Francisco de Avila ([1608] 1966, cap. 29), the Incas' model of the celestial sphere was as a solid. Thus, we can suppose that their choice to use the ecliptical coordinates was reasonably convenient for this theory. But when it came to practical observation, the measurement of ecliptical coordinates was more complicated than the equatorial coordinates. According to Guaman Poma de Ayala (Figure 4, 1980 [1615], pp. 829 [883]), one member of the class of Inca officials was specializing in astrological *khipu*. As Figure 4 shows, the Inca astrologer is represented as a man carrying a fork-like observation instrument, and a *khipu*.

The C*eque* System

The so-called Cuzco *ceque* system was first studied by Polo de Ondegardo, in Cuzco whose inhabitants gave him drawings of their plans (Durán, 1981) [1559]. He also explicitly refers to the calendrical use of the system in Cuzco but does so with giving only a few clues for solving that problem. However, no chronicler provides a sufficiently full account of that use.

According to many accounts of Indian life and lore written in Spanish shortly after the conquest of Cuzco was planned in such a way that the main temple was located at the confluence of 42 lines, called *ceques* (Quechua for "lines"), each containing a varying number of *huacas*. The *ceque* system in Cuzco had 328 *huacas* (shrines), which is the same number as the number of days in 12 sidereal months. The *ceque* system included a complex series of shrines and imaginary lines that radiated out from the centre of Cuzco and had astronomical, calendrical, and sacred connotations. The *ceques* were grouped into four quadrants, each corresponding to one of the four "parts" of the state. This system provided the foundation for Zuidema's (1964, 1977, 1982) interpretation of the Incan calendar.

Tom Zuidema (1964, 1977) was the first to recognize the relevance of astronomy in the structure of the *ceque* system. The *ceque* system as a whole is connected with the Inca lunar-stellar calendar, and several of the line orientations have an astronomical origin and mark relevant astronomical phenomena at the horizon, such as the rising of the sun at the solstices. In Guaman Poma's drawings (1980 [c. 1615]), grids were also employed in Andean spatial organization.

In his article, Gartner (1998, p. 2) asks the following question concerning gridlike geometric structures: "How does one identify a map from a culture whose conceptions of space, geographic relations, mode of representation, and media are very different from Western experience?". In the present study, I will attempt a review of Gartner's work and mention some relevant facts that are of importance to the origin and the planning of the coordinate system.

In the of *Exsul Immeritus*' diagram (Miccinelli Collection), Zuidema (2004) has assigned to each *huaca* one degree in a circle of 365° (and thus not of 360°). During the remaining period of 37 days, as indicated, this primarily agricultural calendar did not operate. I would suggest that the Incas divided the celestial sphere into 365 degrees. As a tropical year has 365 days, each day the sun moves one degree on the celestial sphere. Yet most ancient astronomers adopted the Babylon division, dividing the celestial sphere into 360 degrees.

Pachacuti Yamqui's Diagram of Inca Cosmology

The chart in question (see Figure 2) shows objects depicted on the Temple of the Sun, Qorikancha, in Cuzco, adding Spanish and Quechua notations. These objects are: Orion on the upper left, the southern cross at the center, seven Pleiades; solar system objects: the Sun, the Moon, Venus as morning and evening star; meteorological objects: the rainbow; natural "earthly" objects: the surface of the earth, the sacred tree; and finally, a rectangular grid depicted below the seven Pleiades.

The extraordinarily crafted Temple of the Sun at Qorikancha (Cuzco) was the most sumptuous temple in the Inca Empire. Some 4,000 priests and their attendants once lived within its confines. Qorikancha also served as the main astronomical observatory for the Incas. In addition to hundreds of gold panels lining its walls, there were life-size gold figures, solid-gold altars, and a huge golden sun disc. The golden sun disc reflected the sun and bathed the temple with light. During the summer solstice, the sun still shines directly into a niche where only the Inca chieftain was permitted to sit.

The Qorikancha 's Rectangular Grid and the Incan pseudo Longitudes and Latitudes

At the bottom of the Pachacuti Yamqui's diagram we can see seven points corresponding to the seven visible stars of the Pleiades. Below the representation of the 7 stars we can clearly see a rectangular grid. The Pachacuti Yamqui's rectangular grid consists of 7 horizontal and 18 vertical lines, meeting each other at right angles and thus forming a rectangular grid, like a Cartesian coordinate system in two dimensions. If we observe the position of the rectangular grid, we notice that it was depicted between two key words, such as *collca* (Pleiades) and *pata* (Quechua for step, range, edge, terrace, a height), i.e., on the left side of the grid we can read the word "*collca*", while on the opposite side the word "*pata*" is easily readable.

According to Pachacuti Yamqui (1968), the Temple of the Sun at Qorikancha was the location of a plaque that depicted the Inca cosmos, providing a map of this cosmos depicted on a paper sheet. As Figure 2 shows, an important part of this image is the rectangular grid depicted by Pachacuti Yamqui at its bottom.

