IMPROVING ALGEBRA I STAR TEST SCORES THROUGH MULTIPLICATION TABLES PRACTICE AND RATIONAL NUMBER PRACTICE

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ABSTRACT

IMPROVING ALGEBRA I STAR TEST SCORES THROUGH MULTIPLICATION TABLES PRACTICE AND RATIONAL NUMBER PRACTICE

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Only 28% of California students score proficient or higher on their Algebra I STAR test (CDE, 2009). The standards-based movement and the emphasis on enabling all students to score proficient or advanced on the California STAR test have created a high-stakes environment for teachers. Many students enter eighth grade not knowing their multiplication facts and fractions. Statistics also show that many eighth-grade students in California score poorly on the Algebra I STAR test. Though there has been minimal research on the connection between these two phenomena, a strong correlation between not knowing "math basics" and scoring poorly on the Algebra I STAR test would seem plausible. Accordingly, the central research question for this study was: What is the impact of practicing multiplication facts and fractions problems twice a week on a selected group of eighth-grade students' Algebra I STAR test scores?

The author (an eighth-grade math teacher) and another teacher divided the participating students into an experimental group and a control group. The study began on Monday, November 15, 2010, and ended on Friday, February 18, 2011. The control group simply took a pretest at the beginning of the study and then a posttest 11 academic

weeks (weeks that each school was in session) later. The experimental group took the same pretest and posttest, but also practiced multiplication facts and fraction problems during the intervening 11-week period. The experimental group took timed multiplication fact and fraction problem quizzes twice per week, each of them lasting 2.5 minutes. These quizzes counted toward the students' overall math grade as well.

There are many ways to improve students' Algebra I STAR test scores. Practicing multiplication facts and fractions is an already-known, specific strategy to help students better understand math basics. Researchers agree that practicing multiplication facts and fractions is good math pedagogy, but it has not been isolated as a particular practice to help students score higher on problems similar to those on the eighth-grade Algebra I STAR test.

Analysis of the results suggests that practicing multiplication facts and fractions problems twice per week helps students score better on problems similar to those on the eighth-grade Algebra I STAR test, although longer-term effects have yet to be studied. I do not know if this practice is effective as a year-round remediation activity.

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CHAPTER ONE

INTRODUCTION

In 2009, only 28% of California students scored at the proficient level or higher on their Algebra I STAR (State Testing and Reporting) test (California Department of Education [CDE], 2009). Students take Algebra I as early as eighth-grade or as late as twelfth-grade in their academic careers. Many Algebra I teachers find that students are not prepared to take this course, and many observers believe that California students are behind their peers in most other states in mathematical competence.

Passing Algebra I is a graduation requirement at any high school in California. The Algebra I curriculum has 25 multi-layered standards with many concepts that require foundational-level mathematics skills (National Mathematics Advisory Panel [NMAP], 2010). As a teacher of eighth-grade Algebra I, I have found that many students enter eighth grade not knowing their multiplication facts and fraction skills. Both of these skills are necessary for success in Algebra I.

While taking the California Standardized Testing and Reporting (STAR) test is not a requirement, not all students may opt out of taking the test without repercussions. The Federal No Child Left Behind (NCLB) Law only allows 5% of students to opt out of the STAR test with written parent permission, and regulations also allow schools to use alternate assessments to declare up to 1% of all students proficient (Virginia Department of Education [VDOE], 2008). If a school has more than the allotted amount of students opt out of taking the test, the entire school's Academic Performance Index (API) score is reduced. California schools are punished for not making Adequate Yearly Progress (AYP) on their STAR test scores as well. AYP measures if the school's STAR test scores have improved from the previous year (CDE, 2009).

The consequences for schools not meeting AYP on their STAR test scores increases in severity. Over time, schools that miss AYP for a second consecutive year must create a school-wide improvement plan to show how they will improve their test scores (CDE, 2009). Schools that miss AYP for a third consecutive year must then offer free tutoring and other supplemental educational services to struggling students (CDE, 2009). Missing AYP for the fourth consecutive year labels the school as "corrective action," where the staff can be replaced, the school day may be extended, or new curriculums may be adopted (CDE, 2009). After the sixth year of missed AYP the school either must restructure or close (CDE, 2009).

Schools are under much pressure to have their students take the STAR test and score proficient or higher in all categories and meet their annual AYP. Therefore, the purpose of this quantitative research study was to determine the impact of practicing multiplication facts and fraction skills two times per week on Algebra I STAR test scores. The sample consisted of eighth-grade Algebra I students in two schools, Fern Canyon Elementary School and Fort Apache Middle School, in Northern California. Chapter Two of this study discusses issues related to the teaching and learning of Algebra I in eighth grade. This chapter provides information on prior math knowledge, rational number knowledge, rationales for teaching Algebra I, politics of teaching Algebra I, students' cognitive development in relation to learning Algebra I, teaching methods for Algebra I, and STAR testing with Algebra I.

Chapter Three describes the quantitative research methods. This study used 74 participants at two different schools in Northern California. The sample was composed of students in my own eighth-grade Algebra I class and the classes instructed by another local teacher who was interested in my research. Researcher 1 and Researcher 2 could easily stay in contact throughout the study as we lived just 10 miles apart.

The research participants were split into two groups: an experimental group and a control group. The control group took a pretest at the beginning of the study and a posttest 10 weeks later. The experimental group took the same pretest and posttest; however, during the intervening 10 weeks the experimental group took timed, 10-question quizzes on multiplication facts and fractions twice a week. After administration of the posttest, data on test results were collected and analyzed.

