Algorithms and Data Structures Final Exam

100 Points

Fill in the Blank (1 point each)

1. **After many insertions and deletions in a hash table, it is possible that every unused node is marked as "having been deleted". The process used to fix the problem is termed _____.**

2. **A _____ is an example of a search tree which is multi-way (allows more than two children).**

3. **A _____ is a path through the graph beginning and ending at the same node such that all vertices are visited exactly once.**

4. **A directed graph is termed _____ if there is a directed path from any node to any other node.**

5. **In a graph, when removal of a node makes the graph fall apart, the node is termed _____**

6. **I want a search tree in which the worst-case time for a find is log n. A tree that would work is _____.**

7. **A tree in which every node is no smaller than its children is termed _____.**

8. **I want to find a way of re-painting all the lines on all the roads in the county in minimal cost. The algorithm I need is termed _____.**

9. **The _____ algorithm is used when I want to determine if two entities are in the same group or not.**

10. **A _____ is a priority queue that is implemented not as a single tree but as a collection of heap-ordered trees.**

True-False (1 points each) (Circle the correct answer)

T F 1. A heap can be a useful tool for sorting.

T F 2. If an operation takes $O(n)$ worst case time, then it takes $O(n)$ amortized time.

T F 3. Greedy algorithms do not always find the optimal solutions.

T F 4. If two algorithms for the same problem have the same complexity then they will take nearly the same amount of time to run on the same input.

T F 5. An $O(n^2)$ algorithm can always be slower than a $O(n^3)$ algorithm if the constant is large enough.

T F 6. Let G be an undirected weighted graph, and let T be a minimum spanning tree of G. If all the edge weights on G are increased by a constant number c, then T is still a minimum spanning tree.

T F 7. In any heap (considered as a binary tree), every branch from the root to a leaf has the same length**.**

Multiple choice – 3 points each.

1. What is the expected number of operations needed to visit all the edges terminating at a particular vertex given an **adjacency matrix representation** of the graph? (Assume n vertices are in the graph and m edges terminate at the desired node.)

A. O(m) B. O(n) C. O(m²) D. O(n²)

2. Here is an array of ten integers:

5 3 8 9 1 7 0 2 6 4

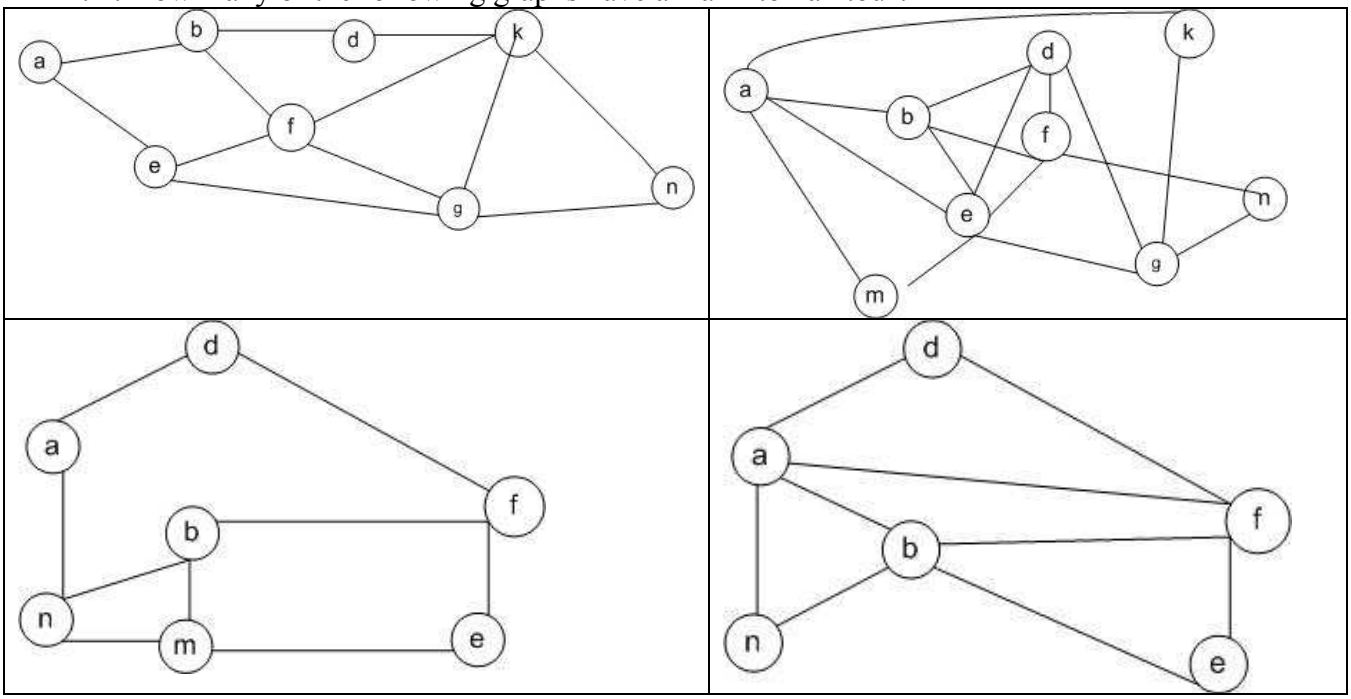
Suppose we partition this array using quicksort's partition function and using 5 for the pivot. Which shows the array after partition finishes:

