

Looking Ahead to Chapter 9

Focus

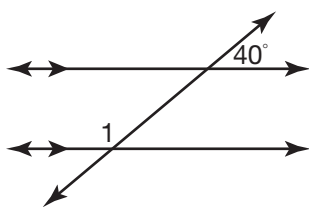
In Chapter 9, you will learn the properties of quadrilaterals, including kites, trapezoids, parallelograms, rhombi, rectangles, and squares. You will also learn the sums of exterior and interior angle measures in polygons.

Chapter Warmup

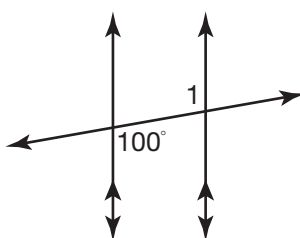
Answer these questions to help you review skills that you will need in Chapter 9.

Find the measure of $\angle 1$.

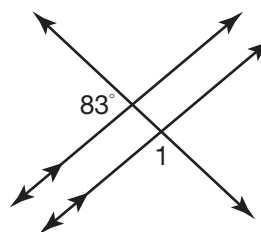
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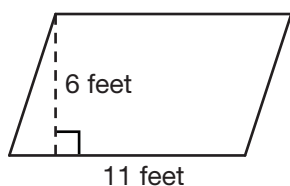
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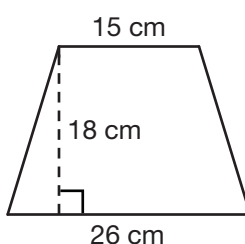
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Find the area of the quadrilateral.

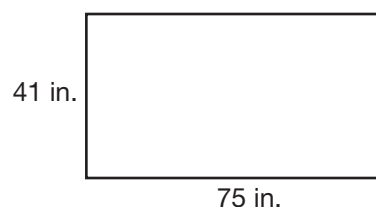
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5.



6.



Read the problem scenario below.

Your friend cuts a triangle out of a piece of poster board. She measures each angle of the triangle and tells you that the measure of the first angle is twice the measure of the second angle, and the measure of the third angle is 16 degrees more than the measure of the second angle.

7. Find the measures of the three angles of the triangle.

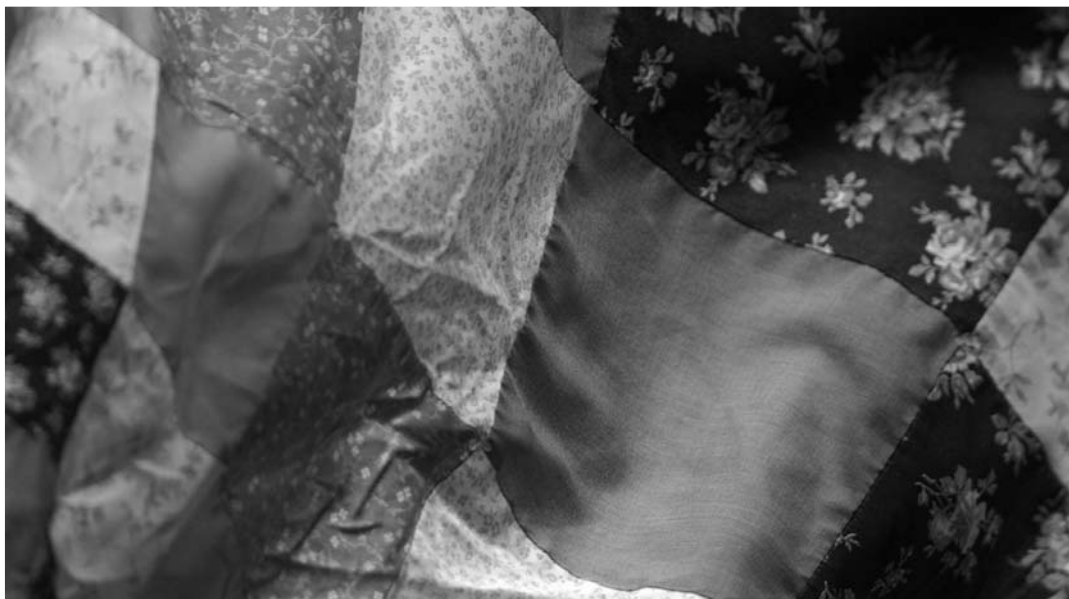
Key Terms

tessellation ■ p. 351
parallelogram ■ p. 352
rhombus ■ p. 352
rectangle ■ p. 352, 369
square ■ p. 353, 369
trapezoid ■ p. 353
kite ■ p. 353

Venn diagram ■ p. 355
diagonal ■ p. 357
isosceles trapezoid ■ p. 360
base ■ p. 360
base angles ■ p. 360
opposite sides ■ p. 363
consecutive sides ■ p. 363

consecutive angles ■ p. 363
opposite angles ■ p. 363
convex polygon ■ p. 375
interior angle ■ p. 375
regular polygon ■ p. 377
exterior angle ■ p. 377

Quadrilaterals



The earliest evidence of quilting is an ivory carving from the 35th century BC. It shows the king of the Egyptian First Dynasty wearing a quilted cloak. In Lesson 9.1, you will examine quilts formed by using tessellations.

