Introduction

In this section, the lessons focus on solving problems that use whole number operations, prime and composite numbers, odd and even numbers, perfect squares and square roots.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning

The following Standards of Learning are addressed in this section:

5.3 The student will
- a) identify and describe the characteristics of prime and composite numbers; and
- b) identify and describe the characteristics of even and odd numbers.

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

7.1 The student will
- b) determine scientific notation for numbers greater than zero;
- d) determine square roots; and

8.5 The student will
- a) determine whether a given number is a perfect square; and
- b) find the two consecutive whole numbers between which a square root lies.

Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

* SOL 5.3a

Lesson Summary

Students identify and describe prime and composite numbers. (30 minutes)

Materials

Attached worksheets
Scientific calculators

Vocabulary

composite number. A natural number that has more than two different factors.
factor. An integer that divides evenly into a number with a remainder of zero.
multiple. The product of the number and any natural number.
prime number. A natural number that has exactly two different factors, one and the number itself.

Warm-up

Distribute the "Warm-up" worksheet, which reviews even and odd numbers. If students have difficulty with this, using variables, offer them the strategy of substituting numbers for the variables.

Lesson

1. Explain to students that they will be investigating characteristics of certain types of numbers. Distribute the "Where Are the Prime Numbers?" worksheet. Define the term *divisible* for the class.
2. Ask students to circle the 2 in the first row. Then, ask them to go through the rest of the chart and cross out each number that is divisible by 2. Remind them to keep in mind the meaning of the term *divisible.*
3. Tell students to circle the 3 in the first row and cross out all numbers that are divisible by 3 if they are not already crossed out. You may wish to allow students to use a calculator to determine divisibility.
4. Continue this same procedure, having students circle 5 and cross out all numbers divisible by 5, circle 7 and cross out all numbers divisible by 7, circle 11 and cross out all numbers divisible by 11, etc. This will take most of the class period.
5. When the exercise is complete, ask the students to examine the numbers that are circled and decide what these numbers have in common. Once students realize that each circled number has exactly two factors—1 and the number itself—define *prime number.* Note that the definition of a prime number does not allow 1 to be a prime number because 1 has only one factor—namely, 1.
6. Ask the students what all of the numbers that are crossed out have in common. Define *composite number.*
7. Have students make a list of all the prime numbers less than 100. Tell them that they have just completed the Sieve of Eratosthenes, and explain that Eratosthenes was one of the ancient Greek mathematicians who figured out this method for identifying prime numbers. Ask students whether this method works for finding prime numbers between 100 and 200. Why, or why not?
8. Distribute the "Prime and Composite Numbers" worksheet, and have the students complete it.

Reflection

Ask students to write the answers to the following questions:

- What are the characteristics of prime numbers?
- How do prime numbers compare to and contrast with composite numbers?
- What process did you use to find the prime numbers between 1 and 100?

Name: _____

Warm-up

1. Complete each of the following statements, using the word *even* or *odd*.

   a. If n is an even whole number, then $n + 1$ is an _____ whole number.

   b. If $m > 1$ and m is an odd whole number, then $m - 1$ is an _____ whole number.

   c. If e is an even whole number and o is an odd whole number, then $e + o$ is an _____ whole number.

   d. If o is an odd whole number, then $o + o$ is an _____ whole number.

2. If p is an odd whole number, how do you know that $4p$ is an even whole number?

   _____

3. Is this statement true or false? "Zero is an even whole number." _____ Justify your answer.

   _____

Name: ANSWER KEY

Warm-up

1. Complete each of the following statements, using the word *even* or *odd*.

   a. If n is an even integer, then $n + 1$ is an __odd__ integer.

   b. If $m > 1$ and m is an odd integer, then $m - 1$ is an __even__ integer.

   c. If e is an even integer and o is an odd integer, then $e + o$ is an __odd__ integer.

   d. If o is an odd integer, then $o + o$ is an __even__ integer.

2. If p is an odd integer, how do you know that $4p$ is an even integer?

   The product of an odd integer times an even integer (4) is always even.

3. Is this statement true or false? "Zero is an even integer." __True__ Justify your answer.

   _____

Name: _____

Where Are the Prime Numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name: ANSWER KEY

Where Are the Prime Numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name: _____

Prime and Composite Numbers

Select the correct answer for each of the following SOL-related questions. Underline your answer.

1. Which of the following statements best explains why 13 is a prime number?

   A It has more than two factors.

   B It cannot be divided.

   C It has exactly two different factors.

   D It is odd.

2. How many factors does a prime number have?

   F 0

   G 1

   H 2

   J 3

3. Which of the following statements best explains why 24 is a composite number?

   A All even numbers are composite numbers.

   B All odd numbers are prime numbers.

   C All composite numbers have exactly two factors.

   D All composite numbers have more than two factors.

4. Which of the following numbers is *not* prime?

   F 11

   G 19

   H 21

   J 31

Name: ANSWER KEY

Prime and Composite Numbers

Select the correct answer for each of the following SOL-related questions. Underline your answer.

1. Which of the following statements best explains why 13 is a prime number?

   A It has more than two factors.

   B It cannot be divided.

   C It has exactly two different factors.

   D It is odd.

2. How many factors does a prime number have?

   F 0

   G 1

   H 2

   J 3

3. Which of the following statements best explains why 24 is a composite number?

   A All even numbers are composite numbers.

   B All odd numbers are prime numbers.

   C All composite numbers have exactly two factors.

   D All composite numbers have more than two factors.

4. Which of the following numbers is *not* prime?

   F 11

   G 19

   H 21

   J 31

* SOL 5.3a

Lesson Summary

Students practice finding factors and identifying prime and composite numbers. (2 class periods)

Materials

Copies of the attached hundreds chart
Overhead or display hundreds chart
Copies of the attached worksheet

Vocabulary

composite number. A positive integer that has factors other than 1 and the number itself.
factor. Any integer that divides evenly into a given integer.
multiple. The product of a number and any other whole number.
prime number. A positive integer that has exactly two positive integer factors, 1 and the number itself.

Warm-up

Ask students what the term *factor* means. Discuss, and have students agree on a class definition.

Play a game that requires students to discover factors. Put two columns on the board: "Examples of My Idea," and "Non-examples of My Idea." Let your first idea be "Factors of 24," but do not reveal this to the students. Tell them that you are going to give them examples and non-examples of your idea as clues that can lead them to guess your idea. When they think they know your idea, they should raise their hand and help you give more examples and non-examples. Begin by listing 2 and 12 as examples of your idea and 5 and 14 as non-examples. Continue adding examples and non-examples of factors slowly until you have listed all factors of 24. At that point, ask a student to reveal your idea. If another round is needed to reinforce the concept of factors, play the game again with factors of 100.

Lesson

1. Play the Factors Game with the class, as follows. Display a hundreds chart. Select a number on the chart, and cross it out. List on the board all the factors of that number. Add all the factors to get the number of points you receive. Finally, cross out on the chart all the factors of your selected number.
2. Have the students, as a class, then take their turn by selecting a number that is not already crossed out, crossing it out, listing all the factors of that number, and adding all the factors to get their score. If a factor is already crossed out because it was used previously, it may not be included in the sum.
3. Continue taking turns until all of the numbers on the chart are crossed out. Total the points to find the winner. Play the Factors Game again if students need more practice finding factors.
4. Discuss strategy for the game, asking questions such as, "How did you decide which number to select? Are some numbers better to select than others? If so, why? Are some numbers not good to pick? If so, which ones are they?"
5. After playing the game several times, focus on the "worst" numbers to pick. Ask students which numbers they should try to avoid in the Factors Game. Lead them to discover that a number that has only 1 and itself as factors does not yield many points. This is called a *prime number*. Highlight the prime numbers from 1 to 100 on the chart, noting that the definition of a prime number does not allow 1 to be a prime number because 1 has only one factor, namely, 1. Discuss the meaning of the term *composite numbers*.
6. Give each pair of students a hundreds chart, and allow them to play the Factors Game.

Reflection

Have students complete the "Reflection" worksheet.

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name: _____

Reflection

Dave is trying to decide which numbers to pick for the Factors Game. He is considering picking the numbers listed below. Make a list of all factors of each of these numbers to help Dave win the game!

6	12	16	20	25	45	50

Of all the numbers that Dave is considering, which one should he pick first?_____

Why?

_____

_____

_____

* SOL 5.3b

Lesson Summary

Students identify and describe the characteristics of even and odd numbers. (30 minutes)

Materials

Large bag of counters
Attached worksheets

Vocabulary

even integer. An even number has 2 as a factor or is divisible by 2.
odd integer. An odd number does *not* have 2 as a factor or is *not* divisible by 2.

Warm-up

Walk around the room with a bag full of counters. As you walk by each student (or pair of students), ask him/her to take a handful of counters from the bag. Ask the students to separate their counters into two groups with an equal number of counters in each group. Some students will have one counter left over. Ask those with a leftover to raise their hands, and then do the same for those with no leftover.