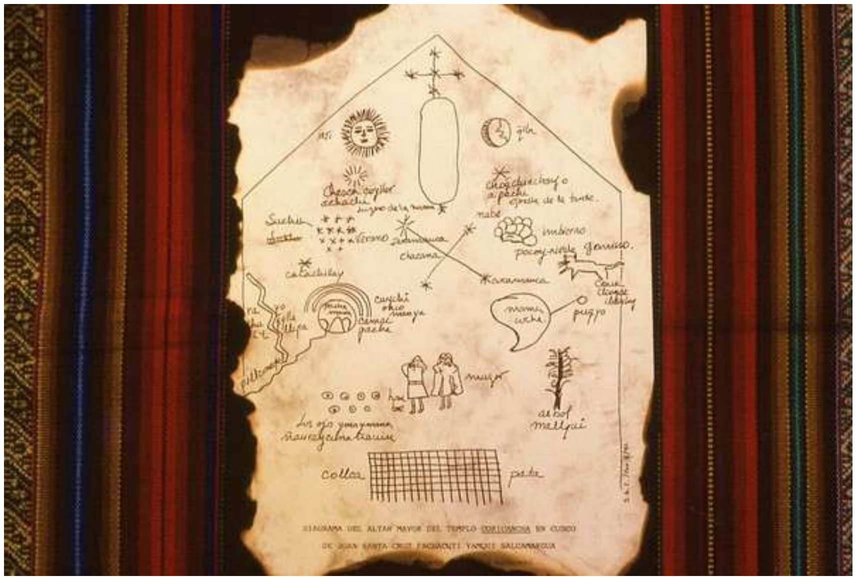

Figure 2. A Peruvian cosmological chart from circa 1613 by Pachacuti Yamqui. Adapted from Earth Institute News, Columbian University. Science and Folklore Converge in Andean Weather Forecasts Based on the Stars, 2000.

Three different ‘arbitrary interpretations’ have been advanced to explain the rectangular grid pattern depicted at the bottom of the Pachacuti Yamqui's diagram. The left side of the grid is labelled *collca*, while the right side is labelled *pata*.

Steele and Allen (2004, p.148) describes the rectangular grid as “…a rectangular stone that retained that produce of the harvest”.

Dean (2001, p. 243) identifies the grid at the bottom of the drawing as the wall of the Qorikancha covered in gold plates.

In the Silverblatt’s (1987, p.43) Schematic version of Pachacuti Yamqui's diagram the rectangular grid is interpreted as “storehouse-terrace” (*collca* + *pata* = *collcapata*). This is the most common interpretation of the rectangular grid depicted at the bottom of the diagram.

I should emphasize here that the words *collca* and *pata* are not written on the rectangular grid as two successive words. Why then, was the rectangular grid centered between two words, such as *collca* (Pleiades, storehouse) and *pata* (step, range, edge, terrace, a height)? I suggest that Pachacuti Yamqui knew what he was writing, and he wrote these two words on both sides of the rectangular grid, not because he transposed them, or simply there was not enough space on the diagram sheet to insert them together into a single label.

Thus, written on both sides of the rectangular grid, i.e., written as two separate words in the cosmological context provided by the Pachacuti Yamqui's diagram, these two words should be translated into English as two isolate words.

Here arises an interesting question: Why did Pachacuti Yamqui depict a "storehouse-terrace" or the wall of the Qorikancha as a rectangular grid of horizontal and vertical lines, meeting each other at right angles?

The term *collca* may have different senses or meanings: granary, storehouse, warehouse, including the name of the Pleiades in Quechua.

The word *pata* refers to an agricultural terrace. It is also well known that the word *pata* has the additional meaning of step, range, edge, a height. However, *Inti-Pata* is the Sun Gate marking the entrance to Machu Picchu. The word *pata* is also a suffix added to words to describe their edges (e.g., *nawinpata* = eyelid, and *mayupata* = riverbank), which was described by Diego González Holguín in the early 17th century (1952 [1608]). Even if these two Quechua words might be associated with "storehouse" and "terrace", it does not fit well with the context of an almost perfect grid pattern. *Collcapata* is a compound Quechua word also meaning checkerboard motif, i.e., *collcapata* motif (Rowe, 1999, p. 608).

Narrative records spring solely from Spanish chronicles taken from oral testimony. Julien (2000) notes that at least one of the chroniclers, Pedro de Cieza de Leon, was conscious of cultural and linguistic barriers to his understanding of the Inca collective memory; Cieza also notes "the controlled, edited quality of the transmissions and the conscious forgetting of individuals whose deeds did not measure up to some standard" (Julien 2000, p. 11; MacCormack 1997, pp. 288-289 in Julien 2000, p. 12).

I suggest that the rectangular grid pattern depicted at the bottom of the Pachacuti Yamqui's diagram is a blank coordinate grid with grid lines shown (7 horizontal and 18 vertical). The axes are, thus, labelled with two different Quechua names, such as *collca* and *pata*. We can't imagine that the measurement of equatorial coordinates used by ancient Inca

Astronomers was much simpler. The instrument need be installed once. That was probably the reason why the Incas preferred equatorial coordinates.

Hypothesis

I hypothesized that the coordinate system was the kind of idea that may have been developed by Incas from the *ceque* system (see Zuidema, 1964, for details). According to this hypothesis, a coordinate system could be developed using the *ceque* system. The Inca made a similar coordinate system which was "fixed to the sky". To be more concrete, the Incas used the coordinate plane to create a map of the Pleiades star cluster. This hypothesis is novel because it addresses three fundamental questions:

Q1-Why would the Incan astronomers have wanted to produce such a coordinate grid representation of the organization of the Pleiades on a plane?