9.1 Quilting and Tessellations

Introduction to Quadrilaterals ■ p. 351

9.2 When Trapezoids Are Kites

Kites and Trapezoids ■ p. 357

9.3 Binocular Stand Design

Parallelograms and Rhombi ■ p. 363

9.4 Positive Reinforcement

Rectangles and Squares ■ p. 369

9.5 Stained Glass

Sum of the Interior Angle Measures in a Polygon ■ p. 373

9.6 Pinwheels

Sum of the Exterior Angle Measures in a Polygon ■ p. 377

9.7 Planning a Subdivision

Rectangles and Parallelograms in the Coordinate Plane ■ p. 383

Objectives

In this lesson, you will:

- Classify quadrilaterals.
- Name quadrilaterals and parts of quadrilaterals.
- Draw a Venn diagram that shows the relationships among quadrilaterals.

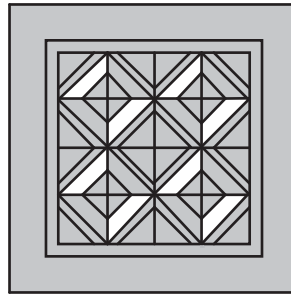
Key Terms

- tessellation
- parallelogram
- rhombus
- rectangle
- square
- trapezoid
- kite
- Venn diagram



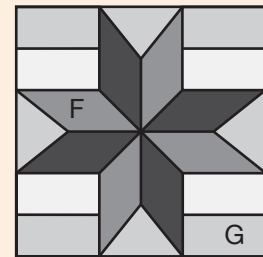
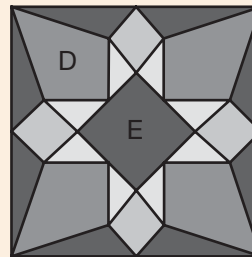
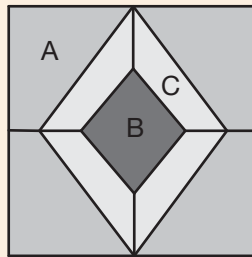
SCENARIO Quilts are often made of repeating geometric shapes that form *tessellations*. A **tessellation** of a plane is a collection of polygons that are arranged so that they cover the plane with no holes or gaps.

Some quilts are created in a block pattern, such as the one shown below. Copies of these blocks are created by sewing different patterns or colors together. Then the blocks are sewn together to form the quilt.



Problem 1

Describing Quadrilaterals



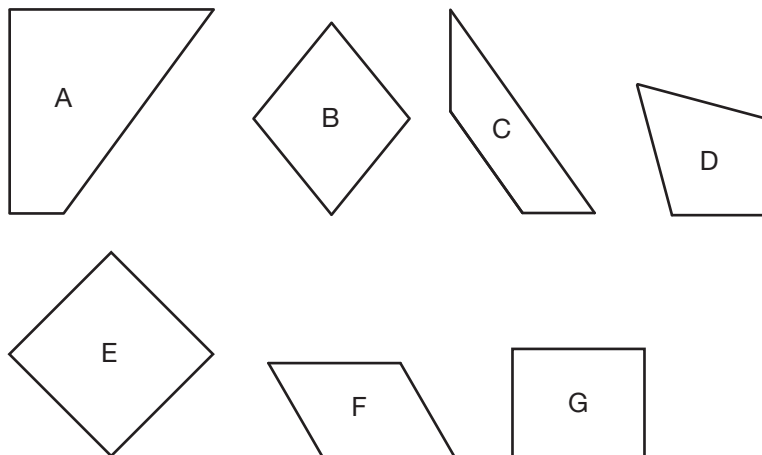
A. Shapes A through G labeled in the quilt squares above are polygons. What is the classification for these polygons by the number of sides in the polygon?

B. How are these polygons the same? How are they different? Use complete sentences in your answer.



Investigate Problem 1

Use the quadrilaterals below to answer the following questions.



1. For each quadrilateral, use your protractor to determine which interior angles are right angles. Mark these angles as right angles on the quadrilaterals.
2. For each quadrilateral, use your protractor to determine which interior angles are congruent. Mark the congruent angles on the quadrilaterals.
3. For each quadrilateral, determine which sides are congruent. Mark the congruent sides on the quadrilaterals.
4. For each quadrilateral, determine which sides are parallel. Mark the parallel sides on the quadrilaterals.
5. Name the quadrilaterals in which both pairs of opposite sides are parallel.

These quadrilaterals are **parallelograms**.

6. Name the quadrilateral(s) in which both pairs of opposite sides are parallel and all the sides are congruent.

These quadrilaterals are **rhombi**. The singular form of rhombi is *rhombus*.

7. Name the quadrilateral(s) in which both pairs of opposite sides are parallel and the interior angles are right angles.

These quadrilaterals are **rectangles**.

Investigate Problem 1

8. Name the quadrilateral(s) in which both pairs of opposite sides are parallel, the interior angles are right angles, and the sides are congruent.

These quadrilaterals are **squares**.

9. Name the quadrilateral(s) in which only one pair of opposite sides are parallel.

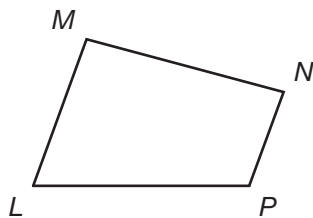
These quadrilaterals are **trapezoids**.

10. Which of the quadrilaterals has yet to be classified by its sides or angles?

Describe this quadrilateral in terms of its sides. Use complete sentences in your answer.

This quadrilateral is a *kite*. A **kite** is a quadrilateral in which two pairs of adjacent sides are congruent, but the opposite sides are not congruent.

11. Quadrilaterals are named by their vertices. For instance, the quadrilateral below can be named quadrilateral $LMNP$, quadrilateral $MLPN$, but not quadrilateral $NLMP$.