Lesson

1. Make two columns on the board, one labeled "Numbers with a Leftover" and the other labeled "Numbers with No Leftover." Have each student call out the number of counters he/she took from the bag and whether or not there was a leftover when the counters put into two groups. Record each number on the board in the appropriate column.
2. After recording all the students' numbers, ask students whether they notice any pattern in the numbers in the "No Leftover" column. Explain that these numbers are referred to as *even integers*. Have students examine the integers in the "with a Leftover" column and describe the pattern in these numbers. Explain that these numbers are referred to as *odd integers.* You may wish to review the definition of *integer* at this time.
3. Ask the class to select a number not already on the board. Count out that many counters, and separate them into two groups with an equal number of counters in each group. If there is no counter leftover, add the number to the "No Leftover" column on the board. If there is a leftover, add the number to the "with a Leftover" column. Allow students to continue adding numbers to the board in this manner.
4. When you feel you have enough numbers to establish a pattern in the ones place, conduct a class discussion. Ask students leading questions to help them arrive at the conclusion that all of the numbers in the "No Leftover" (even integers) column end in 0, 2, 4, 6, or 8, while all the numbers in the "with a Leftover" (odd integers) column have a 1, 3, 5, 7, or 9 in the ones place.
5. Have the students work in pairs to explore the answers to the following questions. Have them use the counters and/or drawings to justify their responses.
   - Is the sum of two even numbers even or odd? (even)
   - Is the sum of two odd numbers even or odd? (even)
   - Is the sum of one odd number and one even number even or odd? (odd)
   - Is the product of two even numbers even or odd? (even)
   - Is the product of two odd numbers even or odd? (odd)
   - Is the product of one odd number and one even number even or odd? (even)

Reflection

Have students write the characteristics of even and odd integers, using drawings and/or symbols to explain their reasoning.

Name: _____

SOL Questions

1. Which set of numbers contains only odd numbers?

   A {0, 1, 2, 3}

   B {1, 2, 3, 4}

   C {1, 3, 7, 9}

   D {3, 4, 5, 6}

2. Which number is even?

   F 0

   G 1

   H 3

   J 5

3. Which set of numbers does *not* contain any even numbers?

   A {0, 1, 2, 3}

   B {0, 1, 3, 5}

   C {11, 13, 15, 17}

   D {13, 14, 15, 16}

Name: ANSWER KEY

SOL Questions

1. Which set of numbers contains only odd numbers?

   A {0, 1, 2, 3}

   B {1, 2, 3, 4}

   C {1, 3, 7, 9}

   D {3, 4, 5, 6}

2. Which number is even?

   F 0

   G 1

   H 3

   J 5

3. Which set of numbers does *not* contain any even numbers?

   A {0, 1, 2, 3}

   B {0, 1, 3, 5}

   C {11, 13, 15, 17}

   D {13, 14, 15, 16}

* SOL 5.4

Lesson Summary

Students explore and discuss the importance of remainders in division. (30 minutes)

Materials

Copies of the attached worksheets

Vocabulary

None

Warm-up

Have students complete the "What Do I Do with This?" worksheet. Hold a class discussion in which students explain their responses.

Lesson

1. Ask the class whether remainders are important to consider when solving division problems. Encourage them to be as specific as possible in their responses.
2. Have students complete the "Remainders" worksheet, allowing them as much time as they need to complete the problem set. Instruct them not to write the summary sentence yet.
3. Review students' responses, and discuss and correct any errors.
4. Give students a few minutes to write their summary statements.
5. Encourage the students to discuss their summary statements. Highlight any similarities and/or differences among the statements.

Reflection

Have students complete the "Exit Ticket" worksheet.

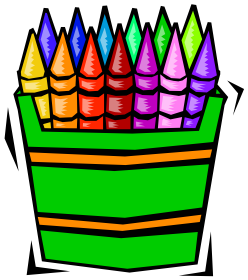
Name: _____

What Do I Do with This?

Problem	Do the math	Is the remainder important? (Y/N)	Explain why, or why not.
There are 26 seventh-grade students going to the big game against County Middle School. If 4 students can ride in one car, how many cars do they need?			
The Foreign Language Club is selling raffle tickets. Each ticket costs $3. The Club wants to make $110. How many tickets will the club need to sell?			
The volleyball coach wants to buy new warm-up suits for her team. The suits cost $20. The coach has $210. How many warm-up suits can she buy?			
County Middle School is buying pizza for all of its students. Each pizza has 8 slices. Each student will receive 1 slice. There are 700 students at County Middle School. How many pizzas will the school need to buy?			
The homecoming game is next week, and the students want to hang banners around the school. The students want to put up 110 banners. The banners come in packs of 40. How many packs will the students need to buy?			

Name: _____

Remainders



1. A full box of crayons contains 8 crayons. If each of the 26 students in a class needs to use 1 crayon simultaneously in a class activity, how many full boxes of crayons will be used?

   How many more crayons will be needed in addition to the ones in these full boxes?

2. Juice boxes are sold in packs of 6. If 35 students eat lunch at one time, how many packs of juice boxes will the cafeteria need to open in order to serve them?

3. The school is buying extra juice boxes for their annual Field Day. Each pack of juice boxes costs $3. How many packs of juice boxes can the school purchase with $212?

Summary Statement

Are remainders important? Explain why, or why not.

_____

_____

_____

Name: _____

Exit Ticket

1. How are these word problems different?

Problem 1	Problem 2
A box can hold 6 baseballs. There are 50 baseballs. How many boxes can be filled?	A team needs 12 bats. There are 5 bats in a pack. How many packs will the team need?

Explain:

_____

_____

_____

2. Write two word problems involving division — one in which the remainder matters and one in which it does not.

Problem 1	Problem 2

Explain:

_____

_____

_____

* SOL 5.4

Lesson Summary

Students find the quotients and remainders of division problems, given dividends of four digits or fewer and divisors of two digits or fewer. (45 minutes)

Materials

Copies of the attached worksheets

Vocabulary

None

Background

This lesson is designed to help students sharpen their division skills. The problem set in this activity is appropriate for students who understand the basic division algorithm but who may have difficulty completing a long-division exercise.

Warm-up

Division can also be thought of as repeated subtraction, although with long division, the dividends and divisors grow too large to make repetitive subtraction a viable problem-solving strategy. Ask students to complete the attached warm-up worksheet, "Dividing Differently," in which they must subtract a number repeatedly until they reach zero or a remainder. Continue to discuss this process in the following lesson.

Lesson

1. Ask the class what the warm-up activity has to do with division. How is subtraction like division? Allow time for students to share their thoughts.
2. Continue the discussion by asking students to describe the difference between problems 1 and 2 versus problems 3 and 4 on the warm-up worksheet. Students may offer various ideas, but guide them toward recognizing the difference in the level of difficulty of the problems and the difference in the time it took to reach the solutions. Ask whether repeatedly subtracting a number to find the quotient is a good strategy to choose when dividing. Students may say "yes" for smaller dividends and divisors but "no" for larger ones, although they should already be accustomed to dividing single digits, using basic arithmetic facts.
3. Distribute copies of the process flow chart "Division: My Steps," and have students write a division problem in the oval at top center, for example, 712 ÷ 32. Allow the students to tell you how they would solve the problem. Listen carefully for them to identify the specific steps of a division algorithm, and have them write the steps they describe in their chart. Note that there are many different versions of the division algorithm that are mathematically correct. Encourage a class discussion in which students share their algorithms.
4. Once the students have explained their division strategies, help them summarize their main points, and have them write the main points in their chart.
5. Distribute copies of the "I Dare You to Divide!" worksheet, and give students time to work through each problem. Have them circle parts of any problem that they find difficult. Discuss these parts individually with students as you walk around the room.
6. Review the solutions with the students.

Reflection

Have students write the letter for the prompt on the "Reflection" worksheet.

Name: _____

Dividing Differently

Division can separate a large group of objects into smaller groups of objects. Repeated subtraction can do the same thing.

Example: Subtract 4 from 24 repeatedly until you reach 0 or have a remainder. Then, write the corresponding division equation.

Repeated subtraction:

24
− 4
20
− 4
16
− 4
12
− 4
8
− 4
4
− 4
0

Division equation: **24 ÷ 4 = 6**

In the following problems, write the corresponding division equation after doing the subtraction.

