Formation of runaway electron distribution function during gas breakdown in high electric fields

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This paper presents a simple kinetic equation for the analysis of the runaway electron distribution function (EDF) during a dense gas breakdown in high electric fields. Characteristic scales (in energy and space) are obtained from a formal solution of the kinetic equation, and an analytical solution for the EDF is derived in the plasma region where the EDF does not depend on spatial coordinates and depends on electron velocity and time via a combination $\xi = t - v/a$, where a is the net electron acceleration. Kinetic analysis of the cathode sheath dynamics confirms the absence of the cathode directed ionisation wave in the absence of electron emission from the cathode.

1. Introduction

Breakdown of dense gases by high voltage pulses with sharp fronts is currently being investigated for a variety of applications [1]. The theory of pulsed breakdown in dense gases under different conditions is at early stages of development. The peculiarities of pulsed gas breakdown and basic scaling laws are discussed in [2]. The appearance of runaway electrons during pulsed breakdown is rather beneficial for the production of uniform plasma at high pressures.

We have previously developed an analytic theory of pulsed breakdown at low *E/N*, under conditions where no runaway electrons appear in the gap [3]. It was possible to better understand the physics of the breakdown process using the assumption of a sharp front separating a cathode sheath and a plasma region. It was shown that during the breakdown the sheath expands a distance comparable to the gap length. In the presence of electron emission from the cathode, the sheath collapses at a later stage of the breakdown development. With no electron emission, the sheath remains thick.

Comparison with numerical experiments has shown that it is extremely difficult to reproduce the analytical results under conditions with no electron emission at the cathode. The computational fluid models of different levels of sophistication show the sheath collapse due to numerical diffusion.

The goals of this paper are to obtain analytical solutions of a simplified kinetic equation for runaway electrons in high electric fields and to confirm the main conclusions of the pulsed breakdown theory [3] at the kinetic level, for higher electric fields.

2. Kinetic Equation for Runaway Electrons

Electron collision cross-sections with neutral atoms fall with energy at energies exceeding 100 eV. That is why strong electric fields can continuously accelerate electrons to high energies leading to *electron runaway*. In spatially non-uniform fields, the runaway electrons can produce an intense non-local ionization and luminosity in the areas with no electric field. Understanding of these phenomena is far from complete because simultaneous account of electron scattering and deceleration in collisions is a very challenging task. Additional difficulties appear due to lack of reliably cross-section data for the description of electrons with energies 10-100 eV, especially on angular dependences of inelastic cross-sections. Simple approximations of weak EDF anisotropy and diffusion, both in space and energy, are violated in high electric fields and/or for the energetic electrons [4].

State-of-the-art models for simulations of runaway electrons in weakly ionised plasmas can be found in [5,6]. A simplified kinetic equation for fast electrons was proposed in [7] in the form:

$$\frac{\partial F}{\partial t} + \mathrm{v}\frac{\partial F}{\partial x} + \frac{\partial}{\partial v}\left[aF\right] = S \quad . \qquad (1)$$

Here $a = \left(eE - NL(w)\right)/m$ is the net electron acceleration due to energy gain in the electric field and continuous energy loss in collisions with atoms. This equation neglects electron scattering (see Figure 1) and describes the energy loss as dynamic friction with an energy-loss function $L(w)$, where w is the electron kinetic energy.

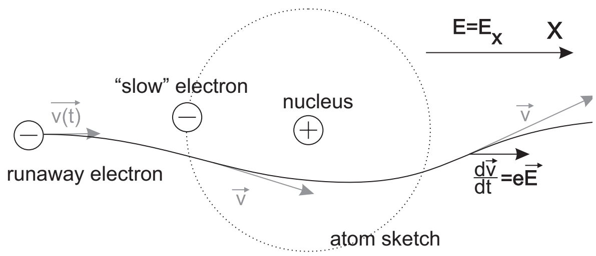

Fig. 1 Electron scattering is neglected thanks to the continued net acceleration.

This function has a smooth maximum at energies $w \sim 10^2$ eV and corresponds to the Bethe-Bloch law at higher electron energies [7]. The source of newly born electrons is approximated in the form:

$$S = c\delta(v)\int_0^\infty v' F(t,x,v')\,dv' \quad , \qquad (2)$$

where $c = NL/\varepsilon_0$. ε_0 is the energy loss per creation of ion-electron pair, and N is the neutral particle density. It is assumed that new electrons are born with low energy and described by the $\delta(v)$ - Dirac delta function.

For the analysis of gas breakdown, we assume the initial distribution function in the form:

$$F(t=0) = F_0(x,v) = n_0\delta(v)\eta(x) \quad , \qquad (3)$$

where n_0 is the initial electron density, η is the step function. In this paper, we assume that no electrons are emitted at the cathode (located at $x \le 0$).