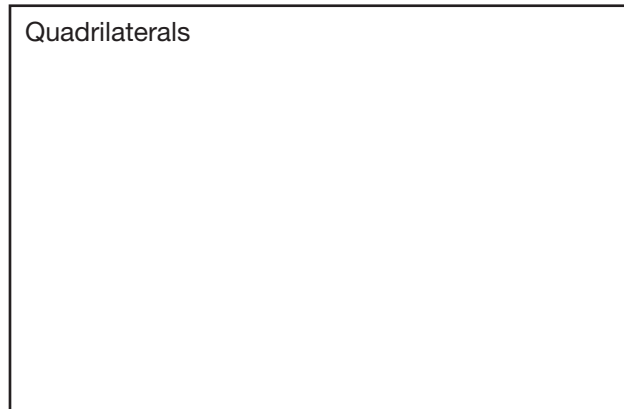
What does this tell you about how a quadrilateral must be named? Use a complete sentence in your answer.

Investigate Problem 1

12. Draw quadrilateral $WXYZ$ so that the quadrilateral is a parallelogram that is not a rectangle. Then name the pairs of parallel sides. Name any congruent angles.
13. Decide whether the following statements are true or false. Explain your reasoning. Use complete sentences in your answer.
- All rectangles are squares.
- All squares are rectangles.
- All trapezoids are parallelograms.
- All rectangles are parallelograms.
- All squares are rhombi.
- All quadrilaterals are parallelograms.

Investigate Problem 1

14. You can use a Venn diagram to show the relationship between different kinds of quadrilaterals.



First, inside the rectangle, draw a large circle that represents all parallelograms.

Now add a circle to the diagram that represents all rhombi. If no rhombus is a parallelogram, then draw the circle that represents the rhombi so that it is outside of the circle that represents parallelograms. If every rhombus is a parallelogram, draw the circle that represents the rhombi inside the circle that represents parallelograms. If some, but not all rhombi are parallelograms, draw the circle that represents the rhombi so that it intersects the circle that represents parallelograms.

Draw the circle that represents all rectangles. Then draw a circle that represents all kites. Then draw a circle that represents all trapezoids.

Complete the Venn diagram by labeling the part of the diagram that represents all squares.



Objectives

In this lesson,
you will:

- Determine properties of kites.
- Determine properties of isosceles trapezoids.

Key Terms

- diagonal
- isosceles trapezoid
- base
- base angles

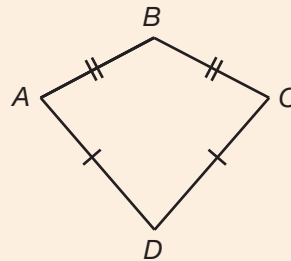
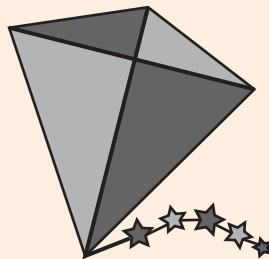


SCENARIO Simple kites are often in the shapes of simple geometric forms. A diamond kite is in the shape of a (geometric) kite, and a delta conyne kite is in the shape of a trapezoid.

Problem 1

Diamond Kites

A diamond kite and the outline of the kite are shown below.



- A.** Draw a *diagonal* of the kite from vertex B to vertex D . A **diagonal** is a line segment that is drawn from a vertex to a nonadjacent vertex.
- B.** What is the relationship between $\triangle ABD$ and $\triangle CBD$? Explain your reasoning. Use complete sentences in your answer.
- C.** Are $\angle A$ and $\angle C$ congruent? Explain your reasoning. Use a complete sentence in your answer.
- D.** Can you determine whether $\angle B$ and $\angle D$ are congruent without measuring the angles? Explain your reasoning. Use a complete sentence in your answer.

Problem 1 Diamond Kites

- E.** What do you know about $\angle ABD$ and $\angle CBD$? What do you know about $\angle ADB$ and $\angle CDB$? Explain your reasoning. Use complete sentences in your answer.
- F.** What does part (E) tell you about \overline{BD} ? Use a complete sentence in your answer.
- G.** Suppose that $\angle B$ and $\angle D$ are congruent. Then how does $m\angle ABD$ compare to $m\angle ADB$? Explain your reasoning. Use complete sentences in your answer.

Because $m\angle ABD = m\angle ADB$, what kind of triangle is $\triangle ABD$? Use a complete sentence to explain.

How does AB compare to AD ? Use a complete sentence to explain.

Is this possible? Why or why not? Use complete sentences in your answer.

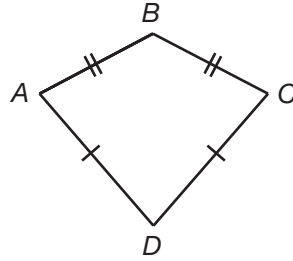
- H.** Complete the following statement:

If a quadrilateral is a kite, then only _____ of _____ angles are congruent.

Investigate Problem 1



1. The outline of the diamond kite is again shown below. Draw both diagonals of the kite on the figure and label the point of intersection as point E .



From Problem 1, we know that $\triangle ABD \cong \triangle CBD$. We also know that \overline{BD} bisects $\angle B$. What does this tell you about the relationship between $\triangle ABE$ and $\triangle CBE$? Explain your reasoning. Use complete sentences in your answer.

What do you know about the relationship between $\angle AEB$ and $\angle CEB$? Explain your reasoning. Use a complete sentence in your answer.

Complete the following statement:

Angle AEB and $\angle CEB$ form a _____ pair.

What can you conclude about $m\angle AEB$ and $m\angle CEB$? Explain your reasoning. Use a complete sentence in your answer.

Complete the following statement:

The diagonals of a kite are _____.