1. Subtract 7 from 56 repeatedly.

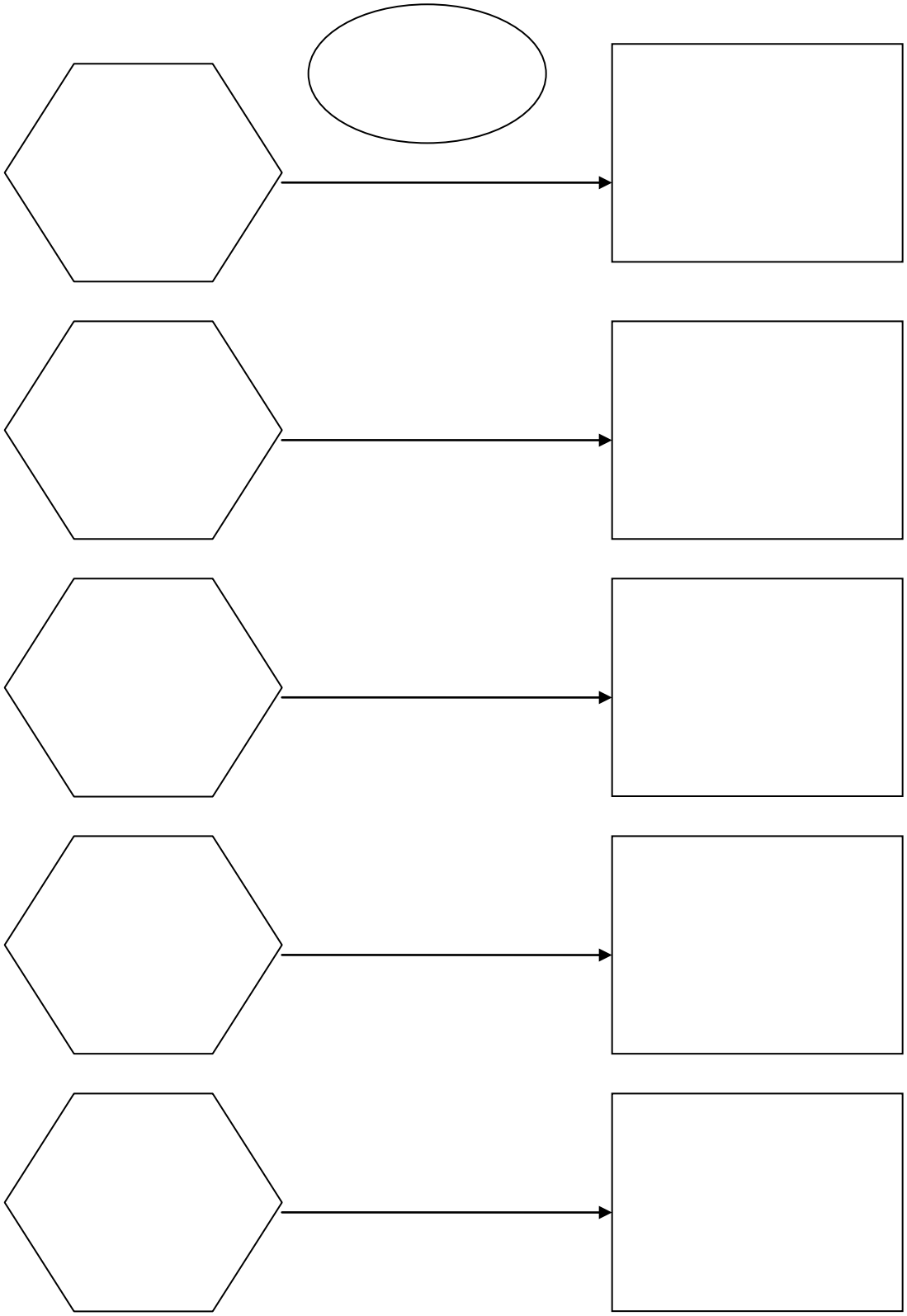
2. Subtract 9 from 36 repeatedly.

3. Subtract 8 from 144 repeatedly.

4. Subtract 16 from 212 repeatedly.

Name: _____

Division: My Steps



Name: _____

I Dare You to Divide!

Find the quotients and remainders for the problems below by finding the missing numbers. There may or may not be a remainder.

1. $33\overline{)627}$ — $1\_$, $-\ 33$

Remainder: __

2. $41\overline{)894}$ — $2\_$, $-\ 82$

Remainder: __

3. $18\overline{)440}$ — $2\_$, $-\ 36$

Remainder: __

4. $31\overline{)868}$ — $2\_$

Remainder: __

5. $23\overline{)970}$ — $4\_$

Remainder: __

6. $24\overline{)1,776}$ — $7\_$

Remainder: __

7. $16\overline{)4,056}$ — $2\_\ \_$, $-\ 32$

Remainder: __

8. $44\overline{)2,178}$ — $4\_$, $-\ 176$

Remainder: __

9. $48\overline{)3,192}$ — $6\_$

Remainder: __

10. $62\overline{)3,375}$ — $5\_\ \_$

Remainder: __

11. $6\overline{)2,1\_\ \_}$ — 57

Remainder: __

12. $1\_\overline{)3,\_\ \_\ \_}$ — $\_\ 26$

Remainder: __

Name: _____

Reflection

A new student has started at your school, and he is not yet very comfortable doing division. Write him a short explanation of how to divide, using as many of the vocabulary words (*dividend, divisor, quotient, remainder, multiple*) as you can. Also, create two division problems, and solve them for him so that he has examples to use in the future.

Examples of two division problems, with solutions, for future reference:

* SOL 5.4

Lesson Summary

Students solve multistep practical problems involving whole numbers by using the operations of addition, subtraction, multiplication, and division. (30–45 minutes)

Materials

Copies of the attached worksheets
Calculators (optional)
Highlighters or colored pencils

Warm-up

Students should realize that all numbers in a word problem are not always needed for solving it. Have students complete the "Number Search" worksheet by choosing the appropriate numbers from each set to answer the questions.

Ask the class to explain the relevance of this warm-up activity. Accept all reasonable responses. Ask students what they learned from having many given numbers but not using all of them in each word problem. Give students a chance to explain and share their opinions with each other.

Lesson

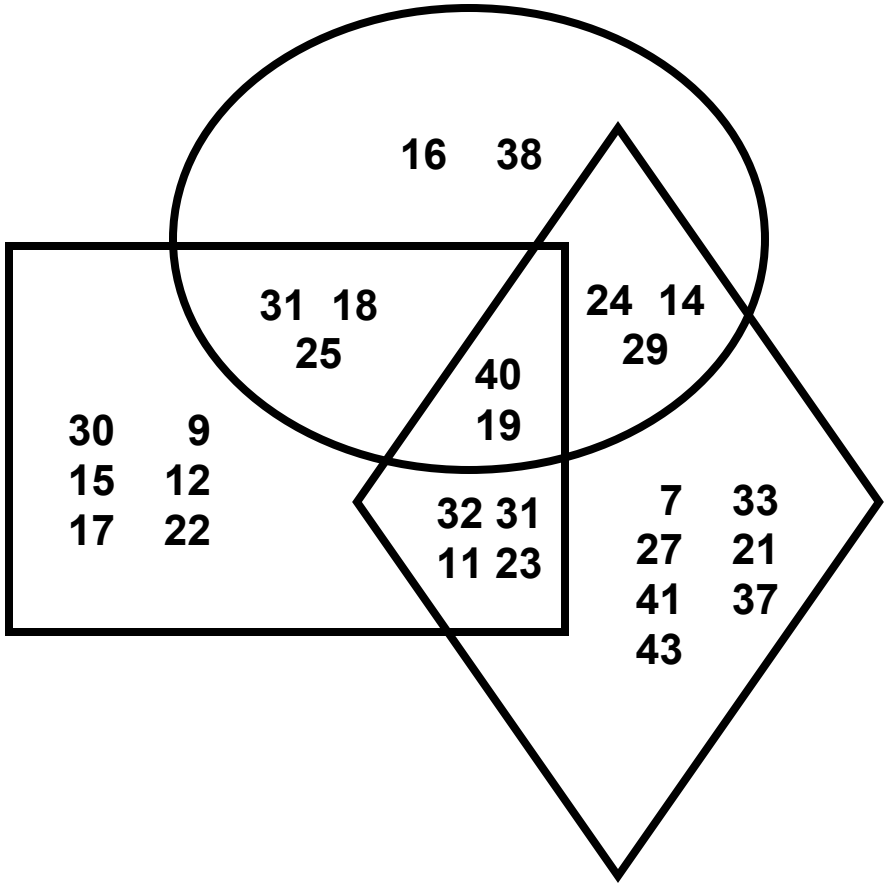
1. Tell the class that they will be solving real-life word problems involving numbers that may or may not be needed for solving the problems. Distribute the "Who'll Paint the Hotel?" worksheet, and go over the proposal data with the class, encouraging questions about painting and the information in the proposals.
2. Allow students to work in pairs to solve the word problems. Have each student circle or underline the essential parts of each word problem and the numbers they will use to solve it. Encourage discussions, monitor the students' work carefully, and answer any questions they may have.
3. When students have finished, ask them to share their solutions and describe the strategies they used to solve the problems. Encourage all students to respond, as a discussion on different approaches to solving word problems will be helpful to all.

