Lesson 20

Quadratic Formula and Its Proof

CA Standard Alg 1 19.0

The quadratic formula is used to find the roots of a quadratic function. The formula was generated by completing the square on the standard form of a quadratic function.

Proving the Quadratic Formula

The standard form of a quadratic function is $y = ax^2 + bx + c$, where a, b, and c are constants. Follow the steps to complete the square on this function to find the solutions. The expression that equals x is known as the quadratic formula. You can substitute values of a, b, and c to find the roots of a quadratic function without factoring or completing the square.

$ax^2 + bx + c = y$	Standard form
$ax^2 + bx + c = 0$	Substitute 0 for y.
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a so that the coefficient on x^2 is 1.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$ from both sides of equation.
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Make the left side of the equation a perfect square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Factor left side of the equation. Simplify right side.
$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$	Take square root of both sides of the equation.
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	Simplify each side.
$x + \frac{b}{2a} - \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$	Isolate x term. Subtract $\frac{b}{2a}$ from both sides of equation.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Simplify right side of equation.

Using the Quadratic Formula

EXAMPLE 1

Values from a quadratic function were substituted into the quadratic formula to get $x = \frac{-5 \pm \sqrt{25 - 4(1)(-3)}}{2}$. Name the function.

Determine the values of *a, b,* and *c.* Use those values to write the function in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4(1)(-3)}}{2}$

$b = 5, a = 1,$ and $c = -3$

The function is $y = x^2 + 5x - 3$.

CA Standards Check 1

1a. Values from a quadratic function were substituted into the quadratic formula to get $x = \frac{2 \pm \sqrt{4 - 4(5)(1)}}{10}$. Name the function.

1b. Values from a quadratic function were substituted into the quadratic formula to get $x = \frac{1 \pm \sqrt{1 - 4(4)(-4)}}{8}$. Name the function.

EXAMPLE 2

Use the quadratic formula to find the roots of the quadratic function $2x^2 + 6x - 1 = y$.

Identify the values of a, b, and c. Substitute those values into the quadratic formula.

$a = 2, b = 6, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{38 + 8}}{4} = \frac{-6 \pm \sqrt{44}}{4} = \frac{-6 \pm 2\sqrt{11}}{4} = \frac{-3 \pm \sqrt{11}}{2}$$

$$x = \frac{-3 + \sqrt{11}}{2} \quad \text{or} \quad x = \frac{-3 - \sqrt{11}}{2}$$

CA Standards Check 2

2a. Use the quadratic formula to find the roots of the quadratic function $-3x^2 - 2x + 5 = y$.

2b. Use the quadratic formula to find the roots of the quadratic function $x^2 - 8x - 3 = y$.

California PRACTICE

Name_____Class_____Date_____

1 Which shows the use of the quadratic formula for $y = x^2 + x + 1$?

A $x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{1}$

B $x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2}$

C $x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{1}$

D $x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2}$

2 What do you do to the standard form of a quadratic function to change the coefficient of x^2 to 1?

A Multiply both sides of the equation by -1.

B Divide both sides of the equation by a.

C Multiply both sides of the equation by a.

D The coefficient is already 1.

3 What does $\sqrt{b^2 - 4ac}$ equal when using the quadratic formula to solve the quadratic function $y = -2x^2 - 5x + 7$?

A $\sqrt{31}$

B 9

C $\sqrt{-31}$

D $\sqrt{-81}$

4 What term is added to both sides of the standard form when completing the square?

A $\left(\frac{b}{a}\right)^2$

B $\frac{b^2}{2a}$

C $\frac{b}{2a}$

D $\left(\frac{b}{2a}\right)^2$

5 Which shows the use of the quadratic formula for $y = -6x^2 - 2x + 10$?

A $x = \frac{6 \pm \sqrt{6 - 4(-2)(10)}}{-2}$

B $x = \frac{-2 \pm \sqrt{4 - 4(-6)(10)}}{12}$

C $x = \frac{2 \pm \sqrt{4 - 4(-6)(10)}}{-12}$

D $x = \frac{6 \pm \sqrt{36 - 4(-2)(10)}}{-4}$

6 What is the denominator when using the quadratic formula to solve the quadratic function $y = 10x^2 - x + 4$?

A -20

B -2

C 8

D 20