A. 5 3 4 2 1 0 7 9 6 8
B. 0 3 4 2 1 5 7 9 6 8
C. 3 1 0 2 4 5 8 9 6 7
D. 3 1 0 2 4 5 8 9 7 6
E. None of the above

3. To avoid so many recursive calls in quicksort, the best idea is to

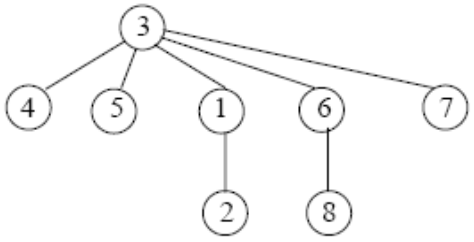
A. Have a single recursive call in the code.
B. Stop before the slices get too small and use an insertion sort at the end.
C. Have a base case which can handle slices of size five or less.
D. Use one pass of shell sort before calling the quick sort.

4. . How many of the following graphs have a Hamiltonian tour?



(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. The following disjoint set was generated without path compression. Assume the union does a find of its arguments before the union.



Which of the following is a sequence of unions that could have generated this tree?

a. union(3,7); union(3,6); union(6,8); union(3,1); union(1,2); union(3,5); union(3,4)

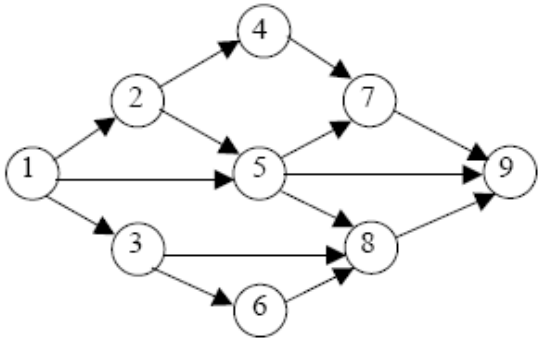
b. union(4,3); union(5,3); union(2,1); union(2,3); union(8,6); union(8,3); union(7,3)

c. union(1,3); union(3,4); union(4,5); union(2,5); union(6,8); union(6,2); union(7,6)

d. None of the above.

6. In listing nodes in topological ordering, which is NOT a step of the algorithm.

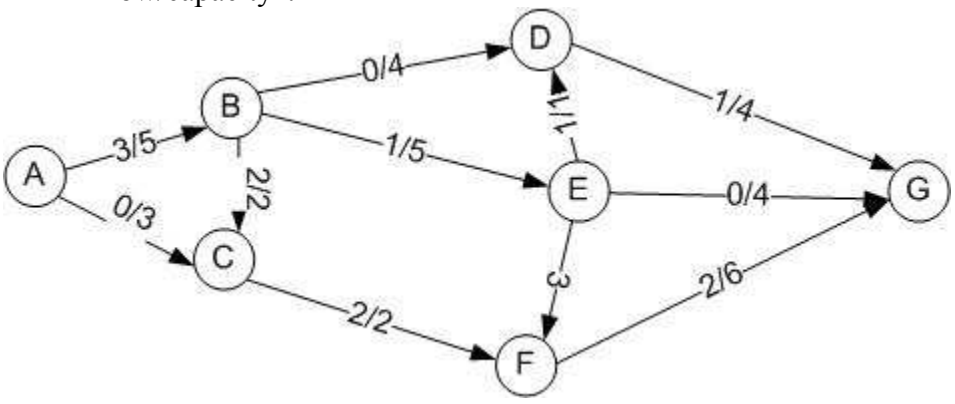
a. get a predecessor count for each node
b. verify that the predecessor has been listed
c. decrement the predecessor count of all a node's successors
d. list the node which has zero predecessors



