6-5 Study Guide and Intervention

Operations with Radical Expressions

Simplify Radicals

Product Property of Radicals	For any real numbers *a* and *b*, and any integer *n* > 1: 1. if *n* is even and *a* and *b* are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. 2. if *n* is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

To simplify a square root, follow these steps:

1. Factor the radicand into as many squares as possible.

2. Use the Product Property to isolate the perfect squares.

3. Simplify each radical.

Quotient Property of Radicals	For any real numbers *a* and *b* ≠ 0, and any integer *n* > 1, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

To eliminate radicals from a denominator or fractions from a radicand, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

Example 1: Simplify $\sqrt[3]{-6a^5b^7}$.

$\sqrt[3]{-16a^5b^7} = \sqrt{(-2)^3 \cdot 2 \cdot a^3 \cdot a^2 \cdot (b^2)^3 \cdot b}$

$= -2ab^2\ \sqrt[3]{2a^2b}$

Example 2: Simplify $\sqrt{\frac{8x^3}{45y^5}}$

$\sqrt{\frac{8x^3}{45y^5}} = \frac{\sqrt{8x^3}}{\sqrt{45y^5}}$	Quotient Property
$= \frac{\sqrt{(2x)^2 \cdot 2x}}{\sqrt{(3y^2)^2 \cdot 5y}}$	Factor into squares.
$= \frac{\sqrt{(2x)^2} \cdot \sqrt{2x}}{\sqrt{(3y^2)^2} \cdot \sqrt{5y}}$	Product Property
$= \frac{2\lvert x\rvert\sqrt{2x}}{3y^2\sqrt{5y}}$	Simplify.
$= \frac{2\lvert x\rvert\sqrt{2x}}{3y^2\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}}$	Rationalize the denominator.
$= \frac{2\lvert x\rvert\sqrt{10xy}}{15y^3}$	Simplify.

Exercises

Simplify.

1. $5\sqrt{54}$

2. $\sqrt[4]{32a^9b^{20}}$

3. $\sqrt{75x^4y^7}$

4. $\sqrt{\frac{36}{125}}$

5. $\sqrt{\frac{a^6b^3}{98}}$

6. $\sqrt[3]{\frac{p^5q^3}{40}}$

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6-5 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Operations with Radicals When you add expressions containing radicals, you can add only like terms or **like radical expressions.** Two radical expressions are called *like radical expressions* if both the indices and the radicands are alike.

To multiply radicals, use the Product and Quotient Properties. For products of the form $(a\sqrt{b} + c\sqrt{d}) \cdot (e\sqrt{f} + g\sqrt{h})$, use the FOIL method. To rationalize denominators, use **conjugates.** Numbers of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called conjugates. The product of conjugates is always a rational number.

Example 1: Simplify $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$.

$2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125} = 2\sqrt{5^2 \cdot 2} + 4\sqrt{10^2 \cdot 5} - 6\sqrt{5^2 \cdot 5}$	Factor using squares.
$= 2 \cdot 5 \cdot \sqrt{2} + 4 \cdot 10 \cdot \sqrt{5} - 6 \cdot 5 \cdot \sqrt{5}$	Simplify square roots.
$= 10\sqrt{2} + 40 + \sqrt{5} - 30\sqrt{5}$	Multiply.
$= 10\sqrt{2} + 10\sqrt{5}$	Combine like radicals.

Example 2: Simplify $(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$.

$(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$

$= 2\sqrt{3} \cdot \sqrt{3} + 2\sqrt{3} \cdot 2\sqrt{2} - 4\sqrt{2} \cdot \sqrt{3} - 4\sqrt{2} \cdot 2\sqrt{2}$

$= 6 + 4\sqrt{6} - 4\sqrt{6} - 16$

$= -10$

Example 3: Simplify $\frac{2 - \sqrt{5}}{3 + \sqrt{5}}$.

$\frac{2 - \sqrt{5}}{3 + \sqrt{5}} = \frac{2 - \sqrt{5}}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$

$= \frac{6 - 2\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2}{3^2 - (\sqrt{5})^2}$

$= \frac{6 - 5\sqrt{5} + 5}{9 - 5}$

$= \frac{11 - 5\sqrt{5}}{4}$

Exercises

Simplify.

1. $3\sqrt{2} + \sqrt{50} - 4\sqrt{8}$

2. $\sqrt{20} + \sqrt{125} - \sqrt{45}$

3. $\sqrt{300} - \sqrt{27} - \sqrt{75}$

4. $\sqrt[3]{81} \cdot \sqrt[3]{24}$

5. $\sqrt[3]{2}\left(\sqrt[3]{4} + \sqrt[3]{12}\right)$

6. $2\sqrt{3}(\sqrt{15} + \sqrt{60})$

7. $(2 + 3\sqrt{7})(4 + \sqrt{7})$

8. $(6\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + \sqrt{2})$

9. $(4\sqrt{2} - 3\sqrt{5})(2\sqrt{20} + 5)$

10. $\frac{5\sqrt{48} + \sqrt{75}}{5\sqrt{3}}$

11. $\frac{4 + \sqrt{2}}{2 - \sqrt{2}}$

12. $\frac{5 + 3\sqrt{3}}{1 - 2\sqrt{3}}$

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6-5 Skills Practice
Operations with Radical Expressions

Simplify.

1. $\sqrt{24}$

2. $\sqrt{75}$

3. $\sqrt[3]{16}$

4. $-\sqrt[4]{48}$

5. $4\sqrt{50x^5}$

6. $\sqrt[4]{64a^4b^4}$

7. $\sqrt[3]{-8d^2f^5}$

8. $\sqrt{\frac{25}{36}r^2t}$

9. $-\sqrt{\frac{3}{7}}$

10. $\sqrt[3]{\frac{2}{9}}$

11. $\sqrt{\frac{2g^3}{5z}}$

12. $(3\sqrt{3})\ (5\sqrt{3})$

13. $(4\sqrt{12})\ (3\sqrt{20})$

14. $\sqrt{2} + \sqrt{8} + \sqrt{50}$

15. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$

16. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$

17. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$

18. $(2 + \sqrt{3})\ (6 - \sqrt{2})$

19. $(1 - \sqrt{5})\ (1 + \sqrt{5})$

20. $(3 - \sqrt{7})\ (5 + \sqrt{2})$

21. $(\sqrt{2} - \sqrt{6})^2$

22. $\frac{3}{7 - \sqrt{2}}$

23. $\frac{4}{3 + \sqrt{2}}$

24. $\frac{5}{8 - \sqrt{6}}$