Lesson 7.1 • Polynomial Degree and Finite Differences

Name ______________________________ **Period** __________ **Date** ____________

1. Identify the degree of each polynomial.

a. $3x^4 - 2x^3 + 3x^2 - x + 7$

b. $x^5 - 1$

c. $0.2x - 1.5x^2 + 3.2x^3$

d. $250 - 16x^2 + 20x$

e. $x + x^2 - x^3 + x^4 - x^5$

f. $5x^2 - 6x^5 + 2x^6 - 3x^4 + 8$

2. Determine which of the expressions are polynomials. For each polynomial, state its degree and write it in general form. If it is not a polynomial, explain why not.

a. $1 + x^2 - x^3$

b. $0.2x^3 + 0.5x^4 + 0.6x^2$

c. $x - \dfrac{1}{x^2}$

d. 25

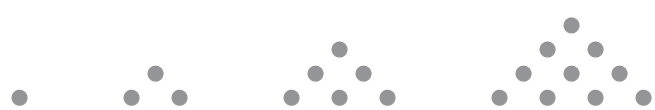
e. $-\dfrac{2}{3}x^2 + \dfrac{3}{5}x^3 + \dfrac{5}{12} + \dfrac{5}{8}x$

f. $\sqrt{x} + 3x^2 + 5$

3. For the data set below, find each set of common differences until the common differences are constant. State the degree of polynomial that models the data.

x	-3	-2	-1	0	1	2	3
y	22	22	14	4	-2	2	22

4. The figures below show why the numbers in the sequence 1, 3, 6, 10, . . . are called *triangular numbers.*



a. Complete the table.

n	1	2	3	4	5	6	7
nth triangular number	1	3	6	10			

b. Calculate the finite differences for the completed table.

c. What is the degree of the polynomial function that you would use to model this data set?

d. Write a polynomial function t that gives the nth triangular number as a function of n. (*Hint:* Create and solve a system of equations to find the coefficients.)

Lesson 7.2 • Equivalent Quadratic Forms

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1. Identify each quadratic function as being in general form, vertex form, factored form, or none of these forms. Give all answers that apply.

a. $y = 3x^2 - 4x + 5$ **b.** $y = (x - 2.5)^2 + 7.5$

c. $y = -0.5(x + 3)^2$ **d.** $y = 2(x - 8)(x + 6)$

e. $y = -1.5x(x - 2)$ **f.** $y = x^2 - 7$

2. Convert each quadratic function to general form.

a. $y = 2x(x - 5)$ **b.** $y = (x - 3)^2$ **c.** $y = 1.5(x + 2)^2 - 3$

d. $y = 2(x - 5)(x + 7)$ **e.** $y = -5(x + 3)(x - 2) - 30$ **f.** $y = 3(x - 1.5)^2 - 10$

g. $y = -\frac{1}{2}(x - 6)^2$ **h.** $y = \frac{2}{3} - \left(x - \frac{1}{2}\right)^2$ **i.** $y = -2.5(x - 4)(x + 6)$

3. Find the vertex of the graph of each quadratic function.

a. $y = -x^2$ **b.** $y = x^2 + 5$ **c.** $y = (x - 4)^2$

d. $y = (x + 3)^2 - 5$ **e.** $y = -(x - 1)^2 + 6$ **f.** $y = 10 - (x + 6)^2$

g. $y = 6.5 + 0.5(x + 4)^2$ **h.** $y = -2\left(x - \frac{2}{3}\right)^2 + \frac{1}{4}$ **i.** $y = \frac{1}{2}\left(x + \frac{5}{6}\right)^2 - \frac{7}{12}$

4. Find the zeros of each quadratic function.

a. $y = (x + 5)(x - 3)$ **b.** $y = -2(x - 1)(x + 6)$

c. $y = 0.5x(x - 5)$ **d.** $y = (x - 7.5)^2$

e. $y = -0.2(x + 3.6)(x - 4.8)$ **f.** $y = 6\left(x + \frac{2}{3}\right)\left(x - \frac{1}{2}\right)$

5. Consider this table of values generated by a quadratic function.

x	-3	-2.5	-2	-1.5	-1	-0.5	0
y	-0.5	-3	-4.5	-5	-4.5	-3	-0.5

a. What is the line of symmetry for the graph of the quadratic function?

b. Identify the vertex of the graph of this quadratic function, and determine whether it is a maximum or a minimum.

c. Use the table of values to write the quadratic function in vertex form.