7. Which graph algorithm does a DFS numbering, reverses the edges and does another depth first traversal?

a. Finding articulation points
b. Finding strongly connected components
c. Finding the maximum flow
d. Finding an Eulerian tour.

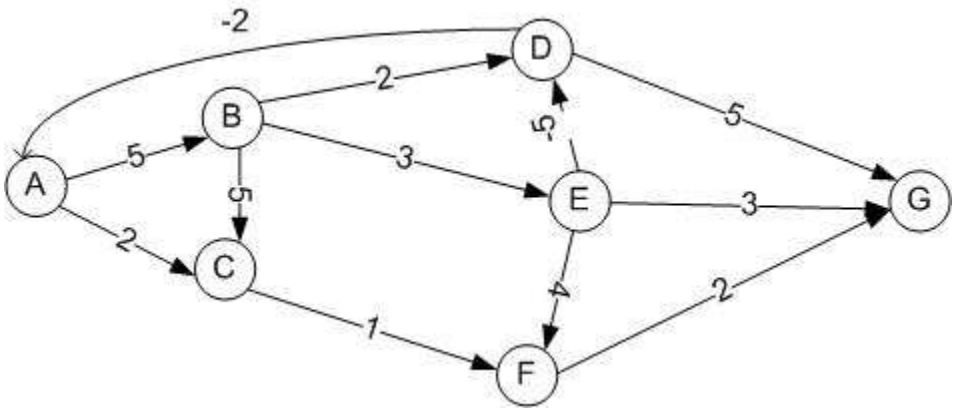
8. In the graph below, which is NOT an augmenting path? Each edge is labeled with "current flow/capacity".



(a) a b e g
(b) a c b d e g
(c) a c f g
(d) a b d g
(e) all are augmenting paths

9. The graph below has negative edge weights. What is the best way of solving the single source shortest path problem in the presence of negative edge weights?

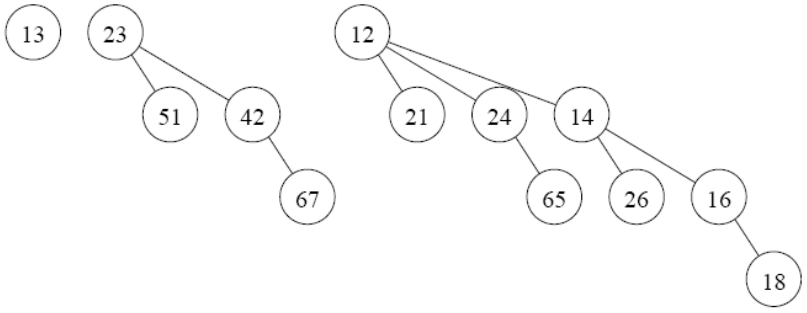
(a) add 5 to each edge and then proceed with Dijkstra's algorithm.
(b) add 5 to each edge and then proceed with the Floyd-Warshall algorithm.
(c) keep a queue of nodes to be examined and anytime the distance to a node changes, put it back on the queue
(d) normal shortest path algorithms will solve the problem because there are no negative cycles.
(e) none of the above



10. A d-heap (stored as an array) is like a binary heap except for each node has d children (except for possibly the last two levels). What advantage does this have:
(a) less space is required
(b) insertions are faster
(c) deletions are faster
(d) merging is faster

11. In performing deleteMin on the binomial queue below, what is the result?

a. 13 becomes the parent of 21
b. 13 becomes the parent of 24
c. 13 may become the parent of 23
d. all of the above
e. none of the above



12. Consider a hash table with linear probing and a size of 9. Use the hash function "k%9". Insert the keys: 5, 29, 20, 0, 27 and 18 into your table (in that order). What is the result?

a.