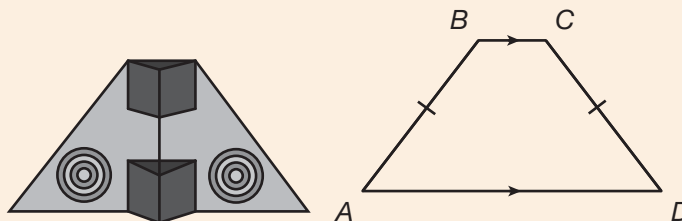
2. Consider \overline{AC} , the diagonal that connects the vertices whose angle measures are congruent. Where does \overline{BD} intersect \overline{AC} ? How do you know? Explain your reasoning. Use complete sentences in your answer.

What relationship does this give between \overline{BD} and \overline{AC} ? Use a complete sentence in your answer.

Problem 2 Delta Conyne Kites



A delta conyne kite and the outline of the kite is shown below.



- A. What do you notice about the nonparallel sides of the trapezoid? Use a complete sentence in your answer.

This trapezoid is called an **isosceles trapezoid**.

- B. How do you think $m\angle A$ compares to $m\angle D$? How do you think $m\angle B$ compares to $m\angle C$? Use complete sentences to explain your reasoning.

Use a protractor to verify your answers.

- C. Recall that the parallel sides of a trapezoid are the **bases** of the trapezoid. The pair of angles that share a base as a side are called a pair of **base angles**.

Name the pairs of base angles in the trapezoid above.

Complete the following statement:

The base angles of an isosceles trapezoid are _____.

Label this information on the figure above.

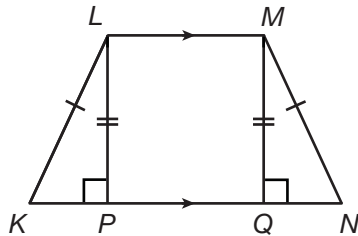
Investigate Problem 2

1. Draw the diagonals of the trapezoid on the figure in Problem 2. Then sketch $\triangle ABC$ and $\triangle DCB$ separately below. Mark any information on your sketch that you know about the relationships between angles and sides of the triangles.

What can you conclude about the triangles? Use a complete sentence to explain your reasoning.

Write a statement that tells what you know about the lengths of the diagonals of an isosceles trapezoid. Use a complete sentence in your answer.

2. Complete the paragraph proof below that shows that the base angles of an isosceles trapezoid are congruent.



We are given that $\overline{KL} \cong$ _____ and _____ $\parallel \overline{KN}$. First draw perpendicular line segments from vertex L and vertex M to _____ to form \overline{LP} and \overline{MQ} . Segment LP and \overline{MQ} are _____ because $\overline{LM} \parallel \overline{KN}$, and the distance between two parallel lines is the same from any point on either line. Angle KPL and $\angle NQM$ are right angles because $\overline{LP} \perp$ _____ and $\overline{MQ} \perp$ _____. So $\triangle KLP$ and $\triangle NMQ$ are _____ triangles with a pair of congruent legs and congruent hypotenuses. By the _____ Theorem, $\triangle KLP \cong \triangle NMQ$. Because $\angle K$ and $\angle N$ are _____ angles of congruent triangles, the angles are congruent. Angle L and $\angle M$ can be shown to be congruent in a similar way.



Objectives

In this lesson, you will:

- Determine properties of parallelograms.
- Determine properties of rhombi.



SCENARIO Sometimes, binoculars are better for viewing stars than telescopes. Because it is not reasonable for a person to hold the binoculars for an extended period of time, there are binocular stands that can be used to hold the binoculars. Part of the structure for this stand is in the shape of a parallelogram.

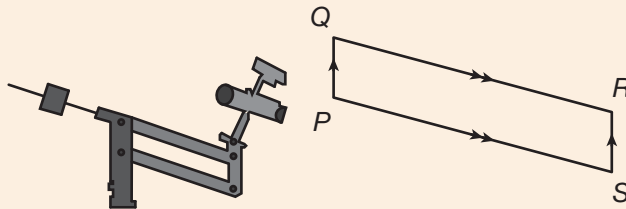
Key Terms

- opposite sides
- consecutive sides
- consecutive angles
- opposite angles



Problem 1 Holding It Steady

A typical binocular stand and an outline of the parallelogram part of the stand are shown below.



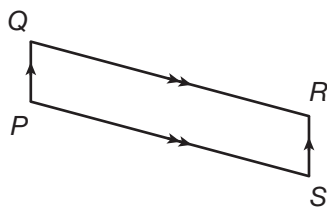
- A. Two sides of a parallelogram that do not intersect are **opposite sides**. Name the pairs of opposite sides in parallelogram $PQRS$.
- B. Two sides of a parallelogram that intersect are **consecutive sides**. Name the pairs of consecutive sides in parallelogram $PQRS$.
- C. Two angles of a parallelogram that have a side in common are **consecutive angles**. Name the pairs of consecutive angles in parallelogram $PQRS$.
- D. Two angles of a parallelogram that do not have a side in common are **opposite angles**. Name the pairs of opposite angles in parallelogram $PQRS$.

Problem 1 Holding It Steady

- E. What do you think is the relationship between the opposite sides of a parallelogram? What do you think is the relationship between the opposite angles of a parallelogram? What do you think is the relationship between the consecutive angles of a parallelogram? Use complete sentences in your answer.

Investigate Problem 1

1. The parallelogram from Problem 1 is shown below. Draw the diagonal that connects vertices P and R .



How does $m\angle PRS$ compare to $m\angle RPQ$? How does $m\angle QRP$ compare to $m\angle SPR$? Explain your reasoning. Use a complete sentence in your answer.