Q2-What would have been its value to them? To be more concrete, what would have been gained for the purposes of the Incan astronomers (or the *khipukamayuqs*), by creating the highly sophisticated, a coordinate grid representation of the seven (or so) stars in the cluster, which I suggest they created?

Q3-How would the intimate knowledge the Incan astronomers had about the positioning of the stars of the Pleiades have served some interest, or need, of the Inca astronomer?

Thus, as a main hypothesis, I postulate that the ancient Incan astronomers might have developed a method of defining the location of points on a plane based on their distances from two fixed, perpendicular, straight lines (axes), that is, concepts that prefigure the two-dimensional Cartesian coordinate system, which knot makers/keepers encoded on *khipu*. I also postulate that if the values in the pairing quadrants, which are determined by the distribution of Z- and S-knots, lie along two perpendicular axes (x, y) and are decomposed into points along Cartesian axes, then our numerical results should provide a theoretical stellar model, representing, for example, the Pleiades star cluster.

To test these hypotheses, I focused on the analysis of the numerical data registered on the *khipu* sample, examined the information contained in the pairing quadrants for the purpose of formulating testable hypotheses, and implemented conventional statistical tools. Correspondence analysis should show how the variables are related, not just that a relationship exists. As opposed to traditional hypothesis testing designed to verify *a priori* hypotheses about the relation between the Pleiades, the word *pata* and the rectangular grid, which was suggested by Pachacuti Yamqui. In the present study, exploratory data analysis is used to identify systematic relations between variables when there are no (or rather incomplete) *a priori* expectations as to the nature of those relations. In particular, the extent to which we can predict some variables by knowing others, or the extent to which some are associated with others.

Figure 3. The Pleiades (M45) star cluster khipu (Courtesy of the Anglo-Australian observatory).

Figure 4. An Inca astrologer with his (Guaman Poma, 1980 , pp. 829 [883]).

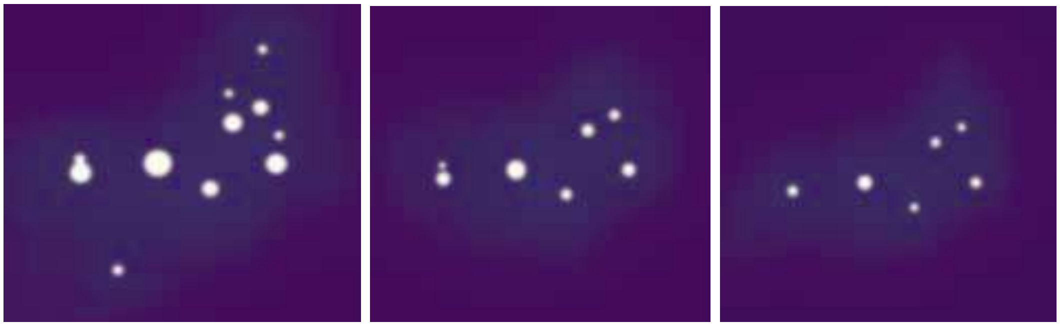

Figure 5. Simulated view of the Pleiades as they might appear during a normal year, when high cirrus clouds do little to obscure the night sky; eleven stars are visible (left). Viewing the cluster during an *El niño* year, when high cirrus clouds are more abundant, would reveal fewer stars (right image). Conditions between these extremes would allow an intermediate number of stars to be seen (middle). Adapted from Orlove et al. (2002).

Figure 6. The *Intihuatana* Stone. Machu Picchu - The Hitching Post of the Sun. (Image courtesy of SacredSites.com).

The order of presentation followed in Method of Investigation will first take into account what I think the reader may need to know in order to understand the explanation of any given set of techniques and operations.

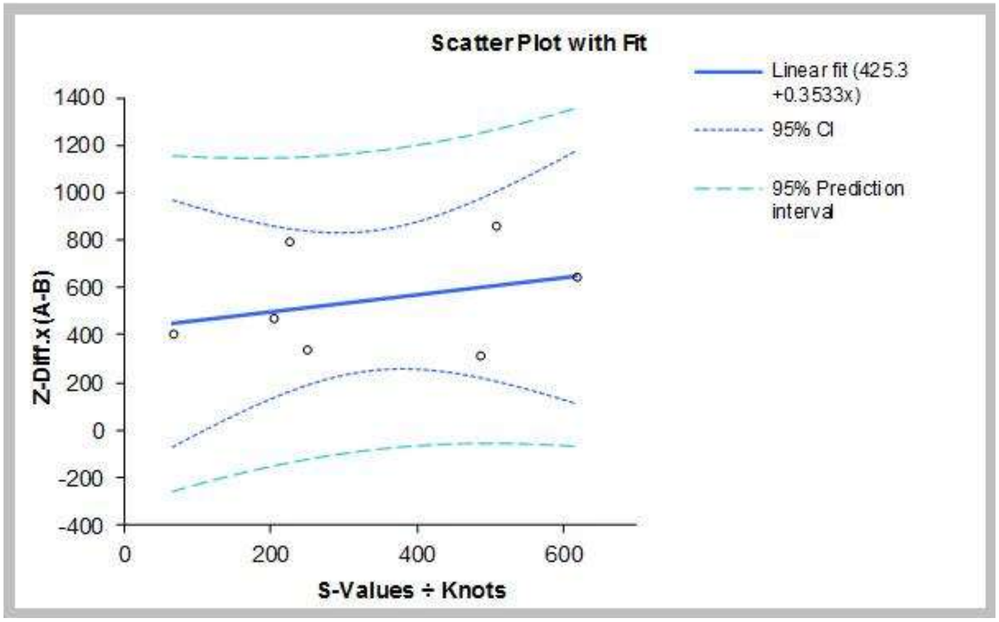

Figure 7. The *x*, *y*-plane defined by two axes at right angles to each other and the locations of the seven stars of the Pleiades.[2]