Reflection

Have students complete the "Reflection" worksheet individually.

Name: _____

Number Search



1. Which numbers are in the oval and the rectangle? _____
2. Which numbers are in the rectangle and the diamond? _____
3. What is the sum of the numbers in only the diamond? _____
4. Which numbers are in the oval but not in the rectangle? _____
5. Which numbers are in the rectangle but not in the diamond? _____
6. Which numbers are in only the oval? _____
7. Which numbers are in the oval and diamond but not in the rectangle? _____
8. Which numbers are in all three shapes? _____

Name: _____

Who'll Paint the Hotel?

The City Hotel has 30 rooms that need to be painted. The hotel manager is considering two painting companies for the job. He has gotten proposals from both companies, and now he's comparing the information in the proposals. Here's the data he's comparing:

PAINT SPECIALISTS

Proposal to paint 30 rooms at The City Hotel:

- Crew of 3 painters
- Each painter can paint 300 square feet per hour.
- Each room has 720 square feet of surface area to paint.
- We buy paint for $16 a gallon.
- One gallon of our paint covers 400 square feet.

PRO PAINTERS

Proposal to paint 30 rooms at The City Hotel:

- Crew of 4 painters
- Each painter can paint 250 square feet per hour.
- Each room has 720 square feet of surface area to paint.
- We buy paint for $14 a gallon.
- One gallon of our paint covers 350 square feet.

Answer the following questions:

Paint Specialists	Pro Painters
1. How many gallons of paint will be needed to paint the 30 rooms in the City Hotel?	1. How many gallons of paint will be needed to paint the 30 rooms in the City Hotel?
2. How much will it cost to buy the paint?	2. How much will it cost to buy the paint?
3. Can the crew of 3 painters finish the job in 2 days if each person works 7 hours a day?	3. Can the crew of 4 painters finish the job in 2 days if each person works 7 hours a day?
4. If not, how many days would it take to finish the job?	4. If not, how many days would it take to finish the job?
5. If each painter makes $8 an hour and works 26 hours, how much money would the entire crew get?	5. If each painter makes $9 an hour and works 22 hours, how much money would the entire crew get?
6. How much will it cost City Hotel to paint the rooms if Paint Specialists does the job?	6. How much will it cost City Hotel to paint the rooms if Pro Painters does the job?
7. City Hotel also needs its lobby painted. Its surface is 25 times greater than one hotel room. What is the amount of surface area to be painted? How much would it cost to buy the paint?	7. City Hotel also needs its lobby painted. Its surface is 25 times greater than one hotel room. What is the amount of surface area to be painted? How much would it cost to buy the paint?

8. Based only on this data, which company should the manager of City Hotel choose to paint the hotel? Explain why.

Name: ANSWER KEY

Who'll Paint the Hotel?

Paint Specialists	Pro Painters
1. How many gallons of paint are needed to paint the 30 rooms in the City Hotel? 54	1. How many gallons of paint are needed to paint the 30 rooms in the City Hotel? 62
2. How much will it cost to buy the paint? $864	2. How much will it cost to buy the paint? $868
3. Can the crew of 3 painters finish the job in 2 days if each person works 7 hours a day? No	3. Can the crew of 4 painters finish the job in 2 days if each person works 7 hours a day? No
4. If not, how many days would it take to finish the job? 4	4. If not, how many days would it take to finish the job? 4
5. If each painter makes $8 an hour and works 26 hours, how much money would the entire crew get? $832	5. If each painter makes $9 an hour and works 22 hours, how much money would the entire crew get? $792
6. How much will it cost City Hotel to paint the rooms if Paint Specialists does the job? $1,696	6. How much will it cost City Hotel to paint the rooms if Pro Painters does the job? $1,660
7. City Hotel also needs its lobby painted. The surface area is 25 times greater than one hotel room. What is the amount of surface area to be painted? How many gallons of paint would be needed? How much would it cost to buy the paint? 18,000 sq. ft.; 45 gal.; $720	7. City Hotel also needs its lobby painted. The surface area is 25 times greater than one hotel room. What is the amount of surface area to be painted? How many gallons of paint would be needed? How much would it cost to buy the paint? 18,000 sq. ft.; 52 gal.; $728

8. Based only on this data, which company should the manager of City Hotel choose to paint the hotel? Explain why.

   Answers will vary.

Name: _____

Reflection

Was this activity easy or difficult? _____ Why?

_____

_____

_____

What did you learn from this activity?

_____

_____

_____

* SOL 5.4

Lesson Summary

Students solve multistep practical problems, using a variety of mathematical strategies. (30–45 minutes)

Materials

Copies of the attached worksheets
Calculators (optional)

Warm-up

Have students complete the "Choose the Correct Operation" worksheet. Identifying the operation to be used will increase students' familiarity with word-problem language and help them identify strategies to solve such problems.

Lesson

1. Lead a class discussion about ways to figure out what to do to solve a word problem. Ask students whether in the warm-up problems, they did anything differently in Step 2 from what they did in Step 1. Have students explain what strategies they used to decide on each operation. Listen carefully to their explanations, which will give valuable insight into the way they approach problem solving.
2. Create and post a class list of the strategies students employ to decide on the correct operations. Give a name to each strategy. Prompt thinking by asking such questions as: "Are there any other strategies you could add to this list? Which strategies do you use often? Which strategies do you use rarely? Why?"
3. Have the student pairs complete the "Using Different Strategies" worksheet, but ask them to leave the last column (Strategy Used) empty for now. Allow them time to complete the entire problem set. Give assistance when needed. When everyone is finished, review responses, and clear up any errors.
4. Refer the students back to the list of strategies discussed previously, and have them write in on the "Using Different Strategies" worksheet the strategy they used to solve each problem. Ask each student to explain his/her choices.

Reflection

Have students complete the "Reflection" worksheet individually.

Name: _____

Choose the Correct Operation

Read each word problem, and circle the operation or operations you'll use to solve it. Then show the steps used to solve it.

Word Problem	Operation	Step 1	Step 2
1. Allison paid for 3 school lunches at $3 each. She gave the office $20. How much change will she receive?	Addition Subtraction Multiplication Division		
2. The Drama Club is sewing their costumes for the play. One type of ribbon is $2 per yard, and they bought 3 yards. A second type of ribbon is $12 per yard, and they bought 5 yards. What was the cost of all the ribbon they bought?	Addition Subtraction Multiplication Division		
3. Don is fencing in his yard. He is also painting the fence white. Each of the 4 sides of the lawn is 15 ft. Each gallon of paint covers 20 feet. How much fencing and gallons of paint will Don need to buy?	Addition Subtraction Multiplication Division		
4. Susan bought 5 highlighters for $2 each and 4 pencils for $1 each. How much change will Susan receive if she pays $15?	Addition Subtraction Multiplication Division		
5. Marissa bought a new pair of jeans. The original price was $56, but she saved $19 by buying them on sale. She paid $40. How much change did Marissa receive?	Addition Subtraction Multiplication Division		

Name: _____

Using Different Strategies

Read each word problem. Then, show the steps for solving it, and write the answer. Finally, describe the strategy you used.

Word Problem	Steps	Answer	Strategy Used
1. Angela's soccer team washed 32 cars on Saturday. Each customer paid $4.75 for the wash. Did her team earn the $80 they needed for their new uniforms? How much did the team earn?			
2. Mark is having a picnic. He will prepare 3 hot dogs for each guest. There are 49 people coming. How many hot dogs will Mark prepare?			
3. Mr. Miller works 8 hours a day. Mrs. Miller works 6 hours a day. If Mr. Miller worked 96 hours in 2 weeks, how many hours did Mrs. Miller work during the same time?			
4. Scott invited his friends over for pizza. Scott ate 6 slices. His friend Billy ate half as much as Scott. David ate twice as many slices as Billy but 4 fewer than Terrence. How many slices of pizza did each boy eat?			
5. Maria had homework in 3 of her subjects. She finished all of it at 6:30 p.m. She spent 30 minutes on English and twice that long on science. She spent as long on math as she did on both English and science combined. At what time did Maria start her homework?			

Name: ANSWER KEY

Using Different Strategies

Read each word problem. Then, show the steps for solving it, and write the answer. Finally, describe the strategy you used.

Word Problem	Steps	Answer	Strategy Used
1. Angela's soccer team washed 32 cars on Saturday. Each customer paid $4.75 for the wash. Did her team earn the $80 they needed for their new uniforms? How much did the team earn?		Yes; $152	
2. Mark is having a picnic. He will prepare 3 hot dogs for each guest. There are 49 people coming. How many hot dogs will Mark prepare?		147	
3. Mr. Miller works 8 hours a day. Mrs. Miller works 6 hours a day. If Mr. Miller worked 96 hours in 2 weeks, how many hours did Mrs. Miller work during the same time?		72 hours	
4. Scott invited his friends over for pizza. Scott ate 6 slices. His friend Billy ate half as much as Scott. David ate twice as many slices as Billy but 4 fewer than Terrence. How many slices of pizza did each boy eat?		Scott: 6 Billy: 3 David: 6 Terrence: 10	
5. Maria had homework in 3 of her subjects. She finished all of it at 6:30 p.m. She spent 30 minutes on English and twice that long on science. She spent as long on math as she did on both English and science combined. At what time did Maria start her homework?		3:30 p.m.	