2.1. The EDF formation during breakdown

The general solution of (1, 2) can be obtained by the method of characteristics. The characteristics are defined as a solution of the system:

$$\frac{dx}{dt} = v, \qquad x(t=0) = x_0, \qquad (4)$$

$$\frac{dv}{dt} = a(t,x,v), \quad v(t=0) = v_0. \qquad (5)$$

Along the characteristics the EDF obeys the equation:

$$\frac{dF}{dt} = S(t,x_0,v_0) \qquad . \qquad (6)$$

The general solution of (2, 4-6) can be written in the form:

$$F(t,x,v) = F_0(x_0,v_0) + \frac{c}{a(\bar{t},x_0,v_0)}\int_0^\infty v' F dv' \ , (7)$$

where $\bar{t}$ is defined as a solution of $v(\bar{t},x_0,v_0) = 0$. We assume that the electric field increases instantaneously to a high value corresponding to $a > 0$.

2.2. The case of constant electric field

During the first breakdown stage, the density of electrons and ions is low and the electric field is unperturbed by the space charge. In this case, the characteristics have the simplest form

$$v = at + v_0, \quad x = at^2/2 + v_0 t + x_0 \ . \qquad (8)$$

We can distinguish two types of characteristics. The first type corresponds to initial electrons starting with zero velocity at $t = 0$ (see Figure 2). For them, $v_0 = 0$. Thus, all primary electrons are located at $x > at^2/2$.

Secondary electrons can appear at any point in the ($x \ge 0, t \ge 0$) spatial domain. A typical characteristic of secondary electrons shown in Figure 3 represents the electrons born at zero velocity at a time t_{se} in a point x_{se}. However, the probability of secondary electron generation decreases sharply at $x < at^2/2$ (see Eq. (2)).

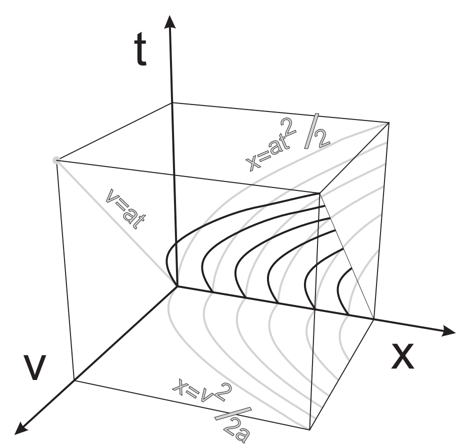

Fig. 2 Characteristics of primary electrons

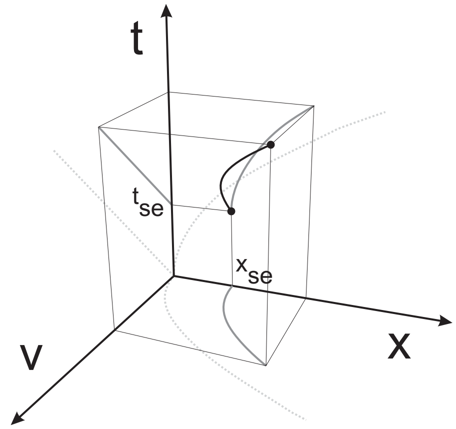

Fig. 3 A typical characteristic of secondary electrons

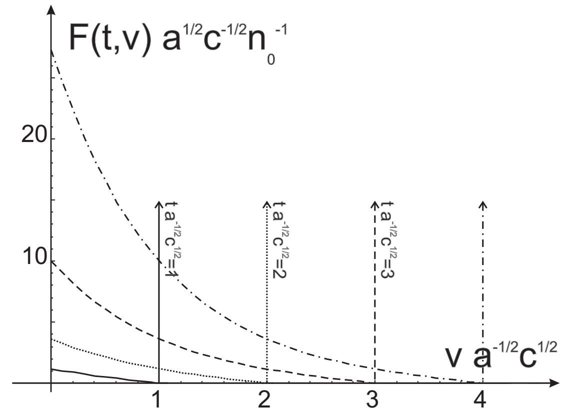

Fig. 4 Electron distribution function at different times

For constant fields, the integral equation (7) has the form:

$$F(t,x,v)=n_0\delta\left(v-at\right)\eta\left(x-at^2/2\right)+ \\ +\frac{c}{a}\left\langle v\right\rangle\Big|_{\substack{x=x-v^2/2a\\ t=t-v/a}} \quad , (9)$$

where the first term corresponds to the initial electrons, and the second term corresponds to the secondary electrons. The symbol $\left\langle v\right\rangle=\int_0^\infty v'F\left(t,x,v'\right)dv'$ denotes the mean electron velocity (flux density), which is a function of x and t. In the integral equation (9) the mean velocity is taken at a previous time $t-v/a$ in a shifted point $x-v^2/2a$.