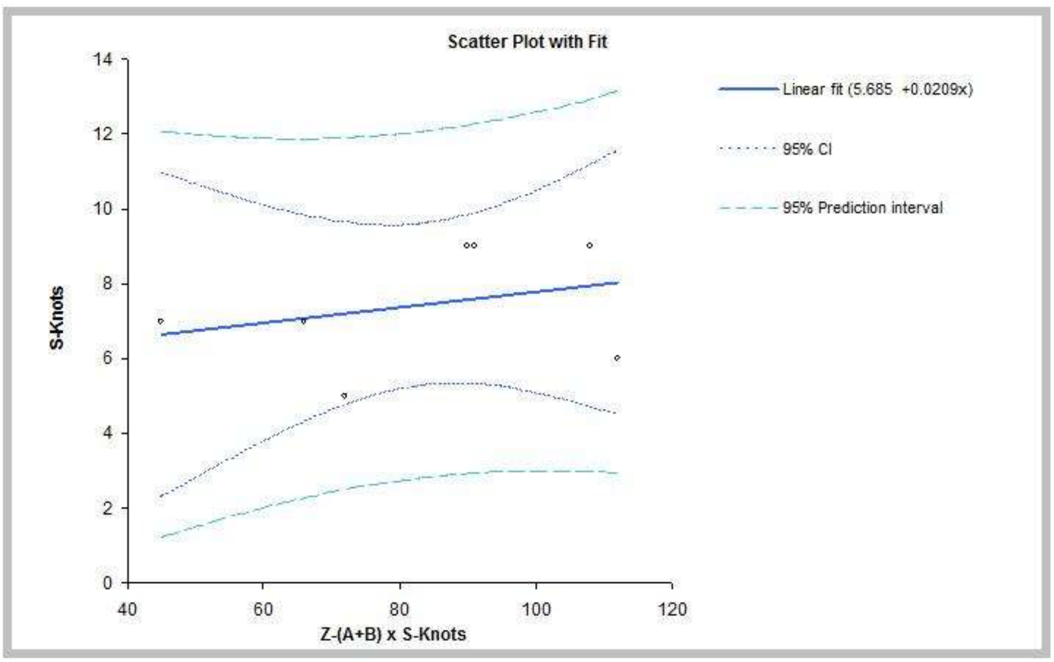

Figure 8. The *x*, *y*-plane defined by two axes at right angles to each other and the locations of the seven stars of the Pleiades.[3]

[2] A set of 7 points (black) indicating the positions of the brightest stars in the Pleiades cluster, compared to the simple linear regression line (blue). Written as ordered pairs (*x*, *y*) according to Table 1, the coordinates of the stars are as follows: Atlas (66, 400); Alcyone (204, 468); Merope (224, 793); Maia (249, 336); Taygeta (488, 315); Electra (508, 858); and Celaeno (618, 640).

Materials and Methods

Star cluster sample

Because of the archaeoastronomical importance of the Pleiades, I have selected a stellar sample that consists exclusively of the seven brightest stars of the Pleiades (M45) star cluster (Figures 3 and 5). Only six stars in the Pleiades cluster can readily be seen by the naked eyen under normal conditions: Alcyone, Atlas, Electra, Maia, Merope, and Taygete. In the Southern Hemisphere, where the sky is completely different than it is North of the Equator, the Inca gazed seven stars in the Pleiades cluster. As was suggested by Levy-Strauss (1964, pp. 246–60) and Urton (1981, p. 200), there is no doubt that some symbolic relations exists between the Pleiades and the rainy period. Orlove's (2002) simulation demonstrates that looking at the Pleiades during an *El Niño* year, when high cirrus clouds are more abundant, would reveal fewer stars (Figure 5). The stars Celaeno und Asterope are at the limit of human visibility. Indigenous farmers in some communities of the high Andes of Peru and Bolivia have been forecasting *El Niño* events for at least 400 years and are able to adjust their planting schedules if poor or late rains are expected (Orlove et al., 2002).

The Pleiades open star cluster is one of the most identifiable of celestial objects. The astronomical knowledge of the Inca was quite detailed, and they considered the Pleiades to be an important celestial feature. Several constellations were recognized as bearing agricultural importance, including the Pleiades (the "Seven Sisters", who preserved the seed). The stars in the Pleiades cluster are not arranged in any definite shape. The Incas called the Pleiades "Seed Scatterer" or "Sower" because they were among the world's first cultures to observe the bright cluster of stars.

Description of the *Khipu* Sample

The *khipu* sample VA 42527 (Museum für Völkerkunde, Berlin) contains an arrangement of 21 pendant strings without subsidiary strings (see Figures 10 and 11). It is one of about

[3] A set of 7 points (black) indicating the positions of the brightest stars in the Pleiades cluster, compared to the simple linear regression line (blue). Written as ordered pairs (x, y) according to Table 3, the coordinates are as follows: Atlas (0, 7); Alcyone (9, 7); Merope (16, 6); Maia (30, 9); Taygeta (35, 9); Electra (63, 5); and Celaeno (72, 9).