0	27	29	20	5	18			

b.

0	27	29	20	18	5			

c.

0	27	29	18	20	5			

d.

5	29	20	0			27		18

e. None of the above

13. You are the county clerk. You receive death records from the towns in the county in batches throughout the year. You need to sort by date 1,000,000 records. Which is the best sort to use:
a. insertion sort b. quicksort c. mergesort d. heap sort

14. The array 10 8 6 2 1 4 5 is organized into a heap (priority queue). Which array represents the heap after two deleteMax operations have been performed?

a.

6	5	4	2	1

b.

6	4	5	2	1

c.

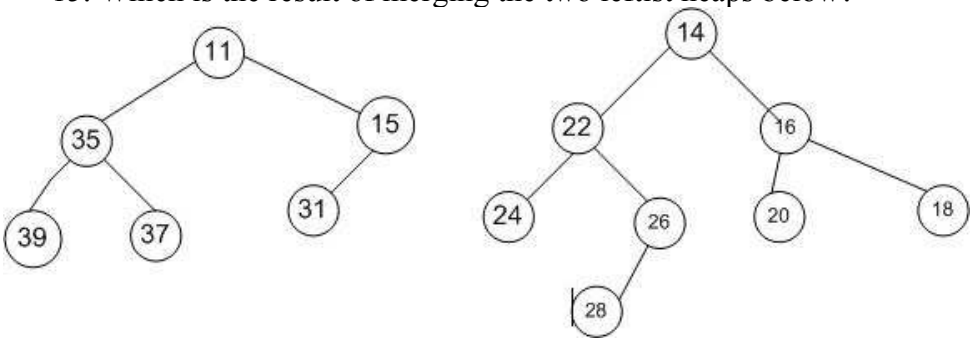
6	4	5	1	2

d.

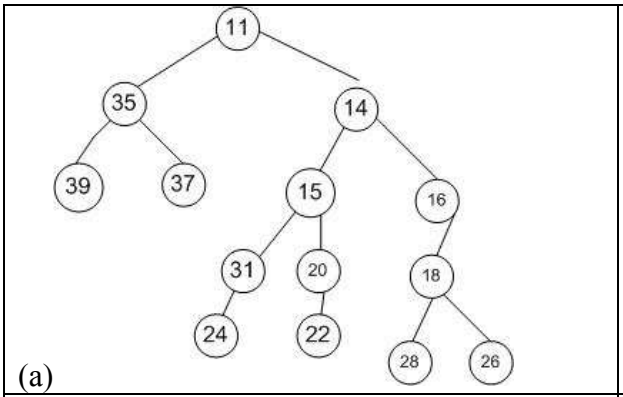
6	5	4	1	2

e. None of the above

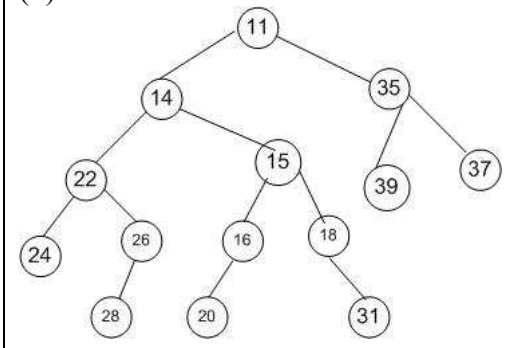
15. Which is the result of merging the two leftist heaps below?



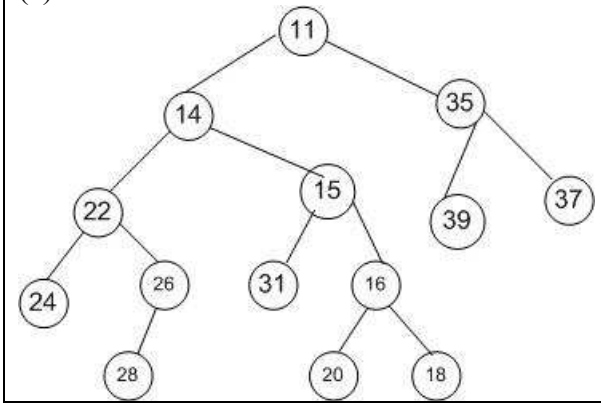
(a)