Lesson 7.3 • Completing the Square

Name ______________________________ Period __________ Date ______________

1. Factor each quadratic expression.

a. $x^2 + 10x + 25$ **b.** $x^2 - 22x + 121$ **c.** $x^2 - x + \frac{1}{4}$

d. $4x^2 - 20x + 25$ **e.** $0.04x^2 + 1.8x + 20.25$ **f.** $9x^2 - 24xy + 16y^2$

2. What value is required to complete the square?

a. $x^2 + 6x +$ _____ **b.** $x^2 - 18x +$ _____ **c.** $x^2 - 5x +$ _____

d. $x^2 + 11x +$ _____ **e.** $x^2 - 0.8x +$ _____ **f.** $x^2 + 4.3x +$ _____

3. Convert each quadratic function to vertex form.

a. $y = x^2 - 8x + 14$ **b.** $y = x^2 + 14x + 50$ **c.** $y = x^2 + 5x + 8$

d. $y = x^2 - 11x + 28$ **e.** $y = 5x^2 - 10x - 3$ **f.** $y = 2x^2 + 5x$

4. Find the vertex of the graph of each quadratic function, and state whether the vertex is a maximum or a minimum.

a. $y = x^2 - 6x + 11$ **b.** $y = (x - 2)(x + 6)$ **c.** $y = -3x^2 + 12x + 17$

d. $y = -3.5x^2 - 7x$ **e.** $y = x^2 + 9x - 10$ **f.** $y = -0.5x^2 + 2.5x + 8$

5. Rewrite each expression in the form $ax^2 + bx + c$, and then identify the coefficients a, b, and c.

a. $5 + x + 4x^2$ **b.** $2x - 5x^2$ **c.** $-6 + 3x^2 + 6x + 8$

d. $-2x(x - 8)$ **e.** $25 - x^2$ **f.** $(2x - 3)(x + 5)$

6. A ball is thrown up and off the roof of a 75 m tall building with an initial velocity of 14.7 m/s.

a. Let t represent the time in seconds and h represent the height of the ball in meters. Write an equation that models the height of the ball.

b. At what time does the ball reach maximum height? What is the ball's maximum height?

c. At what time or times is the ball 30 m above the ground?

d. At what time does the ball hit the ground?

Lesson 7.4 • The Quadratic Formula

Name ______________________________ Period __________ Date ______________

1. Solve.

a. $(x - 5)^2 = 49$ **b.** $(x + 12)^2 = 169$ **c.** $(x + 1.3)^2 = 20.25$

d. $(x - 2.8)^2 = 39.69$ **e.** $\left(x - \frac{2}{3}\right)^2 = \frac{25}{81}$ **f.** $\left(x + \frac{5}{6}\right)^2 = \frac{49}{144}$

2. Evaluate each expression. Round your answers to the nearest thousandth.

a. $\dfrac{-6 + \sqrt{6^2 - 4(1)(-5)}}{2(1)}$ **b.** $\dfrac{4 - \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$

c. $\dfrac{5 + \sqrt{(-5)^2 - 4(4)(-3)}}{2(4)}$ **d.** $\dfrac{-10 - \sqrt{10^2 - 4(2)(5)}}{2(2)}$

3. Solve by any method. Give your answers in exact form.

a. $x^2 + 3x - 10 = 0$ **b.** $x^2 + 12x + 35 = 0$ **c.** $2x^2 - 5x = 12$

d. $x^2 + 3x - 5 = 0$ **e.** $12x^2 - 11x - 5 = 0$ **f.** $25x^2 - 49 = 0$

g. $2x^2 - 4x - 7 = 0$ **h.** $4x^2 + 7x - 1 = 0$ **i.** $6x^2 + 19x = 7$

j. $x^2 = 5.8x$ **k.** $x^2 - 48 = 0$ **l.** $x^2 - 9.6x + 23.04 = 0$

4. Write each equation in factored form, $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the roots of the equation.

a. $y = x^2 - 7x + 12$ **b.** $y = x^2 + 5x - 24$ **c.** $y = x^2 - 7x - 8$

d. $y = 2x^2 - 8x + 6$ **e.** $y = 4x^2 + 2x - 2$ **f.** $y = 5x^2 + 19x + 12$

5. Write a quadratic function in general form that satisfies the given conditions.

a. $a = 1$; x-intercepts of graph are 6 and 9.

b. $a = -1$; x-intercepts of graph are -4 and -2.

c. $a = 2$; x-intercepts of graph are -7 and 5.

d. x-intercepts of graph are 8 and -3; y-intercept is -12.

e. x-intercepts of graph are 0 and 13; graph contains point (2, 22).

f. x-intercept of graph is 4.8; y-intercept is -5.76.