600 *khipus* in quite good condition that exist in collections. This *khipu* has several long (multiple-turn) knots and the single figure-eight knot on lower parts of the cord, and one may posit a horizontal axis separating these from the overhand knots on the upper part. This division is a normal format attribute of Inca *khipus*. Urton (2003) was the first to notice an unusual (though not unique) vertical axis: to one side of the *khipu,* all of the knots on the upper part are tied rightward, whereas on the other side of the *khipu*, all of the knots on the upper part are tied leftward. Below the horizontal axis, the pattern reverses: in each of the two lower quadrants, the long-knot direction is opposite that of the quadrant above it. In the lower right quadrant, the last cord contains a single figure-of-eight knot. Figure 11 reproduces Urton's diagram with added elements.

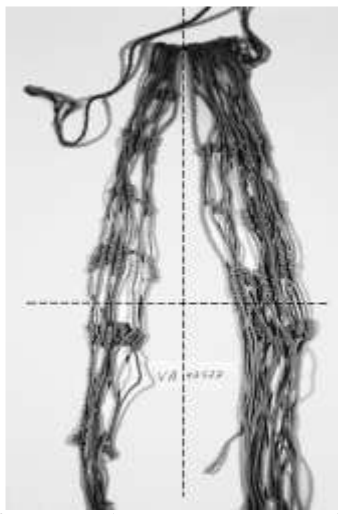

Figure 10. *Khipu* sample from Pachacamac (VA 42527, Museum für Völkerkunde, Berlin). Source: G. Urton (2003).

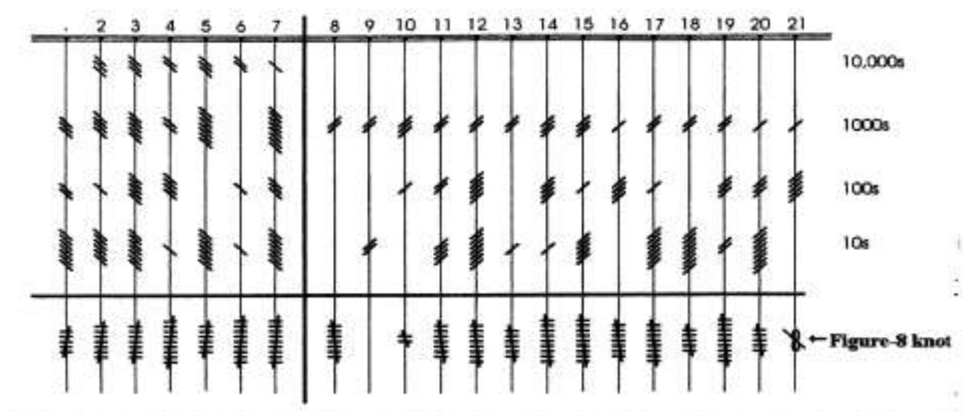

Figure 11. Four-part organization of knot directionality on the *khipu* sample and the distribution of S-knots (1 to 7) and Z-knots (8 to 21). Adapted from Urton (2003).

The Method of Investigation

Linear Regression Analysis—The Incan Celestial Coordinate System

The linear regression analyses performed in this paper provides a tool for a) assessing the numerical data registered on the *khipu* sample, and b) the assessment of the degree of concordance between the Pleiades cluster and their graphical (grid) representation. I have chosen to create a scatter plot with fit using Microsoft EXCEL (see Figures 7, 8 and 9) to test the degree of concordance that exists between the Pleiades cluster and their graphical (grid) representation, i.e., the Incan map of the Pleiades. The line of best fit shows whether these two variables appear to be correlated.

Moreover, I have chosen regression analysis to understand the statistical dependence of S-knot values on Z-knot values. Regression provides the information needed to use the *x* values (S-long knots) to estimate what the corresponding *y* values (Z-long knots) should be. In other words, we can predict the S-knot values from Z-knot values. Tables 1, 2 and 3 show a bivariate plot of two variables (S-knot and Z-knot values). As Tables 1, 2 and 3 show, the long knot values are used as "multiplicands" or “divisors”. The construction proposed in this work lends itself very naturally to a rudimentary Cartesian geometry, where the *y*-axis is vertical and the *x*-axis is parallel to the earth.

***x*-coordinates** S-Values ÷ Long knots[4]	***y*-coordinates** Z- (A–B) by Long knot values[5] Written as an ordered pair
328 ÷ 5 = 65.6	50 x 8 = 400
3,416 ÷ 7 = 488	63 x 5 = 315
3,557 ÷ 7 = 508	78 x 11 = 858
2,241 ÷ 9 = 249	28 x 12 = 336
3,707 ÷ 6 = 617.8	40 x 16 = 640
2,021 ÷ 9 = 224.5	61 x 13 = 793
1,837 ÷ 9 = 204	26 x 18 = 468

Table 1. Bivariate plot as determined by the distribution of the S- and Z-Long knots.

[4] Based on the four-part organization of knot directionality (Table 4 and Figure 12)

[5] Based on the four-part organization of knot directionality (Table 4 and Figure 12).