What can you conclude about $\triangle QRP$ and $\triangle SPR$? Explain your reasoning. Use a complete sentence in your answer.

What can you conclude about the opposite sides of a parallelogram? Explain your reasoning. Use a complete sentence in your answer.

What can you conclude about $\angle Q$ and $\angle S$? Explain your reasoning. Use a complete sentence in your answer.

Investigate Problem 1

Can you draw a conclusion about $\angle R$ and $\angle S$? If so, what is your conclusion? Explain your reasoning. Use a complete sentence in your answer.

Which other pairs of angles have this same relationship?

Are $\angle P$ and $\angle R$ congruent? Explain your reasoning. Use complete sentences in your answer.



2. Complete the following statements:

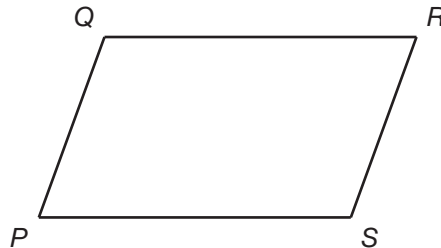
The opposite sides of a parallelogram are _____.

The opposite angles of a parallelogram are _____.

The consecutive angles in a parallelogram are _____
_____.



3. Draw the diagonals on the parallelogram below. Label the intersection point as point T . What do you know about the opposite sides of the parallelogram? Label this information on your figure.



Use what you learned in Question 1 to name the congruent triangles formed by the parallelogram and a diagonal.

On the figure above, label the congruent angles of the congruent triangles you named above. What can you conclude about $\triangle QTR$ and $\triangle STP$? Explain your reasoning. Use complete sentences in your answer.

Investigate Problem 1

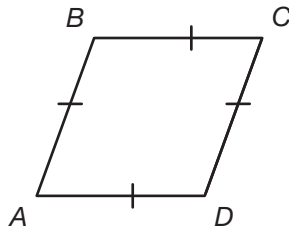
What can you conclude about $\triangle QTP$ and $\triangle STR$? Explain your reasoning. Use complete sentences in your answer.

What can you conclude about \overline{PT} , \overline{RT} , \overline{QT} , and \overline{ST} ? Explain your reasoning. Use a complete sentence in your answer.

Complete the following statement:

The diagonals of a parallelogram _____ each other.

4. Consider the rhombus shown below. Draw a diagonal that connects vertices A and C .



What do you know about $\triangle ABC$ and $\triangle ADC$? Explain your reasoning. Use a complete sentence in your answer.

What can you conclude about $\angle BAC$ and $\angle DAC$? What can you conclude about $\angle BCA$ and $\angle DCA$? Use complete sentences to explain your reasoning.

How does \overline{AC} relate to $\angle A$? How does \overline{AC} relate to $\angle C$? Use a complete sentence in your answer.

Complete the following statement:

The diagonal of a rhombus _____ a pair of opposite angles.

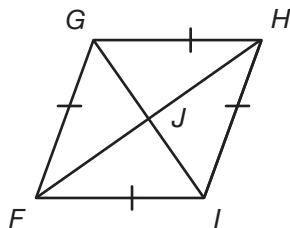
Take Note

Because a rhombus is a parallelogram, the properties of parallelograms are true for rhombi.

9

Investigate Problem 1

5. Consider the rhombus below and its diagonals.



Triangle GHI is congruent to $\triangle IHI$. Why? Use complete sentences in your answer.

How do $m\angle HJG$ and $m\angle HJI$ compare? Use a complete sentence in your answer.

Triangle GJF is congruent to $\triangle IJF$. Why? Use complete sentences in your answer.

How do $m\angle GJF$ and $m\angle IJF$ compare? Use a complete sentence in your answer.

Triangle GHI is congruent to $\triangle GFJ$. Why? Use complete sentences in your answer.

How do $m\angle HJG$ and $m\angle GJF$ compare? Use a complete sentence in your answer.

What is the relationship between $\angle HJG$, $\angle HJI$, $\angle IJF$, and $\angle GJF$? Explain your reasoning. Use complete sentences in your answer.

Investigate Problem 1

What does this tell you about the measures of the angles formed by the intersection of the diagonals? Explain your reasoning. Use complete sentences in your answer.

Complete the following statement:

The diagonals of a rhombus are _____.



9.4

Positive Reinforcement

Rectangles and Squares

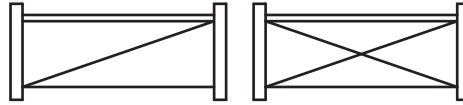
Objective

In this lesson, you will:

- Determine properties of rectangles and squares.



SCENARIO Fences built to keep livestock in enclosed areas are often built in rectangular sections. These sections are made stronger by adding one or two diagonal wire braces as shown below.



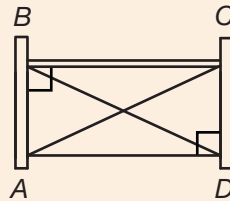
Key Terms

- rectangle
- square

Problem 1

Making the Fence Stronger

A section of rectangular fence with two diagonal braces is shown below.



- A. A rancher is building a section of fence that is 8 feet long and 5 feet tall between two fence posts. Label this information on the figure above. About how much wire does the rancher need for each diagonal brace? Show all your work and use a complete sentence in your answer. Round your answer to the nearest tenth, if necessary.

Take Note

Because rectangles and squares are parallelograms, the properties of parallelograms are true for these quadrilaterals as well.

Because squares are rhombi, the properties of rhombi are true for squares as well.