Lesson 7.5 • Complex Numbers

Name ______________________________ Period __________ Date ______________

1. Add, subtract, or multiply.

a. $(4 - 5i) + (6 + 2i)$

b. $(-5 + 6i) - (1 - i)$

c. $4(2 - 5i)$

d. $\left(\frac{3}{5} - \frac{1}{10}i\right) - \left(\frac{7}{10} - \frac{4}{5}i\right)$

e. $(-2.4 - 5.6i) + (5.9 + 1.8i)$

f. $-4i(-6 + i)$

g. $(3 - 2i)(3 + 2i)$

h. $(2.5 + 1.5i)(3.4 - 0.6i)$

2. Find the conjugate of each complex number.

a. $3 - 2i$

b. $5 - 4i$

c. -2

d. $7i$

e. $\frac{1}{3} + \frac{5}{6}i$

f. $-3.25 + 4.82i$

3. Rewrite each quotient in the form $a + bi$.

a. $\frac{2}{3 + i}$

b. $\frac{1 + i}{1 - i}$

c. $\frac{3 + 2i}{4 - i}$

d. $\frac{3i}{2 + i}$

e. $\frac{3 + 5i}{6i}$

f. $\frac{4 + 5i}{2 - 3i}$

4. Solve each equation. Label each solution as real, imaginary, and/or complex.

a. $x^2 - 2x + 5 = 0$

b. $x^2 + x - 3 = 0$

c. $2x^2 - 3x + 1 = 0$

d. $x^2 + 7 = 0$

e. $3x^2 + 2x + 4 = 0$

f. $x(x - 5) = 1$

g. $x^2 + x + 1 = 0$

h. $4x^2 + 9 = 0$

i. $(x + 7)(x - 3) = 5 - 2x$

5. Write a quadratic function in general form that has the given zeros and leading coefficient of 1.

a. $x = -4, x = 7$

b. $x = 11i, x = -11i$

c. $x = -2 + 3i, x = -2 - 3i$

Lesson 7.6 • Factoring Polynomials

Name ______________________________________ Period ___________ Date ______________

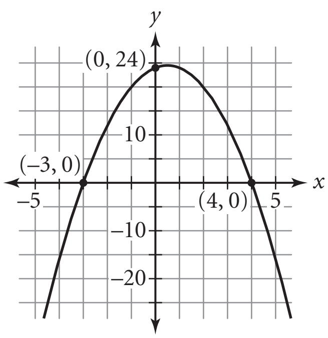
1. Without graphing, find the x-intercepts and the y-intercept for the graph of each equation.

a. $y = (x + 6)(x - 5)$ **b.** $y = -(x - 8)^2$ **c.** $y = 2(x + 1)(x - 1)$

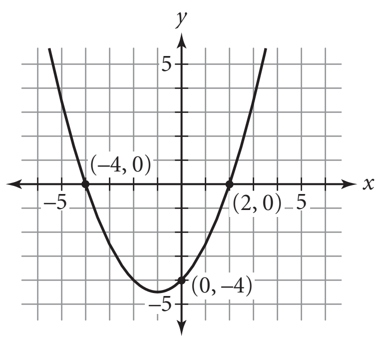
d. $y = 3(x + 4)(x + 2)$ **e.** $y = -(x + 2)(x - 1)(x - 6)$ **f.** $y = 0.75x(x - 2)(x + 6)$

2. Write the factored form of the quadratic function. Don't forget the vertical scale factor.

a.



b.



3. Convert each polynomial function to general form.

a. $y = (x + 5)(x - 3)$ **b.** $y = -2(x - 2.5)(x + 2.5)$ **c.** $y = x(x - 1)(x + 5)$

d. $y = -0.5(x + 3)^2$ **e.** $y = -x(x + 12)(x - 12)$ **f.** $y = 0.8(x + 4)(x - 6)$

4. Write each polynomial as a product of factors.

a. $2x^2 + 4x - 30$ **b.** $x^2 - 14x + 49$ **c.** $x^3 - 3x^2 + 2x$

d. $2x^2 + 3x - 5$ **e.** $x^2 - 169$ **f.** $x^2 + 169$

g. $x^2 - 15$ **h.** $x^2 + 15$ **i.** $x^4 - 10x^2 + 9$

j. $12x^2 - 5x - 3$ **k.** $x^3 + 5x^2 - 17x - 21$ **l.** $3x^3 + 3x^2 - 30x + 24$