At $x>at^2/2$, the distribution does not depend on x, and its dependence on t and v is via the combination $\xi=t-v/a$:

$$F(\xi)=\frac{n_0}{a}\delta\left(\xi\right)+\frac{c}{a}\left\langle v\right\rangle\Big|_{\xi} \quad . \qquad (10)$$

This equation corresponds to the integral equation:

$$F(\xi)=\frac{n_0}{a}\delta\left(\xi\right)+ac\int_0^{\xi}\left(\xi-\zeta\right)F(\zeta)d\zeta \quad ,(11)$$

whose solution (at $\xi\geq 0$) is

$$F(\xi)=\frac{n_0}{a}\delta\left(\xi\right)+n_0\sqrt{\frac{c}{a}}sh\left[\sqrt{ac}\xi\right] \quad . \qquad (12)$$

Figure 4 shows the velocity distribution function of electrons at different times. It is seen that the main part of the distribution contains electrons with velocities $0<v<\sqrt{a/c}$. At $t\gg\left(ac\right)^{-1/2}$ the number of electrons with $v>\sqrt{a/c}$ is exponentially small.

2.3. Dynamics of the cathode sheath

It was shown in [3] that in the absence of electron emission from the cathode, the cathode sheath expands to a maximum length and the field in plasma decays with time. When electrons are emitted from the cathode, the sheath collapses towards the cathode after an initial expansion.

According to the fluid model [3], a sharp boundary $X\left(t\right)$ separates the region with no electrons and the region where the electron density is constant. In the kinetic theory, the characteristic $x=at^2/2$ plays the role of the sharp boundary of the fluid model. Contrary to the fluid model, there are electrons at $0<x<at^2/2$, but the number of these electrons decreases rapidly towards the cathode.

The kinetic theory confirms the absence of the sheath collapse if no electrons are emitted from the cathode. Indeed, according to Eq. (9), the EDF at time t in point x and velocity v is proportional to the mean electron velocity in the point $x-v^2/2a$ at the time $t-v/a$. It is seen at Figure 5a that for any moment $t>0$ at $x>0$ there are always low velocities v such that $x-v^2/2a>0$. If $\left\langle v\right\rangle\neq 0$ in this point at the time $t-v/a$, then $F\left(t,x,v\right)\neq 0$. Similar arguments can be proposed for the EDF at a time $t-v/a$ in the point $x-v^2/2a>0$. By continuing this process, we will arrive at a time $t=0$ and $x>0$, where $\left\langle v\right\rangle\neq 0$ according to the initial condition. So, $F\left(x>0\right)\neq 0$ at any time $t>0$.

It is also seen in Fig 5b that at any time $t > 0$ for $x < 0$, there is no v such that $x - v^2 / 2a > 0$. By continuing to $t = 0$, we conclude that the area $x < 0$ remains electron-free.

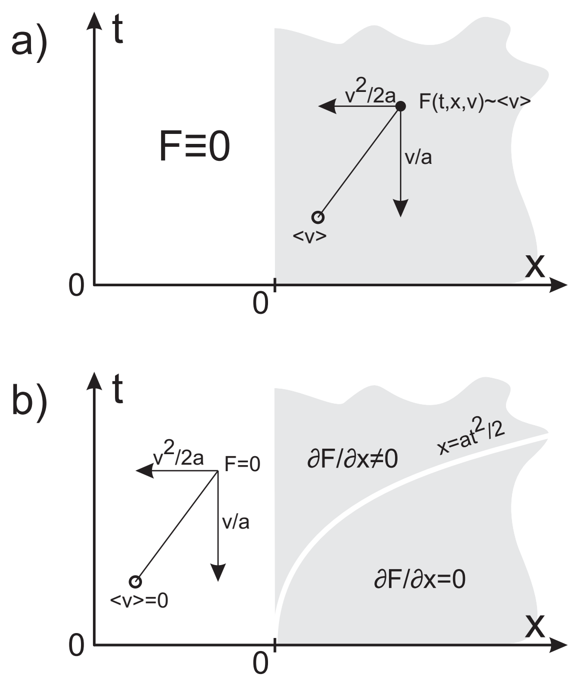

Fig. 5 Electron distribution function in the (*t,x*) plane. a) the domain $x < 0$ remains electron-free while $F(x > 0) \neq 0$ at any time. b) the domain $x > 0$ is subdivided by the boundary characteristics.

It remains to be seen that this conclusion is valid also in the general case $a(x,t) > 0$ when the electric field changes in time and space due to field perturbation by the space charge.

3. Conclusion

A simple kinetic equation was suggested for the analysis of runaway electrons during pulsed gas breakdown in high electric fields. Characteristic scales (in energy and space) are obtained from a formal solution of this kinetic equation. An analytical solution for the distribution function was obtained in the plasma region where the EDF does not depend on spatial coordinate and depends on energy and time in the form $\xi = t - v / a$. Kinetic analysis confirms the absence of the cathode directed ionisation wave in the absence of electron emission from the cathode.

4. Acknowledgements

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