Z-Long knot values Written as an ordered pair (A + B)	***x*-coordinates** Z-(A+B) times S-Long Knots[6]	***y*-coordinates** S-Long knot values	Displayed in inverted order
7 + 1 = 8	9 x 8 = 72	5	9
0 + 5 = 5	9 x 5 = 45	7	9
2 + 9 = 11	6 x 11 = 66	7	6
7 + 5 = 12	9 x 12 = 108	9	9
8 + 8 = 16	7 x 16 = 112	6	7
6 + 7 = 13	7 x 13 = 91	9	7
9 + 9 = 18	5 x 18 = 90	9	5

Table 2. Bivariate plot as determined by the distribution of the S- and Z-Long knots.

***x*-coordinates** Z- (AxB) Crossed strings[7] Written as an ordered pair	***y*-coordinates** S-Long knot values
7 x 9 = 63	5
1 x 9 = 9	7
0 x 7 = 0	7
5 x 6 = 30	9
2 x 8 = 16	6
9 x 8 = 72	9
7 x 5 = 35	9

Table 3. Bivariate plot as determined by the distribution of the S- and Z-Long knots.

In this particular sample (see Table 4 and Figure 12), I observed that half of the values on the 14 pendant strings (nos. 8 to 14 in the upper right quadrant) were higher than the remaining values (nos. 15 to 21). As Figure 12 shows, we can thus arrange pairs of pendant strings (A and B) such that pendant string B represents a smaller (or equivalent) quantity than its partner (pendant string A). Pendant string A should therefore indicate values greater than or equal to those in pendant string B (A ≥ B). Figure 12 thus indicates the existence of a preferential pairing assortment of strings. More careful observation revealed the knot maker/keeper apparently developed a more efficient rationale for arranging pendant strings in pairs, pairing the first pendant string with the last pendant string, the second with the next to last, and so.

[6] According to the Four-part organization of knot directionality (Table 4 and Figure 12)

[7] Based on the four-part organization of knot directionality (Table 4 and Figure 12).

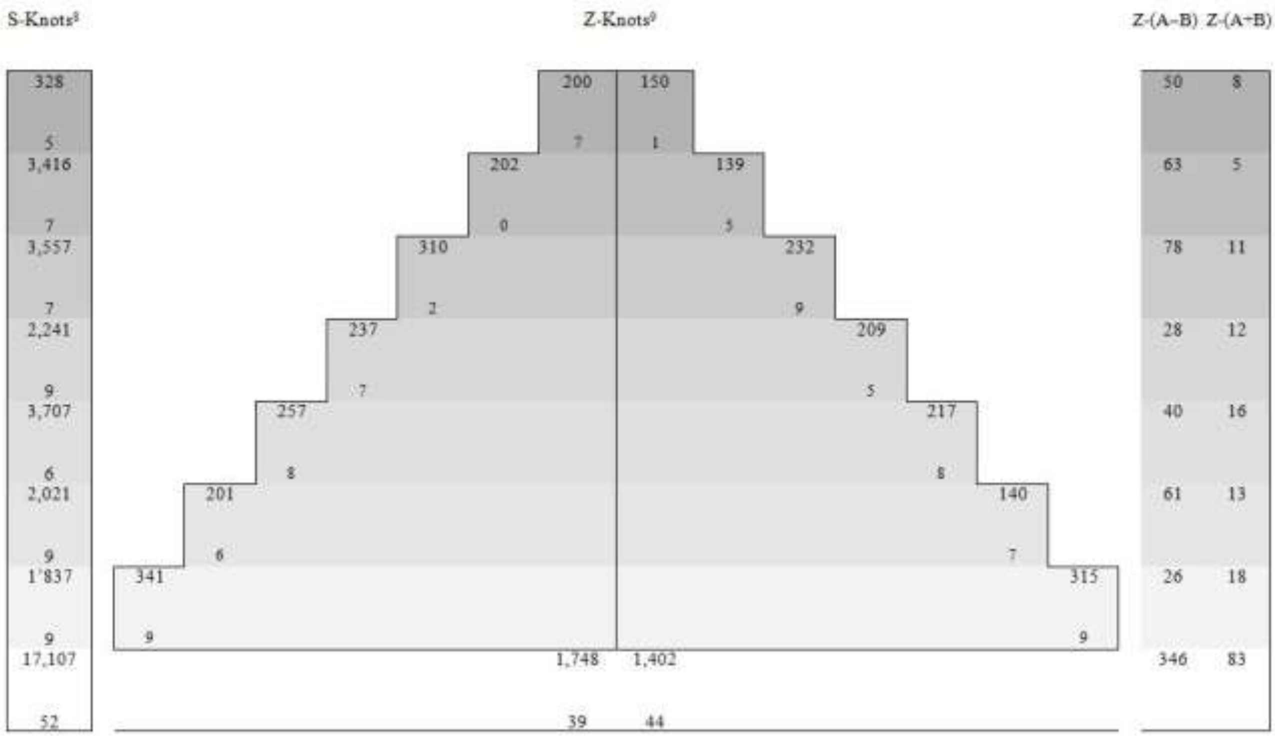

Figure 12. Z-knot values (pendant strings nos. 8 to 21) written as ordered pairs (A, B), resulting in a seven-step pyramid.