Name: _____

Reflection

Solve each released SOL test question. Then, give a brief explanation of the strategy you used.

1. A flight engineer for an airline flies an average of 2,923 miles per week. Which is the best estimate of the number of miles she flies in 3 years?

   F 150,000

   G 300,000

   H 450,000

   J 600,000

   Explain your strategy:

2. Last season, Ellen and Janet together won 32 tennis matches. Ellen won 6 more matches than Janet. How many matches did Ellen win?

   A 13

   B 16

   C 19

   D 25

   Explain your strategy:

Name: ANSWER KEY

Reflection

Solve each SOL released test question. Then, give a brief explanation of the strategy you used.

1. A flight engineer for an airline flies an average of 2,923 miles per week. Which is the best estimate of the number of miles she flies in 3 years?

   F 150,000

   G 300,000

   H 450,000

   J 600,000

   Explain your strategy:

   Answers will vary.

2. Last season, Ellen and Janet together won 32 tennis matches. Ellen won 6 more matches than Janet. How many matches did Ellen win?

   A 13

   B 16

   C 19

   D 25

   Explain your strategy:

   Answers will vary.

* SOL 5.4

Lesson Summary

Students practice mentally estimating and calculating, using the basic operations of addition, subtraction, multiplication, and division. (30 minutes)

Materials

Copies of the attached worksheets
Chart paper
Copies of the attached game cards, cut apart
Counters

Warm-up

Have students practice various problem-solving strategies by completing the "Missing Numbers Square" worksheet. When they are finished, lead a discussion of the *methods* they used to find the missing numbers. As the students offer their strategies, list them on a chart for display throughout this lesson.

Lesson

1. Begin the activity by giving students the opportunity to share what kinds of things they like to shop for at the mall. Ask them how they use math when shopping, and discuss their responses. Tell the students that they will be playing a game similar to Bingo that will give them helpful practice in *estimating* necessary calculations when they go shopping.
2. Give each student a copy of the attached "At the Mall Game Board" handout. Decide what will constitute a winning Bingo game board—traditional row, column, or diagonal; four corners; postage stamp; etc.—and explain it to the students. Pair the students, and give each pair a stack of the attached "At the Mall" game cards and some counters. Have the pairs of players decide which player is Player 1 and which is Player 2.
3. Each pair of players shuffles the game cards and places the stack face down. Player 1 draws the top card, solves the problem, using estimation or mental calculation, and announces the solution to Player 2. If the solution is on the game board (all solutions are on the board), Player 1 places a counter on that number on his/her game board. If the solution reached is not on the board, the solution is wrong, and Player 1 gives the game card to Player 2 for solving.
4. If Player 2 solves the problem correctly, he/she places a counter on that number on his/her game board. If Player 2 does not solve it correctly, he/she must ask the teacher to review with both players the calculations used and discuss the errors.
5. Player 2 draws the next card and follows the same procedure.
6. The game continues in this manner until one player achieves Bingo.

Reflection

Have students complete the "Memo: What I Remember" worksheet. Allow students time to share what they have written with their partners.

Name: _____

Missing Numbers Square

In the square below

- the numbers in each row add up to totals on the right
- the numbers in each column add up to the totals at the bottom
- the six missing numbers are whole numbers, 0 through 9.

Fill in the missing numbers, doing the calculations in your head. You may use a number more than once.

__	__	3	9	20
0	6	9	__	18
1	5	__	__	11
__	9	3	5	23
9	26	19	18	17

At the Mall Game Cards

The mall is having a special *"Buy 2, Save $20!"* sale on tennis shoes. How much will 2 pairs of tennis shoes cost during this sale?	Sandra is going to a dance. She has $60 and wants to buy a new dress. How much change will she get back?
Mrs. White has 3 children. Each of her children needs a new pair of jeans. How much will she spend?	The city basketball coach wants to buy each of his 9 team members a new sweat suit. How much money will the coach need?
Tommy plans to buy one pair of jeans, one T-shirt, and one belt. How much money will he spend?	Devon plans to buy 2 pairs of jeans. How much will he spend?
How much money will Ben save if he buys one pair of shorts instead of one sweat suit?	The mall is having a sale on sweat suits – *Buy 2 sweat suits and save $10 off the total cost!* How much would it cost to buy 2 sweat suits during this sale?
Mr. Jones has $40 and needs to buy a belt and 2 pairs of socks. How much change will he receive?	Jenna needs a new skirt, a new blouse, and a belt. How much money will she need to buy all the items?
Josh plans on buying a new outfit, but he wants to spend as little as possible. Should he buy jeans and a T-shirt or shorts and new tennis shoes? What is the cost of the least expensive combination?	Mary Anna is going to tennis camp. She needs to buy 4 pairs of socks, 1 pair of tennis shoes, and 2 T-shirts. How much money will she spend on these items?
Travis is on the cross country team at his school. He buys 5 pairs of tennis shoes during one year. How much money does Travis spend on tennis shoes?	Which costs more: one dress OR one skirt and one blouse? How much does the more expensive outfit cost?
Mrs. Davis is buying her two boys one pair of new jeans each, but she is spending only $50 of her own money. How much will each of her sons need to pay for his jeans?	Kayla is going to the beach and wants to buy 5 new T-shirts. How much money will she need to spend?
Marcus wants to buy a new pair of tennis shoes. He has $27. How much more money does he need?	Mrs. Johnson is buying herself a new sweat suit and a new pair of tennis shoes. How much money will she spend on these items?
Lisa bought one new pair of jeans and one blouse. She took $80 to the mall. How much change did she receive?	Michael buys a sweat suit and one pair of socks. How much money does he spend?
Matt gets $10 per week allowance. If he saves his money for 5 weeks, how much money will he have after buying a new pair of tennis shoes?	Ellen wants to buy a new skirt and belt, but she has only $37. How much more money does she need to buy both of these items?
Ms. Daniels needs new clothes for work. She buys 2 skirts, 4 blouses, and 1 belt. How much money does she spend?	Mariah bought a pair of shorts instead of a dress. How much money did she save?
Alex got $50 for her birthday. She bought one new pair of jeans. How much of her birthday money does she have left?	

At the Mall Game Cards: Answer Key

The mall is having a special *"Buy 2, Save $20!"* sale on tennis shoes. How much will 2 pairs of tennis shoes cost during this sale? $70	Sandra is going to a dance. She has $60 and wants to buy a new dress. How much change will she get back? $18
Mrs. White has 3 children. Each of her children needs a new pair of jeans. How much will she spend? $108	The city basketball coach wants to buy each of his 9 team members a new sweat suit. How much money will the coach need? $306
Tommy plans to buy one pair of jeans, one T-shirt, and one belt. How much money will he spend? $67	Devon plans to buy 2 pairs of jeans. How much will he spend? $72
How much money will Ben save if he buys one pair of shorts instead of one sweat suit? $13	The mall is having a sale on sweat suits – *Buy 2 sweat suits and save $10 off the total cost!* How much would it cost to buy 2 sweat suits during this sale? $58
Mr. Jones has $40 and needs to buy a belt and 2 pairs of socks. How much change will he receive? $0	Jenna needs a new skirt, a new blouse, and a belt. How much money will she need to buy all the items? $73
Josh plans on buying a new outfit, but he wants to spend as little as possible. Should he buy jeans and a T-shirt or shorts and new tennis shoes? What is the cost of the least expensive combination? $51	Mary Anna is going to tennis camp. She needs to buy 4 pairs of socks, 1 pair of tennis shoes, and 2 T-shirts. How much money will she spend on these items? $123
Travis is on the cross country team at his school. He buys 5 pairs of tennis shoes during one year. How much money does Travis spend on tennis shoes? $225	Which costs more: one dress OR one skirt and one blouse? How much does the more expensive outfit cost? $57
Mrs. Davis is buying her two boys one pair of new jeans each, but she is spending only $50 of her own money. How much will each of her sons need to pay for his jeans? $11	Kayla is going to the beach and wants to buy 5 new T-shirts. How much money will she need to spend? $75
Marcus wants to buy a new pair of tennis shoes. He has $27. How much more money does he need? $18	Mrs. Johnson is buying herself a new sweat suit and a new pair of tennis shoes. How much money will she spend on these items? $79
Lisa bought one new pair of jeans and one blouse. She took $80 to the mall. How much change did she receive? $16	Michael buys a sweat suit and one pair of socks. How much money does he spend? $46
Matt gets $10 per week allowance. If he saves his money for 5 weeks, how much money will he have after buying a new pair of tennis shoes? $5	Ellen wants to buy a new skirt and belt, but she has only $37. How much more money does she need to buy both of these items? $8
Ms. Daniels needs new clothes for work. She buys 2 skirts, 4 blouses, and 1 belt. How much money does she spend? $186	Mariah bought a pair of shorts instead of a dress. How much money did she save? $21
Alex got $50 for her birthday. She bought one new pair of jeans. How much of her birthday money does she have left? $14	

Name: _____

At the Mall Game Board

Jeans:	**$36**	**Skirt:**	**$29**
T-shirt:	**$15**	**Socks:**	**$12**
Blouse:	**$28**	**Tennis shoes:**	**$45**
Belt:	**$16**	**Sweat suit:**	**$34**
Dress:	**$42**	**Shorts:**	**$21**

$79	$46	$51	$72	$58
$306	$16	$18	$75	$11
$8	$108	$0	$123	$225
$21	$73	$5	$14	$67
$186	$13	$57	$70	$18

Name: _____

Memo: What I Remember

What did I learn today?