Problem 1 Making the Fence Stronger

- B.** What does part (A) tell you about the diagonals of rectangle $ABCD$? Is this true for all rectangles? Use complete sentences to explain your reasoning.
- C.** Complete the following statement:
The diagonals of a rectangle are _____.

Investigate Problem 1

1. Do you think that the diagonals of every rectangle are perpendicular? If so, give an argument that supports your answer. If not, give an example that shows that the diagonals are not perpendicular. Explain your reasoning. Use complete sentences in your answer.
2. Do you think that the diagonals of every square are perpendicular? If so, give an argument that supports your answer. If not, give an example that shows that the diagonals are not perpendicular. Use complete sentences in your answer.

Summary

Properties of Quadrilaterals

In this chapter, you have learned the following properties of quadrilaterals.

- A **parallelogram** is a quadrilateral in which the opposite sides are parallel.

The opposite sides of a parallelogram are congruent.

The opposite angles of a parallelogram are congruent.

The consecutive angles of a parallelogram are supplementary.

The diagonals of a parallelogram bisect each other.

- A **rhombus** is a parallelogram with four congruent sides.

A diagonal of a rhombus bisects a pair of opposite angles.

The diagonals of a rhombus are perpendicular.

- A **rectangle** is a parallelogram in which the angles are all right angles.

The diagonals of a rectangle are congruent.

- A **square** is a rectangle in which all four sides are congruent.

The diagonals of a square are perpendicular.

- A **kite** is a quadrilateral in which two pairs of adjacent sides are congruent but the opposite sides are not congruent.

In a kite, only one pair of opposite angles are congruent.

In a kite, the diagonal that joins the vertices with the congruent angles is bisected by the other diagonal.

The diagonals of a kite are perpendicular.

- A **trapezoid** is a quadrilateral in which exactly one pair of opposite sides is congruent.

- An **isosceles trapezoid** is a trapezoid in which the nonparallel sides are congruent.

The base angles of an isosceles trapezoid are congruent.

The diagonals of an isosceles trapezoid are congruent.



Sum of the Interior Angle Measures in a Polygon

Objectives

In this lesson, you will:

- Determine the sum of the interior angle measures in a convex polygon.
- Determine the measure of an interior angle of a regular polygon.
- Determine the number of sides in a regular polygon given the measure of an interior angle.

Key Terms

- convex polygon
- regular polygon
- interior angle

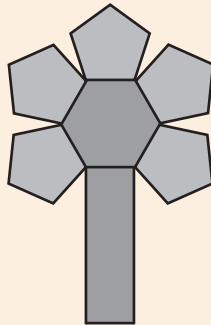


SCENARIO Modern stained glass artwork and windows are created by cutting out pieces of glass and fitting them together with a metal strip that is grooved to hold the glass. All the metal strips that hold pieces of glass in a window or artwork are “glued” together by using molten metal.

Problem 1

Stained Glass Flowers

A stained glass design is shown below.



- A. Identify the different kinds of polygons that are in the stained glass design. Use a complete sentence in your answer.

- B. Draw one diagonal in the quadrilateral. What kinds of polygons are formed by the diagonal and the quadrilateral? Use a complete sentence in your answer.

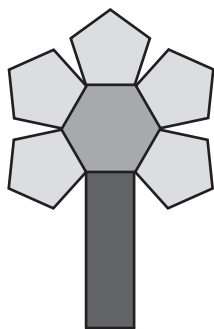
How many triangles are formed? Use a complete sentence in your answer.

What is the sum of the interior angle measures of one triangle? Use a complete sentence in your answer.

Problem 1 Stained Glass Flowers

What is the sum of the interior angle measures of the rectangle? Explain how you found your answer. Use a complete sentence in your answer.

What is the sum of the measures of the interior angles of any quadrilateral? Use a complete sentence in your answer.



- C.** Choose a pentagon from the stained glass design. Then choose one of the vertices from the pentagon and draw all of the diagonals that connect to this vertex.

How many triangles are formed by the diagonals? Use a complete sentence in your answer.

What is the sum of the interior angle measures of one triangle? Use a complete sentence in your answer.

What is the sum of the interior angle measures of any pentagon? Explain how you found your answer. Use a complete sentence in your answer.

- D.** Choose a hexagon from the stained glass design. Then choose one of the vertices from the hexagon and draw all of the diagonals that connect to this vertex.

How many triangles are formed by the diagonals?

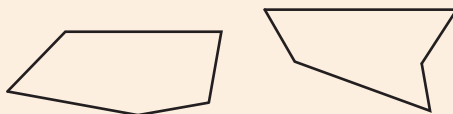
What is the sum of the interior angle measures of one triangle? Use a complete sentence in your answer.

Problem 1

Stained Glass Flowers

What is the sum of the interior angle measures of any hexagon? Explain how you found your answer. Use a complete sentence in your answer.

- E. The polygons that you have been considering so far are *convex polygons*. We are concerned only with convex polygons in this lesson and the next lesson. A **convex polygon** is a polygon in which no segments can be drawn to connect any two vertices so that the segment is *outside* the polygon. The polygon on the left is a convex polygon. The polygon on the right is not a convex polygon. Draw the line segment on the polygon that shows that it is not a convex polygon.



9

Investigate Problem 1



1. Use complete sentences to explain how the sum of the interior angle measures of a triangle can be used to find the sum of the interior angle measures of any polygon.

2. How does the number of diagonals that connect to a single vertex of the polygon relate to the number of sides in a polygon? Use a complete sentence in your answer.

How does the number of triangles that are formed by drawing all of the diagonals that connect to a single vertex of the polygon relate to the number of sides in a polygon? Use a complete sentence in your answer.