Distribution of S-knot values							Distribution of Z-knot values														
1	2	3	4	5	6	7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	15	16	17	18	19	20	21	Decima l units
	30,000	30,000	20,000	30,000	20,000	10,000															10,000s
3,000	4,000	5,000	2,000	7,000		8,000	2,000	2,000	3,000	2,000	2,000	2,000	3,000	3,000	1,000	2,000	2,000	2,000	1,000	1,000	1,000s
200	100	500	400		200	300			100	300	500		400	100	400	100		300	300	500	100s
80	60	70	10	70	10	70		20		70	70	10	10	50		70	90	20	90		10s
3,280	34,160	35,570	22,410	37,070	20,210	18,370	2,000	2,020	3,100	2,370	2,570	2,010	3,410	3,150	1,400	2,170	2,090	2,320	1,390	1,500	Totals
328	**3,416**	**3,557**	**2,241**	**3,707**	**2,021**	**1,837**	**200**	**202**	**310**	**237**	**257**	**201**	**341**	315	140	217	209	232	139	150	Divide d by 10

Table 4. Four-part organization of knot directionality on the *khipu* sample[10]

In this section, I will describe three pairs (*x*/*y*) of linear equations (that is, algebraic equations) in which each value of the S- or Z-knots in the *khipu* is either a constant, the

[8] S-knot values and their corresponding long-knots
[9] Z-knot values and their corresponding long-knots
[10] Pairing quadrants were determined based on the distribution of the S- and Z-knots.

product/quotient, or the addition/subtraction of a constant and (the first power of) a single variable.

A common form for the linear equation in the two variables *x* and *y* is $y = mx + b$, where *m* and *b* designate constants. Suppose we have *n* points (x_i; y_i) with distinct *x*-coordinates; for example, if $n = 7$ we have the points $(x_1, y_1);...;(x_7, y_7)$.

The Incan altitude-azimuth coordinate system

Altitude is the angular distance of a star above the local horizon. Azimuth is the angular distance of a star object from the local (North), measured along the horizon. Reinhard (2002) has written of the importance to the Inca of cardinal directions.

How did the Incas calculate the azimuth and the altitude of an object?

As far as is known, the astronomical instruments and devices of the Incas were of the simplest character. To calculate the approximate altitude (e.g., *pata* means 'above' in Quechua, *orqopata* = above the mountain) and azimuth for a star, the Incas needed to estimate those angular distances. The Incas also needed to keep in mind that since stars change their position with respect to horizon throughout the night, their altitude-azimuth position changes, as this system was based on the observer. The second way of specifying star positions is the equatorial coordinate system. This system is fixed with respect to the stars so, unlike the altitude-azimuth system, a star's position does not depend on the observer's location or time. Because of this, the Incas may have preferred using this system.

After analyzing the values in the pairing quadrants, I have recognized the following three different pairs of equations (*x*, *y*), resulting in three distinct types of stellar configuration (i.e., star maps), which are very similar to the configuration of the brightest stars in the Pleiades cluster.

Star map I

-f(*x*) (Figure 7 and Table 1).

The general solution of the equation in *x*-coordinates (Table 1) is given by

$$x_i = \frac{S_{(\text{knot values})}}{S_{(\text{long knot values})}} \tag{1}$$

-f(*y*) (Figure 7 and Table 1).

The general solution of the equation in *y*-coordinates (Table 1) is given by

$$y_i = Z_{(A-B)} \times Z_{(\text{long knot values})} \tag{2}$$

According to Table 1, from our Regression Equation $y = 425.57 + 0.35x$, the *y*-intercept is 425.57; this means that when *x* is 0, *y* is 425.57.

Star map II

-f(*x*) (Figure 8 and Table 2).

The general solution of the equation in *x*-coordinates (Table 2) is given by

$$x_i = S_{(\text{long knot values})} \tag{3}$$

-f(*y*) (Figure 8 and Table 2).

The general solution of the equation in *y*-coordinates (Table 2) is given by

$$y_i = S_{(\text{Long knot values})} \tag{4}$$

According to Table 2, from our Regression Equation $y = 105.31–2.95x$, the *y*-intercept is 105.31; this means that when *x* is 0, *y* is 105.31.

Star map III

-f(*x*) (Figure 9 and Table 3).

The general solution of the equation in *x*-coordinates (Table 3) is given by

$$x_i = Z_{(A \times B)} \text{Crossed strings} \tag{5}$$

-f(*y*) (Figure 9 and Table 3).

Finally, the general solution of the equation in *y*-coordinates (Table 3) is given by

$$y_i = S_{\text{(Long knot values)}} \tag{6}$$

According to Table 3, from our Regression Equation $y = 425.57+0.35x$, the *y*-intercept is 425.57; this means that when *x* is 0, *y* is 425.57.

As noted above (Figure 11), *khipu* records contain numerical values based on the base-ten system (Locke, 1923), and the numerical values of knots tied onto pendant strings (that is, each series of knots) from 1 to 21 of our sample are shown in Figure 11. The long knots are illustrated using a group of horizontal dashes with a diagonal slash across the group. The value represented by the pendant string is equal to the number of ridges.

Arranging the pendant strings in pairs, from pendant string 8 to pendant number string 21, I observed seven pairs of knots (“A” and “B”), resulting in a seven-step pyramid (see Figure 12). Assuming that “A” and “B” long knots (that is, determined by the distribution of Z-knots) were used as coordinates, I observed that the sum of every pair of Z-long knot values (the *y*-coordinate) multiplied by its corresponding S-long knot value results in an S-coordinate, the *x*-coordinate.