_____

_____

_____

What did I do well?

_____

_____

_____

What am I confused about?

_____

_____

_____

What do I need help with?

_____

_____

_____

How will I use this skill in real life?

_____

_____

_____

* SOL 5.4

Lesson Summary

Students solve multistep practical problems involving whole numbers by using the operations of addition, subtraction, multiplication, and division. (30–45 minutes)

Materials

Copies of the attached worksheets
Math textbooks
Calculators (optional)

Warm-up

Identifying and completing a pattern will help students discern the operation being used and is one method for solving word problems. Have students complete the "Follow That Pattern" worksheet. Review answers with students.

Lesson

1. Ask students how they can figure out what to do to a solve word problem, and make sure that they fully explain the process they follow. Record their strategies, and display this list on the board or a chart.
2. Give each student a copy of the attached "Problem-Solving Checklist." Juxtapose it with the list of strategies just made. Ask students to point out the similarities and differences between their list and the checklist and to compare and contrast the steps in the two lists.
3. Guide students in applying the "Problem-Solving Checklist" to various selected word problems from classroom math textbooks. Ask students to explain how they decided which information to put into the Understand, Plan, Solve, and Check categories. Have students also explain how they chose the operation(s) to use in each problem.
4. Select additional word problems from the textbooks, and have the students independently apply the checklist to them. Review each category with them, the operation(s) chosen, and the reasonableness of the solution.

Reflection

Pair the students, and assign one student in each pair to be Person A and the other Person B. Direct the A's to explain to the B's the most important idea in this lesson. Allow the A's one minute to think about this and make the explanation. Then, have the partners reverse roles.

Name: _____

Follow That Pattern

Finish the patterns in the rows below, and identify the pattern in the last column.

Pattern	First term	Second term	Third term	Fourth term	Fifth term	Sixth term	What is the pattern?
1.	7	5	7	5			
2.	1	3		27	81		
3.	192	96	48		12		
4.	2	5	8	11			
5.	5	10		20		30	
6.	17	16	13	12			
7.	8	11		17		23	
8.	1	10	2	20			
9.	64	32		8		2	
10.	2	10	50	250			

Name: _____

Problem-Solving Checklist

Complete each step below, and put an **X** in the box when each is complete.

Understand

☐ What do you know?

_____

☐ What do you need to find?

_____

Plan

☐ How can you find the answer? (+, −, ×, ÷)

_____

Solve

☐ Calculate the answer.

_____

_____

The answer is _____

Check

☐ How do you know your answer is correct?

_____

_____

* SOL 5.4

Lesson Summary

Students practice identifying appropriate strategies to solve problems. (30–45 minutes)

Materials

Copies of the attached worksheets
Calculators (optional)

Warm-up

Have students complete the "What Do YOU Think?" worksheet. Ask the students to explain their responses. Encourage discussion about the similarities and differences in their responses.

Lesson

1. Ask students how thinking about the *meaning* of addition can help when solving word problems. Give students a chance to explain individually and share their thoughts with each other. Repeat this process for subtraction, multiplication, and division.
2. Ask students what different strategies are used to solve word problems. List their answers, and keep these strategies on display throughout the lesson. Ask whether they are more comfortable using certain strategies then others. If they answer yes, ask them to explain.
3. Distribute the "What's the Story?" worksheets, and review the directions, noting that there is a combination of multiple-choice and free response problems. Also emphasize the importance of explaining solutions thoroughly when asked to do so.
4. Have students complete the worksheets, offering assistance as needed while they work. Their responses will help you help them with any misunderstanding or errors. You may want to work on the theater problem one day and the camping problem the next, depending on your the time available.
5. Once students are finished, review answers with the class. Clear up any errors or confusion.

Reflection

Have students complete the "Reflection" worksheet. You may want to collect, check, and return these to give individual feedback to students.

Name: _____

What Do YOU Think?

1. In your own words, write what it means to add.

_____

_____

2. In your own words, write what it means to subtract.

_____

_____

3. In your own words, write what it means to multiply.

_____

_____

4. In your own words, write what it means to divide.

_____

_____

Name: _____

What's the Story? 1

Read the story carefully. Then, read each question, and consider all choices before choosing a solution.

A Day of Theater

The Martinez family is going to a play in Richmond. A round trip train ticket costs $8.00. Children under 7 years old travel for half price. The bus from the train station to the theater costs $2.00 per person each way. Tickets for the play are $14.00 for an adult and $6.00 for a child.

Mr. and Mrs. Martinez will go with their sons, Ricky and Thomas, and their daughter, Ramona. Ramona and Ricky are 12 years old. Thomas is 6 years old.

1. How much will it cost the entire Martinez family to take the train to Richmond and back?
   - A $16.00
   - B $8.00
   - C $40.00
   - D $36.00

2. It is most expensive for
   - A Mr. and Mrs. Martinez to travel to and from the play.
   - B Mr. Martinez, Ricky, and Thomas to buy theater tickets.
   - C Mr. and Mrs. Martinez to buy theater tickets.
   - D Mrs. Martinez, Ricky, and Ramona to travel to and from the play.

3. A class of children is also going to the play. It spends $158 on tickets for the students and the teacher. How many students are in the class? Explain your solution.

4. What would be the *entire* cost of the trip for 12 students if 5 of the students are younger than 7 years old? Explain your solution.

Name: _____

What's the Story? 2

Read the story carefully. Then, read each question, and consider all choices before choosing a solution.

A Camping Trip

Scouting Troop A is going on a hike. There are 8 scouts and 2 adults on the trip, each with a backpack. Each adult weighs about 160 pounds. Each scout weighs about 100 pounds. Each backpack weighs about 20 pounds.

The troop is getting ready to cross a river in a small boat. The boat can hold up to 360 pounds without sinking.

1. The boat will be filled to its limit by
   - A 1 scout and 7 backpacks.
   - B 1 adult and 2 scouts.
   - C 3 scouts and 3 backpacks.
   - D 2 scouts and 3 backpacks.

2. The weight of 1 adult is equal to
   - A 8 backpacks.
   - B 1 scout and 5 backpacks.
   - C 1 scout and 4 backpacks.
   - D 2 scouts.

3. What weighs the same as 3 scouts and 1 backpack?
   - A 1 adult and 5 backpacks
   - B 1 adult and 8 backpacks
   - C 2 adults
   - D 2 adults and 1 backpack

4. What is the combined weight of all scouts, adults, and backpacks? Explain your solution.

Name: _____

Reflection

1. Write one word problem involving addition in which the answer is 4.

2. Write one word problem involving subtraction in which the answer is 6.

3. Write one word problem involving multiplication in which the answer is 16.

4. Write one word problem involving division in which the answer is 5.

Name: _____

Released SOL Test Questions

1. A bag of chocolate candy holds exactly 408 pieces. Eight friends plan to share the chocolate pieces equally. How many chocolate pieces should each friend receive?

   F 3,264

   G 416

   H 400

   J 51

2. A flight engineer for an airline flies an average of 2,923 miles per week. Which is the *best* estimate of the number of miles she flies in 3 years?

   F 150,000

   G 300,000

   H 450,000

   J 600,000

Name: ANSWER KEY

Released SOL Test Questions

1. A bag of chocolate candy holds exactly 408 pieces. Eight friends plan to share the chocolate pieces equally. How many chocolate pieces should each friend receive?

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   F 150,000

   G 300,000

   H 450,000

   J 600,000

* SOL 6.5, 8.5a

Lesson Summary

Students build, describe, and explore color-tile patterns that represent square numbers and square roots. (45 minutes)

Materials

Color tiles
"Build a Square Recording Sheet" handouts
"Reflection" worksheets

Vocabulary

square number. A number that results from multiplying any whole number by itself.
square root. A number which, when multiplied by itself, produces a square number.
area. The number of square units needed to cover a two-dimensional figure.