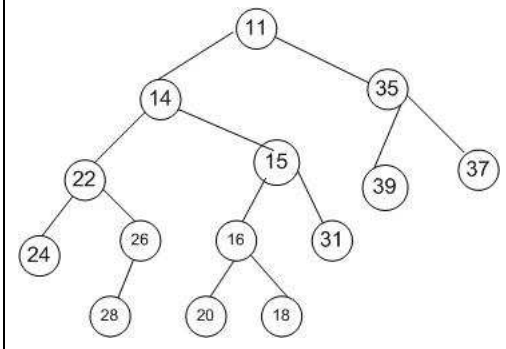
(b)



(c)

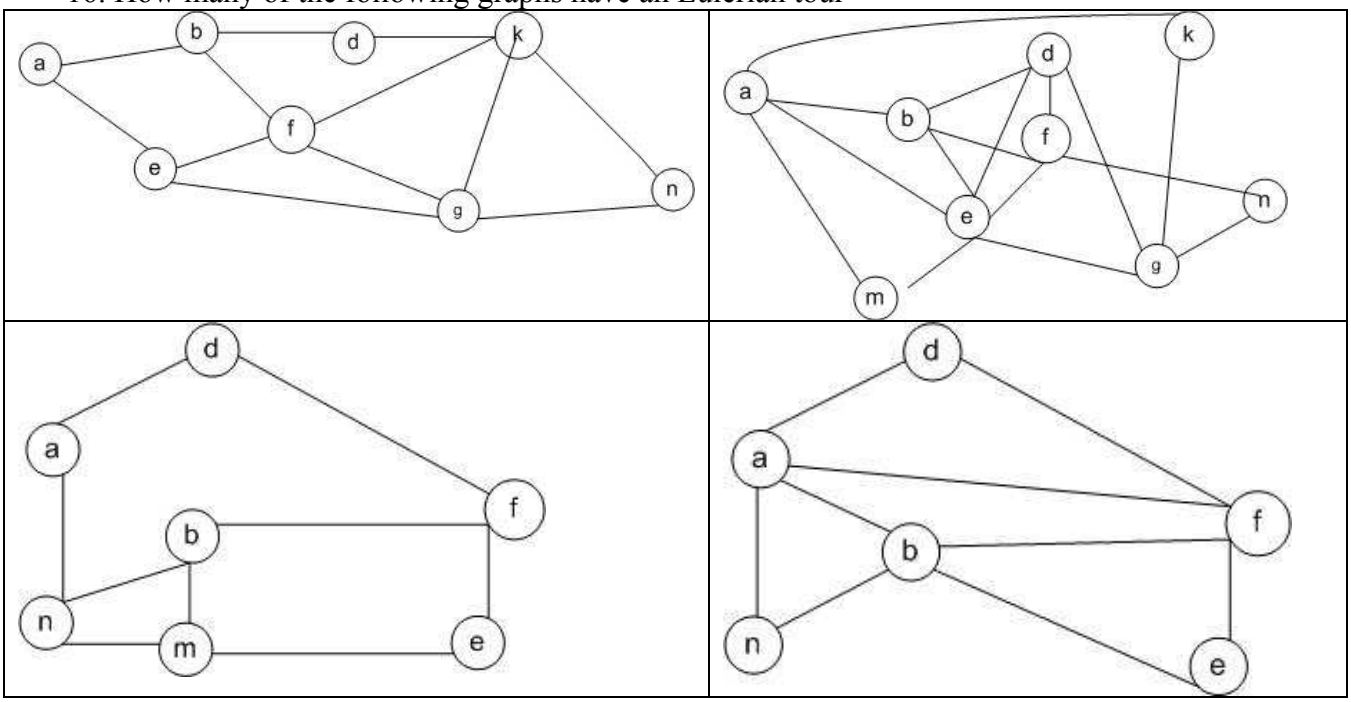


(d)