Results

Because two correlated dependent variables (which are determined by the distribution of the S- and Z-knots) were used, I performed a linear regression analysis for translating the values recorded on the *khipu* sample into an apparently realistic representation and describing a general graphical representation of the information recorded in the pairing quadrants (as determined by the distribution of the S- and Z-knots) to obtain an overall view of the *khipu*'s subject matter. In this regard, it is worth noting that the observed distribution of the long-knot values, that is, valued according to their number of turns, from 2 to 9 along two perpendicular axes (x, y), by considering a linear relationship between the x-axis (the S-knot values) and the y-axis (the Z-knot values) corresponds to the seven brightest stars in the Pleiades cluster. The values for the "S" and "Z" knots are shown in Figures 11 and 12 and Tables 2 and 4. The values represent the Incan calendar of 12 sidereal lunar months of 27.32 days (328 days in total). That is, the total number of *huacas* ("holy things" = 328), providing the foundation for Zuidema's (1964, 1977) interpretation of the Inca calendar.

Degree of Concordance

An analysis using Pearson's correlation coefficient indicates a statistically significant linear relationship between the relative positions of the seven optically brightest stars in the Pleiades star cluster (Figures 3 and 5) and the S- and Z-long knot values as scatter point sets used for interpolation (see Figures 7, 8 and 9). The results of the Pearson's correlation provided statistical support for there being a high correlation for the X position $r(7) = 0.73$ ($df = 5$; $p < 0.0617$) and a moderate correlation for the Y position $r(7) = 0.43$ ($df = 5$; $p < 0.3397$). Three graphical representations (scatter diagrams) of the data pairs were drawn. As Figures 7, 8 and 9 show, this *khipu* indicated that two stars of the Pleiades (Maia and Taygeta) changed their positions on the coordinate grid, whereas the positions for each of the remaining stars (Alcyone, Atlas, Electra, Merope, and Celaeno) remained almost unchanged on the coordinate grid.

Discussion

Interpretation of the coordinate grids

In my main hypothesis, I postulated that the knot makers/keepers could have developed a method of defining the location of points on a plane. The findings above support my hypothesis, which suggests that *khipu* sample VA 42527 records a map of the Pleiades star cluster. After comparing the seven stars of the Pleiades represented by points on the Cartesian coordinate plane shown in Figures 7, 8 and 9 with the Pleiades cluster (Figures 3 and 5), we can conclude that they are indeed similar. According to Randall (1987), documentary, ethnographic, and archaeological evidence suggests that the Incas may have related the Pleiades celestial position in spring and fall with the seasonal limits of the frost-free season; however, the differences between these three types of grids (Figures 7, 8 and 9) suggest probable past observations made by Incas of the brightening of a star (that is, a nova) in the Pleiades cluster. One interpretation of this *khipu* sample is that it is an accurate representation of a close conjunction of Venus with the Pleiades. Although Venus passes the Pleiades every year, it passes particularly close by every 8 years. The Incas adopted the equatorial coordinate system, or the right ascension and the declination to show position of the 7 brightest stars in the Pleiades cluster on the celestial sphere.

Conclusion

What is most valuable is that the Incas provided values of the coordinates of the 7 brightest stars in the Pleiades cluster. The idea of the celestial north pole as the only center of the sky was never abandoned by the Incas. This affected the type of instruments installed, and consequently the coordinate system employed. While not exactly like the *x*- and *y*-axes we use today, the Incas seemingly used an idea similar to the modern coordinate plane. The Inca officials specializing in astrological *khipu* appear to have learned to plot points on the coordinate plane and to read the coordinates from a graph because that information could be numerically encoded on a *khipu*. Apparently, the Incas developed a method of determining the position of stars very accurately. The present study confirms that the Incas recorded measurements of the positions of the brightest stars of the Pleiades (Figures 7, 8 and 9) with sufficient accuracy to be able to make these kinds of comparisons. I conclude from these results that the Inca astronomers developed a novel coordinate plane in which the seven stars of the Pleiades could be described.

Although there is no other confirmatory evidence from other works that knot makers/keepers invoked plots of the Pleiades star cluster, this apparently inconsequential detail has developed into a key theme, that the Incas developed a celestial coordinate system. It is clear that the knot makers/keepers introduced the novel idea of specifying the position of a point or object on a surface using two intersecting axes as measuring guides. Like the Mayas, the adoption of the equatorial coordinate system was a unique contribution of the Incas to pre-Columbian astronomy, as ancient astronomers all used the ecliptic coordinate system to mark the position of fixed stars. Based on their own observations (see Figures 7, 8 and 9), the Incas deduced that the stars were constantly changing their position in the celestial sphere. The Inca thus became the first astronomers in the pre-Columbian America to study the movement of stars, about 200 years before English astronomer Edmund Halley (1656–1742). The Incan celestial coordinate system was, thus, an old coordinate system for mapping positions on the celestial sphere. I can conclude that the organization of information according to knot directionality, that is to say, the variability in the construction of knots resulting in Z- and S-knots, is an important element of the *khipu* information system.

It seems likely that at the time of the Spanish Conquest some astronomical knowledge of the Incas had either been lost by the Spanish chronicles, or more likely they had simply deliberately withheld important astronomical knowledge from the Inquisitors. It is important to remember that Galileo's book, *Dialogue Concerning the Two Chief World Systems*, was published in 1632, with formal authorization from the Inquisition and papal permission (Pope Paul V), exactly 100 years after the Spanish conquest of the Inca Empire (1532).

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