Warm-up

Give students a collection of color tiles, and ask them to build as many different rectangles as possible, using 12 tiles at a time. All rectangles must be made with exactly 12 tiles. Ask: "What are the dimensions of each of the rectangles?" (1 x 12, 2 x 6, 3 x 4) Ask: "What is the area of each of the rectangles?" (12)

Lesson

1. Tell the students that you want them to use the color tiles to build the smallest square figure possible. Because the smallest square is simply one tile, if a student builds a square that is larger than one tile, challenge him/her to think about how to represent a *smaller* square. Ask students to explain how they know whether the figure is a square. (Answers should include all or some of the following: All four sides are congruent. All four corners are right angles. The opposite sides of the figure are parallel. Adjacent sides of the figure are perpendicular.) If a student builds a rectangle, review the properties of a square.
2. Give students recording sheets, and have them record the data from the smallest square. Explain that the length of one side of the tile will be considered 1 unit. The first column will be used to record the number of units on one side of the square figure. (1) The dimensions column will be used to record the number of units along the width times the number of units along the length. (1x1) The last column will be used to record the area of the square—the total number of tiles needed to build the square. (1)
3. Ask students to build the next smallest square possible. (This should be a 2 x 2 square, using four tiles. Again, if a student builds a rectangle, review the properties of a square. If a student builds a larger square, challenge him/her to find one that is smaller. Have students fill in the data for the second square on their recording sheet.
4. Continue this process, having students build successively larger squares and fill in the recording sheet until they have completed a 6 x 6 square.
5. Ask the students what patterns they see in the table. (Answers might include, but are not limited to, the following: The length of one side of each square is increasing by one unit as the squares get larger. The length and width dimensions of each square are always the same. The area of each square is always the product of the length of one side times itself. The area of each successive square is increasing by consecutive odd numbers.)
6. If no one mentions that the numbers in the last column are all square numbers, extend the discussion to square roots, and point out on the chart that the square root of an area measure of a square is equal to the length of one side of the square (column 1). Ask the students to explain why this relationship exists. Explain that "squaring a number" and "taking the square root of a number" are inverse operations. Have students predict what the next square number (and the square root of that number) on their recording sheet will be.

Reflection

Have students complete the “Reflection” worksheet.

Name: _____

Build a Square Recording Sheet

Length of One Side of the Square	Dimensions of the Square	Area of the Square (Total Number of Tiles)

What patterns do you see?

What is your prediction of the numbers for the next row?

Name: ANSWER KEY

Build a Square Recording Sheet

Length of One Side of the Square	Dimensions of the Square	Area of the Square (Total Number of Tiles)
1	1 by 1	1
2	2 by 2	4
3	3 by 3	9
4	4 by 4	16
5	5 by 5	25
6	6 by 6	36

What patterns do you see?

The length of one side of each square is increasing by one unit each time.
The length and width dimensions of each square are always the same.
The area of each square is always the product of the length of one side times itself.
The area of each successive square is increasing by consecutive odd numbers.

What is your prediction of the numbers for the next row?

Length of One Side: 7
Dimensions: 7 by 7
Area: 49

* SOL 7.1b

Lesson Summary

Using the data from books or the internet, students practice writing numbers in scientific notation. (30 minutes)

Materials

A book with large numbers, such as *On Beyond A Million: An Amazing Math Journey* by David M. Schwartz.

Vocabulary

exponent. A number that represents the number of times a base is used as a factor, i.e., *x* in the expression a^x.

base. A factor that is multiplied by itself as many times as indicated by its exponent, i.e., *a* in the expression a^x.

scientific notation. A system for writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10.

Warm-up

Have students extract some very large numbers from a book such as *On Beyond a Million: An Amazing Math Journey* by David M. Schwartz, or pull number facts from *The Guinness Book of World Records* or *Earth Facts*. Tell the students to write the number "googol" (1 followed by 100 zeros, or 10^{100}), and time how long it takes them to do this. Ask students to explain the value of using exponents for writing large numbers. (It is a faster way of writing the number.) Review using a base and exponent for writing other large numbers (e.g., million, billion, and trillion) as powers of 10.

Lesson

1. Using examples of large numbers from the selected book, explain that many people work with huge numbers. An example are scientists who measure distances among stars. Very large numbers are so big that they are hard to read and are cumbersome to write. Therefore, when scientists write very big numbers, they take a shortcut and use *scientific notation,* which is different from the usual way of writing numbers (standard notation).
2. Explain the process of changing a huge number into scientific notation. For example, to write in scientific notation the number of Tootsie Rolls® manufactured daily (37,000,000), do the following:
    - Determine the new number between 1 and 10: Move the decimal point in the original number so that the new number is between 1 and 10. (3.7000000, or 3.7)
    - Determine the power of 10: Count the number of places you moved the decimal point. (7) This number equals the exponent. Write the power of 10 that you would need to multiply the new number by in order to get the original number. (10^7)
    - Write the two parts as a multiplication expression. (3.7×10^7)
3. Have students practice writing other numbers from the selected book in scientific notation, for example: the number of people in the U.S. (2.5×10^8); the number of stars in the Milky Way (3×10^{11}); the weight of the earth in pounds (13×10^{25}).
4. Have students practice changing numbers in scientific notation into standard notation.
5. Ask students to find a very large number that interests them, such as the number of M&M's manufactured in one day, and write it in scientific notation.

Refection

Have students complete the questions on the "Reflection" worksheet.

Name: _____

Reflection

1. Write the number 243,000,000,000 in scientific notation.

2. Write the number 3.6×10^{12} in standard notation.

3. Explain the advantage of writing numbers in scientific notation.

4. Explain the process for writing numbers in scientific notation.

Name: ANSWER KEY

Reflection

1. Write the number 243,000,000,000 in scientific notation.

   2.43×10^{11}

2. Write the number 3.6×10^{12} in standard notation.

   3,600,000,000,000

3. Explain the advantage of writing numbers in scientific notation.

   Writing numbers in scientific notation makes it possible to write very large or very small numbers quickly and makes them easier to read.

4. Explain the process for writing numbers in scientific notation.

   Answers will vary, for example: “When writing a very large number, break the number into two parts. First move the decimal point so that there is one digit to the left of it. Next, multiply that number by a power of 10 that is determined by the number of spaces that the decimal point was moved.

* SOL 7.1b

Lesson Summary

Students discover patterns for converting very large and very small numbers into scientific notation. They determine equivalent relationships between numbers written in standard notation and numbers written in scientific notation. (45 minutes)

Materials

"Number Notation Table" handouts
"Planet Distance Table" handouts
Computers with Internet access

Vocabulary

scientific notation. A system for writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10.

Warm-up

1. Write the following on the board: "There are approximately 6,000,000,000 people on Earth. Can you explain how many people this is? Do you think there is room for 6,000,000,000 more people?" Allow students to share their thoughts.
2. Tell students to assume that every person on Earth has 10 fingers and 10 toes. Have students figure out how many human fingers and toes there are. (20 × 6,000,000,000 = 120,000,000,000) Allow students to share their answers and the techniques they used to arrive at their answers to see if any students solved the problem in a way other than standard multiplication—e.g., multiply 6 times 20 and add 9 zeros. Ask, "Why might you add 9 zeros? What does adding 9 zeros mean?"
3. Distribute the "Number Notation Table" handouts. Lead the class in completing the first few rows of the table together. Discuss the pattern that emerges.
4. Have the students finish the table individually or in groups. Reinforce to the students that they should rely on the pattern just discussed. Review the answers when everyone is finished.

Lesson

1. Give each student a copy of the "Planet Distance Table" handout. Demonstrate how to change the distance Mercury is from the sun, 35,000,000 miles, into scientific notation. (3.5×10^7)
2. Discuss how the number was changed, and have students compare the pattern they discovered in the warm-up to the number. Ask students how the exponent is related to the decimal shift. Students may assume that the exponent number equals the number of zeroes; however, the exponent number (power of ten) equals the number of places the decimal moves, e.g., 1,400 = 1.4×10^3: the decimal moves three places to the left. Ask, "Why does the decimal move three places to the left?" Demonstrate this decimal move by multiplying 1.4 × 1,000 on the board.
3. Demonstrate on the board the process in the previous step. Multiply 3.6 × 100, 3.6 × 1000, and 3.6 × 10. Show all steps. Let the students discover how the decimal point moves. Discuss whether your number is greater than or less than 1. Change 3.5×10^7 back to standard form.
4. Compare 1×10^7 to 3.5×10^7. Emphasize again that <u>the exponent number (power of ten) equals the number of places the decimal moves.</u>
5. Explain to students that scientific notation can also be used to write very small numbers. Write 0.0000046 on the board. Review the steps below:
   - <u>Determine the new number between 1 and 10</u>: Move the decimal point in the original number so that the new number is between 1 and 10. (4.6)
   - <u>Determine the power of 10</u>: Count the number of places you moved the decimal point. (−6) This number equals the exponent. Write the power of 10 that you would need to multiply the new number by in order to get the original number. (10^{-6})
   - Write the two parts as a multiplication expression. (4.6×10^{-6})

- Demonstrate how multiplication of 4.6 × 0.000001 = 0.0000046.