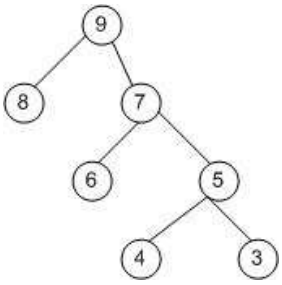
(e) None of the above

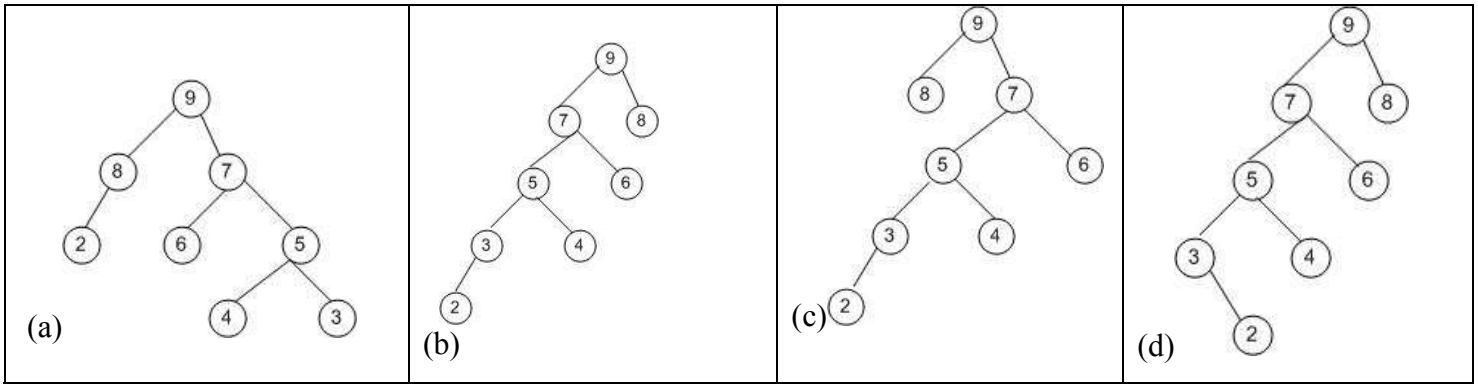
16. How many of the following graphs have an Eulerian tour



(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

17. Consider the skew heap below. Which of the following skew heaps are formed when you add 2 to the heap? [Merging is done by merging the smaller tree into the right subtree of the larger tree and then swapping.]



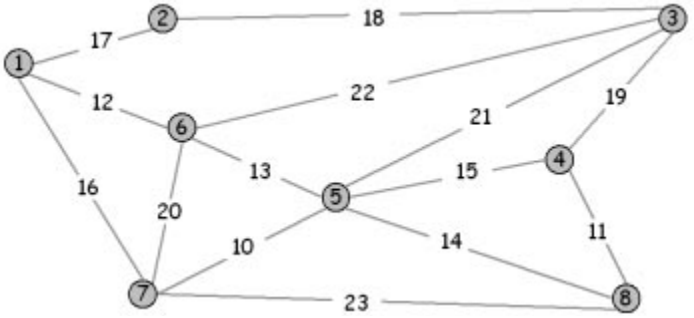


(e) None of the above

Short Answer

1. (13 points) Write a function that finds out if a tree passes the following test for being balanced: for every subtree, the number of nodes in its left and right subtrees differ by at most one

2. **Minimum spanning tree. (7 points)**
Consider the following undirected network with edge weights as shown.



List the *edges* in the MST in the order that Kruskal's algorithm selects them.

3. (5 points) Alan has discovered an amazing new algorithm for the mincost maxflow problem. Alan has done some preliminary computational experiments that appear to indicate the potential utility of the new algorithm, in particular it appears much faster than Beth's classic algorithm. The running time is shown below.
(a) Estimate the asymptotic running time of the algorithms as a function of N.
(b) Which algorithm would you prefer to use?

N	Alan	Beth
5	0.00	0.01
10	0.00	0.05
20	0.01	0.16
40	0.05	0.63
80	0.41	2.51
160	3.27	10.20

4. (5 points) Consider the digraph on eight nodes, labeled 0 through 7, with the 13 directed edges
0->7 0->1 1->4 1->6 2->3 3->4 4->2 5->2 6->0 6->3 6->5 7->1 7->3 .

List the strongly connected components.

5.(6 points) Draw the binomial queue that results when you insert the keys
9 7 5 1 4 3 2 6 8
in that order into an initially empty queue