Investigate Problem 1

3. What is the sum of the measures of the interior angles of a heptagon (seven-sided polygon)? Show all your work and use a complete sentence in your answer.
4. Write a formula that you can use to find the sum of the interior angle measures of an n -gon. Explain your reasoning. Use complete sentences in your answer.
5. Use your formula to find the sum of the interior angle measures of a dodecagon (12-sided polygon). Show all your work and use a complete sentence in your answer.
6. Remember that a **regular polygon** is a polygon in which all sides are equal in length and all angles are equal in measure.
What is the measure of an interior angle of a regular pentagon? Explain how you found your answer. Use a complete sentence in your answer.

The measure of an interior angle of a regular polygon is 144° . How many sides does the regular polygon have? Show all your work and use a complete sentence in your answer.



Sum of the Exterior Angle Measures in a Polygon

Objective

In this lesson, you will:

- Determine the sum of the exterior angle measures in a polygon.

Key Terms

- regular polygon
- exterior angle

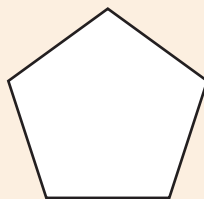


SCENARIO You've probably seen a pinwheel like the one shown below. This pinwheel was made by using a square piece of paper. We will use our knowledge of polygons to create our own pinwheels that are more complicated.

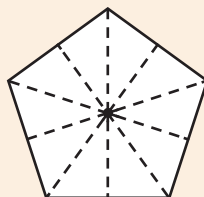
**Problem 1****Making the Cut**

Your pinwheel will be made by using a piece of paper that is cut into the shape of a regular pentagon.

- What is the measure of an interior angle in a regular pentagon? Use a complete sentence in your answer.
- On a sheet of paper, use a protractor and ruler to draw the largest regular pentagon you can. Then cut out the pentagon.

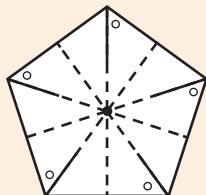


- At each vertex, fold your pentagon so that the fold bisects the vertex angle and then open the pentagon. Mark the point in the center of your pentagon where the folds meet.

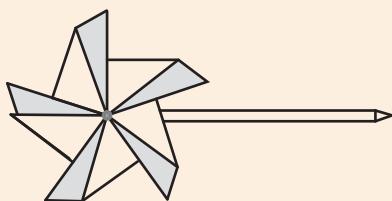


Problem 1 Making the Cut

- D. Cut along the fold at each vertex about halfway to the center. At the upper left corner of each flap, use a hole punch to punch a hole.



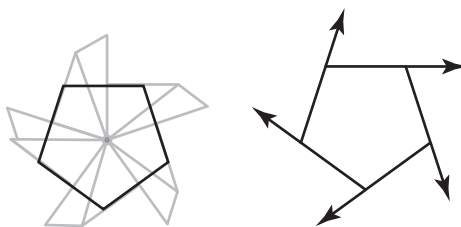
- E. Carefully pull each corner with the hole towards the center of the pentagon. Then carefully put a push pin through the holes and then through the center of the polygon into the eraser head of your pencil. Your pinwheel is complete.



Investigate Problem 1



1. You may have noticed that when you were joining the flaps in the center, a pentagon similar to the one you started with can be seen. Look for the similar pentagon in your pinwheel.



As with triangles, you can consider the exterior angles of convex polygons. Whenever you extend one side at a vertex, you create an exterior angle that is acute, obtuse, or right. Number the exterior angles of the pentagon above on the right.

If one exterior angle is drawn at each vertex, how many exterior angles are there for the regular pentagon? Use a complete sentence in your answer.

Investigate Problem 1

What is $m\angle 1$? Explain your reasoning. Use a complete sentence in your answer.

What is $m\angle 2$? Explain your reasoning. Use a complete sentence in your answer.

What is $m\angle 3$? Explain your reasoning. Use a complete sentence in your answer.

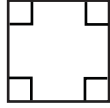
What is $m\angle 4$? Explain your reasoning. Use a complete sentence in your answer.

What is $m\angle 5$? Explain your reasoning. Use a complete sentence in your answer.

What is the sum of the measures of the exterior angles of the regular pentagon? Use a complete sentence in your answer.

Investigate Problem 1

2. Extend each vertex of the square below to create one exterior angle at each vertex.



What is the measure of an interior angle of a square?
Use a complete sentence in your answer.

Find the measure of each exterior angle. Explain how you found your answers. Use a complete sentence in your answer.

What is the sum of the measures of the exterior angles of a square? Use a complete sentence in your answer.

3. Extend each vertex of the regular hexagon below to create one exterior angle at each vertex.



What is the measure of an interior angle of a regular hexagon?
Show all your work and use a complete sentence in your answer.

Investigate Problem 1

Find the measure of each exterior angle. Explain how you found your answers. Use a complete sentence in your answer.

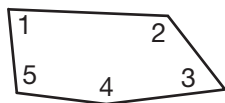
What is the sum of the measures of the exterior angles of a regular hexagon? Use a complete sentence in your answer.

4. Without drawing a regular octagon, find the sum of the measures of the exterior angles of a regular octagon. Show all your work and use complete sentences to explain how you found your answer.

5. Do you think that the sums that you found in Question 1 through Question 4 are the same for any polygon, regular or not?

Investigate Problem 1

6. The pentagon below is not regular. Extend each vertex of the pentagon to create one exterior angle at each vertex.



For each exterior angle, write an expression for its measure in terms of the measure of the adjacent interior angle.