6. Have students complete the "Planet Distance Table," and then review answers. Stress that the sign of the exponent in a power of 10 tells whether the number is less than or greater than 1. Have students write the following rules on their scientific notation handouts:
   - A power of 10 with a **positive exponent,** such as 10^5, means the decimal is greater than 1.
   - A power of 10 with a **negative exponent,** such as 10^{-5}, means the decimal is less than 1.
7. Have students go to http://www.aaamath.com. Seventh-grade students should go to the seventh-grade topics; eighth-grade students should go to the eighth-grade topics. Under each grade's topics, have students choose "Scientific Notation" and do 10 to 20 exercises from the following topics:
   - Converting Numbers to Scientific Notation
   - Converting Scientific Notation to Numbers
   - Comparing Scientific Notation and Standard Numbers
   - Converting Decimals to Scientific Notation
   - Converting Scientific Notation to Decimals

Reflection

Conduct a class discussion around the following questions:

- Why is scientific notation used?
- What are some careers or professions is which scientific notation is regularly used?
- In what professions would very large numbers be used?
- In what professions would very small numbers be used?

Name: _____

Number Notation Table

Number Spelled Out	Number in Standard Notation	Number in Scientific Notation
One		
Ten		
One hundred		
One thousand		
Ten thousand		
One hundred thousand		
One million		
Ten million		
One hundred million		
One billion		
Ten billion		
One hundred billion		
One trillion		

Name: ANSWER KEY

Number Notation Table

Number Spelled Out	Number in Standard Notation	Number in Scientific Notation
One	1	1×10^0
Ten	10	1×10^1
One hundred	100	1×10^2
One thousand	1,000	1×10^3
Ten thousand	10,000	1×10^4
One hundred thousand	100,000	1×10^5
One million	1,000,000	1×10^6
Ten million	10,000,000	1×10^7
One hundred million	100,000,000	1×10^8
One billion	1,000,000,000	1×10^9
Ten billion	10,000,000,000	1×10^{10}
One hundred billion	100,000,000,000	1×10^{11}
One trillion	1,000,000,000,000	1×10^{12}

Name: _____

Planet Distance Table

Complete the table below.

Planet	Miles from the Sun in Standard Notation	Miles from the Sun in Scientific Notation
Mercury	35,000,000	
Venus		6.5×10^7
Earth		9.3×10^7
Mars	137,000,000	
Jupiter	467,000,000	
Saturn	850,000,000	
Uranus		1.7×10^9
Neptune		2.7×10^9
(dwarf planet) Pluto	3,500,000,000	

Name: ANSWER KEY

Planet Distance Table

Complete the table below.

Planet	Miles from the Sun in Standard Notation	Miles from the Sun in Scientific Notation
Mercury	35,000,000	3.5×10^7
Venus	65,000,000	6.5×10^7
Earth	93,000,000	9.3×10^7
Mars	137,000,000	1.37×10^8
Jupiter	467,000,000	4.67×10^8
Saturn	850,000,000	8.5×10^8
Uranus	1,700,000,000	1.7×10^9
Neptune	2,700,000,000	2.7×10^9
(dwarf planet) Pluto	3,500,000,000	3.5×10^9

* SOL 8.5 a, b

Lesson Summary

Using physical representations of square numbers, students investigate the relationship that exists between perfect squares and their square roots. (45 minutes)

Materials

Scissors
"Squares Template" handouts
Colored pencils
Calculators
"Perfect Squares" handouts
White boards (optional)
"Square Roots" worksheets

Vocabulary

perfect square. A whole number whose square root is a whole number.

Warm-up

Write the following incomplete sequences on the board. Ask students to complete the sequences and explain them.

1, 3, 6, 10, __, __, __, __	(Answer: 15, 21, 28, 36. The terms are increasing by 1 more than each previous increase.)
1, 1, 2, 3, 5, 8, __, __, __, __	(Answer: 13, 21, 34, 56. The terms are increasing by the sum of the previous two terms.)
1, 3, 5, 7, __, __, __, __	(Answer: 9, 11, 13, 15. The terms are increasing by 2.)
3/2, 2, 5/2, 3, 7/2, __, __, __, __	(Answer: 4, 9/2, 5, 11/2. The numerator of each fraction is increasing by 1, and fractions are being simplified.)
9, 23, 37, 51, 65, __, __, __, __	(Answer: 79, 93, 107, 121. The terms are increasing by 14.)

When students are finished, have them check their answers with a partner. Help partners resolve any difference between their answers to any sequence problem.

Lesson

1. Make enough copies of the "Squares Template" on card stock or heavy construction paper for each student to have two sheets.
2. Give each student a pair of scissors and two sheets of the "Squares Template." Have the students cut out the squares from one of the templates so that they each have 56 individual squares.
3. Ask students to construct as many different sizes of larger squares out of their individual squares as they can. For each one they make, they should copy and shade it on the intact template, using a different color for each square.
4. After recording their squares, ask students to write the area of each larger square below the square on the template. (They should get 4, 9, 25, 36, and 49.)
5. Conduct a class discussion. Begin by asking the class what areas their squares have, and record the areas on the board. Ask the students if these numbers look familiar.
6. Ask the students to take 15 individual squares and create a large square. Is this possible? (No) Have them try 7, 8, and 3. Once the students are convinced that these numbers of individual squares cannot produce larger squares, write "15, 7, 8, 3" on the board apart from the perfect square numbers. Ask the students what is special about 4, 9, 16, 25, and 36 that is not true of 15, 7, 8, and 3. The answers should be similar to: "4, 9, 16, 25, and 36 squares can make a larger square, but 15, 7,

8, and 3 squares cannot." Once the students come to that conclusion, name the special numbers *perfect squares*.

7. Once the perfect squares are named, point out the factors that are the square roots. Associate the square roots with the length of one side of each larger square. Ask students to identify in writing the perfect square numbers and the square roots for each larger square.
8. Have students work in pairs to answer the following questions: "The first six perfect square numbers are 1, 4, 9, 16, 25, and 36. What are the next five? How did you find your answers?" Assist partners in finding a strategy, if necessary.
9. Have the groups share their answers.
10. Ask students for the square roots of a number you call out. Begin with three or four perfect squares. Then ask for the square root of 15. Students should have difficulty. At this point, pass out the "Perfect Squares" list handouts, and ask the students to find where 15 would fall on the list. (The square root of 15 falls between 4 [the square root of 16] and 5 [the square root of 25].)
11. If time permits or in the next lesson, distribute white boards to students. Call out a number that is not a perfect square, and ask students to estimate the square root of that number by giving the two whole-number square roots it falls between. Allow students to consult their list of perfect squares. Have students write their answers on the white board and hold them up for you to check. Continue for as long as needed. This is a good activity to repeat daily as a brief review.
12. Conclude the lesson by having the students complete the "Square Roots" worksheet for review.

Reflection

Have students explain in writing how a perfect square and a square root relate to the area of a square. Allow them to use drawings in their explanations.

Squares Template

Perfect Squares

1

4

9

16

25

36

49

64

81

100

121

144

Name: _____

Square Roots

Approximate the following square roots. Between which two perfect square numbers does the answer lie?

1. $\sqrt{33}$ _____
2. $\sqrt{46}$ _____
3. $\sqrt{26}$ _____
4. $\sqrt{62}$ _____
5. $\sqrt{87}$ _____

Use a calculator to determine whether each of the following numbers is a perfect square. Support your answer.

1. 96 _____
2. 132 _____
3. 529 _____

Find each of the following measurements.

1. Find the length of the side of a square whose area is 144 $in^{2.}$

   _____

2. Find the area of a square whose side length is 10 cm.

   _____

Name: ANSWER KEY

Square Roots

Approximate the following square roots. Between which two perfect square numbers does the answer lie?

1. $\sqrt{33}$ between 5 and 6
2. $\sqrt{46}$ between 6 and 7
3. $\sqrt{26}$ between 5 and 6
4. $\sqrt{62}$ between 7 and 8
5. $\sqrt{87}$ between 9 and 10

Use a calculator to determine whether each of the following numbers is a perfect square. Support your answer.

1. 96 no
2. 132 no
3. 529 yes (23)

Find each of the following measurements.

1. Find the length of the side of a square whose area is 144 in^2.

   12 in.

2. Find the area of a square whose side length is 10 cm.

   100 cm^2