5. Sketch a graph for each situation if possible.

a. A quadratic function with two real zeros, whose graph has the line $x = 2$ as its axis of symmetry

b. A quadratic function with no real zeros, whose graph has a negative y-intercept

c. A cubic function with three real zeros, whose graph has a positive y-intercept

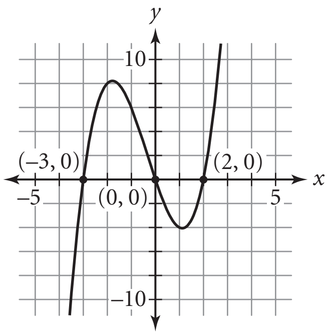
d. A cubic function with two real zeros, whose graph has a negative y-intercept

Lesson 7.7 • Higher-Degree Polynomials

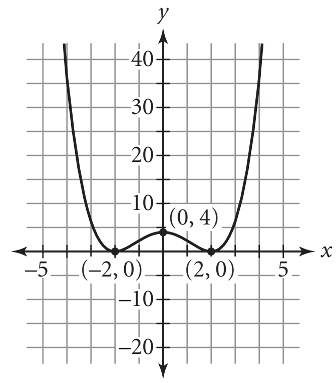
Name ______________________ Period ________ Date __________

1. Refer to these two graphs of polynomial functions.

i.



ii.



a. Identify the zeros of each function.

b. Find the y-intercept of each graph.

c. Identify the lowest possible degree of each function.

d. Write the factored form for each polynomial function. Check your work by graphing on your calculator.

2. Write a polynomial function with the given features.

a. A quadratic function whose graph has only one x-intercept, -4, and whose y-intercept is -8

b. A cubic function with leading coefficient -1 whose graph has x-intercepts 0 and 5, where $x = 5$ is a double root

c. A quadratic function whose graph has vertex $(3, -8)$, which is a minimum, and two x-intercepts, one of which is 5

d. A fourth-degree polynomial function with two double roots, 0 and 2, and whose graph contains the point $(1, -1)$

3. Write the lowest-degree polynomial function that has the given set of zeros and whose graph has the given y-intercept. Write each polynomial function in factored form. Give the degree of each function.

a. Zeros: $x = -3$, $x = 5$; y-intercept: -30

b. Zeros: $x = -2$ (triple root); y-intercept: -8

c. Zeros: $x = -2$, $x = 1$, $x = 3$; y-intercept: 3

d. Zeros: $x = \pm 2i$, $x = -2$ (double root), $x = 5$; y-intercept: 80

Lesson 7.8 • More About Finding Solutions

Name ______________________________ Period __________ Date ____________

1. Divide.

a. $x - 2\overline{)3x^3 - 8x^2 - 11x + 30}$

b. $x - 4\overline{)x^4 - 13x^2 - 48}$

c. $\dfrac{32x^5 - 1}{2x - 1}$

2. Varsha started out dividing two polynomials by synthetic division this way:

$$\underline{-3}\,\big|\; -3 \quad -5 \quad 0 \quad -35 \quad 7$$

a. Identify the dividend and divisor.

b. Write the numbers that will appear in the second line of the synthetic division.

c. Write the numbers that will appear in the last line of the synthetic division.

d. Write the quotient and remainder for this division.

3. In each division problem, use the polynomial that defines P as the dividend and the quotient that defines D as the divisor. Write the result of the division in the form $P(x) = D(x) \cdot Q(x) + R$, where the polynomial that defines Q is the quotient and R is an integer remainder. (It is not necessary to write the remainder if $R = 0$.)

a. $P(x) = x^2 + 8x - 9$; $D(x) = x + 9$

b. $P(x) = 2x^2 - 9x + 2$; $D(x) = x - 5$

c. $P(x) = 2x^3 - 5x^2 + 8x - 5$; $D(x) = x - 1$

d. $P(x) = 6x^3 - 5x^2 + 16x - 8$; $D(x) = 3x - 1$

4. Make a list of the possible rational roots of each equation.

a. $x^3 + x^2 - 10x + 8 = 0$

b. $2x^3 - 3x^2 - 17x + 30 = 0$

5. Find all the zeros of each polynomial function. Then write the function in factored form.

a. $y = x^3 - 6x^2 + 5x + 12$

b. $y = x^3 - 5x^2 + 9x - 45$

c. $y = 6x^3 + 17x^2 + 6x - 8$

d. $y = x^4 - 21x^2 - 100$