Write the sum of your expressions above and simplify the resulting expression.

What is the sum of the interior angle measures of a pentagon? Use a complete sentence in your answer. Then substitute this sum into the expression above.

What is the sum of the measures of the exterior angles of any pentagon? Use a complete sentence in your answer.

7. Consider any n -gon. Write an expression for the sum of the measures of the interior angles of the n -gon.

Complete the following expression for the sum of the measures of the exterior angles of the n -gon.

$$n \cdot \underline{\hspace{1cm}}^\circ - (n - \underline{\hspace{1cm}}) \cdot 180^\circ$$

Simplify the expression to find the sum of the measures of the exterior angles of any convex polygon.



Objectives

In this lesson, you will:

- Classify quadrilaterals in the plane.
- Classify properties of quadrilaterals in the plane.

Key Terms

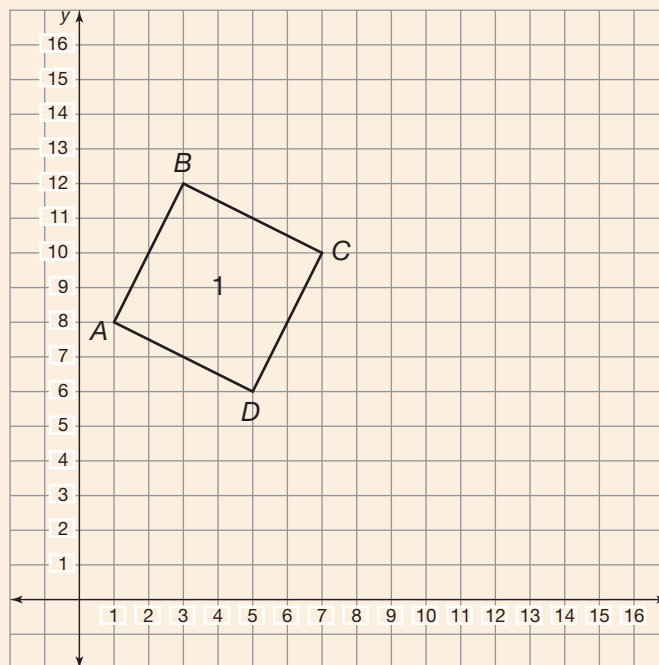
- rectangle
- parallelogram



SCENARIO A land planner is laying out different plots, or parcels, of land for a new housing subdivision. The parcels of land will be shaped like quadrilaterals.

Problem 1 The Lay of the Land

Parcel 1 is shown on the grid below. Each grid square has an area of one acre.



- A.** What kind of quadrilateral do you think parcel 1 is? Use a complete sentence in your answer.
- B.** Find the slopes of each side of the parcel. How many pairs of opposite sides, if any, are parallel? Explain how you found your answer. Use complete sentences in your answer.



Problem 1 The Lay of the Land

Are any of the sides perpendicular? Explain how you know. Use complete sentences in your answer.

Classify the quadrilateral with the information you have so far. Use a complete sentence in your answer.

- C. Find the lengths of the sides that form parcel 1. Show all your work. Are any of the side lengths congruent? If so, describe the sides that are congruent. Use a complete sentence in your answer.

Can you classify parcel 1 further? If so, classify the quadrilateral. Use a complete sentence in your answer.

Take Note

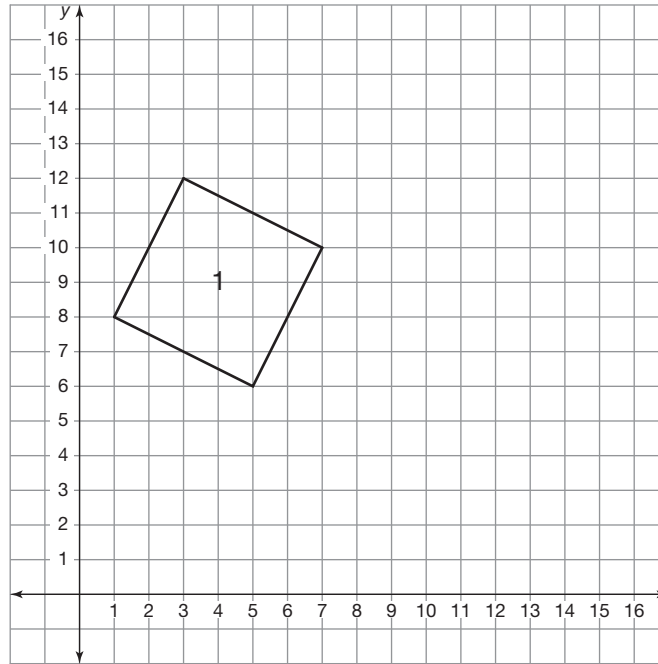
Remember that the Distance Formula is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

You can use this formula to find the lengths of the sides that form parcel 1.

Investigate Problem 1

1. The coordinates of the endpoints of parcel 2 are $E(5, 1)$, $F(5, 6)$, $G(9, 6)$, and $H(9, 1)$. Graph parcel 2 on the grid below. Classify this quadrilateral in as many ways as is possible. Explain how you found your answer. Use complete sentences in your answer.



2. Should the diagonals of parcel 2 be congruent? Find the lengths of the diagonals to verify your answer.

Investigate Problem 1

3. The coordinates of the endpoints of parcel 3 are $I(5, 6)$, $J(7, 10)$, $K(11, 10)$, and $L(9, 6)$. Graph parcel 3 on the grid below. Classify this quadrilateral in as many ways as is possible. Explain how you found your answer. Use complete sentences in your answer.