Chapter Four addresses the study's research question: What is the impact on Algebra I STAR test scores of practicing multiplication facts and fraction skills twice per week in an eighth-grade Algebra I classroom in northern California? Research and experience suggest that lacking simple multiplication facts and fraction skills prevents students from solving multistep mathematics problems. Findings from the study suggest a positive correlation between practicing multiplication facts and fraction skills weekly (in the form of quizzes) and scoring higher on problems similar to those on the eighthgrade Algebra I STAR test.

Chapter Five provides discussion and implications of the research study. The data were organized into categories based on the California Algebra I content standards to see in what specific areas of Algebra I the remediation was helpful.

Chapter Six provides conclusions based on results and analysis of the study. The conclusions are based on deductive reasoning and statistical analysis of the test scores from the research participants

CHAPTER TWO

LITERATURE REVIEW

Algebra is more important now than ever for California students, because passing algebra is a requirement for students to graduate from high school and to be accepted into a four-year college (CDE, 2000). Taking Algebra I in eighth grade is a requirement in some states; in California it is not yet a requirement, but it is preferred that students take Algebra I in eighth grade rather than Pre-Algebra (Flores & Roberts, 2008). Mandating that all eighth-grade students take algebra is a difficult task for many practical reasons.

"It is well-known that California students lag behind students in other states and nations in their mastery of mathematics" (Reese, 1997, p. 4). Between 1970 and 1990, the number of students earning bachelor's and master's degrees in mathematics in California has decreased (National Center for Education Statistics [NCES], 1997). Of all areas tested on California's Standardized Testing and Reporting (STAR), Algebra I consistently has one of the lowest passing rates (CDE, 2009). Algebra requires students to understand basic math fundamentals, so math teachers must equip students with basic math skills in order to ensure their success in algebra (National Council of Teachers of Mathematics [NCTM], 1988). More particularly, students need a solid foundational knowledge of rational numbers and multiplication facts. These critical foundations of Algebra I deserve ample time in any mathematics curriculum (NMAP, 2010). California's math STAR test scores progressively drop in pass rate in the later grades.

Prior Knowledge

Prior knowledge (sometimes referred to as prior learning, previous knowledge, background knowledge, or preexisting knowledge) of basic essentials across curricular areas is directly related to higher student achievement (Flores & Roberts, 2008; Rittle-Johnson & Alibabi, 1999). Students with greater conceptual knowledge tend to have better procedural skills (Rittle-Johnson & Alibabi, 1999). Knowing basic skills helps students comprehend more complex information in later years. Basic and fundamental skills should be emphasized before complex ideas are introduced. (Marquis, 1989). "A large body of information shows that learning proceeds primarily from prior knowledge" (Roschelle, 1995, p. 1). Too many students are pushed beyond their mental capabilities in class, and many students end up failing classes or passing with low grades (Marquis, 1989). With inadequate prior knowledge, they are trying to do complex tasks that require basic skills they do not have.

Prior knowledge influences learning, and learners construct concepts from prior knowledge (Glaserfeld, 1984; Resnick, 1983). If a student does not have a strong base, it is very difficult, if not impossible, to understand the complexities of upper-level mathematics (Sheets & Wallace, 2007).

Prior Knowledge in Learning Mathematics

Advancing to a higher level of mathematics requires a good understanding of the previous math level's material (CDE, 2000, 2009). The State of California has content standards for kindergarten through grade 12 (CDE, 2000, 2009). Standards for kindergarten through seventh grade are presented in five strands: Number Sense; Algebra and Functions; Measurement and Geometry; Statistics, Data Analysis and Probability; and Mathematical Reasoning (CDE, 2000). Standards for grades 8 to 12 are not organized by such strands, but rather fall naturally under the discipline headings of algebra, geometry, and so forth (CDE, 2000). Mathematics rests on the use of four basic functions: addition, subtraction, multiplication, and division (Huber & Hutchings, 2005). As students progress, mathematics content becomes more complex, integrating more difficult operations (CDE, 2000; Huber & Hutchings, 2005).

Math knowledge that constitutes the critical foundations of Algebra I can be broken into three separate categories: (1) fluency with whole numbers, (2) fluency with fractions, and (3) particular aspects of geometry and measurement (NMAP, 2010). In the first category, children should have a robust sense of numbers by the end of fifth or sixth grade (NMAP, 2010). In the second category, students should gain a thorough understanding of positive as well as negative fractions before they begin Algebra I coursework (NMAP, 2010). In the third category, middle-grade experience with similar triangles is most directly relevant for the study of Algebra I (NMAP, 2010). The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions); the teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in Algebra I can be expected (NMAP, 2010). Basic facts and basic procedures, such as multiplication facts and inverse operations, make up complex problems and procedures, such as word problems and factoring (Reys, Reys, Lapan, Holliday, & Wasman, 2003). A large math problem is in effect a collection of small math problems.

Important skills involved in the translation of a mathematical problem into a solvable equation are problem integration and representation. Integration involves putting together different pieces of information that are presented in complex problems, such as multistep problems. However such problems are represented, a wide variety of basic and technical skills is needed to solve problems. Given this need, a mathematics program should include a substantial number of ready-to-solve exercises designed specifically to develop and reinforce such skills (CDE, 2000, p. 11).

Mathematics proficiency is not an innate characteristic; it is achieved through persistence, effort, and practice on the part of students and rigorous and effective instruction on the part of teachers (CDE, 2000). Each year students strive to achieve grade-level benchmarks (U.S. Department of Education, 2001). A benchmark is a clear, specific description of knowledge or skill that students should acquire by a certain time in their schooling (U.S. Department of Education, 2001). A benchmark can also be referred to as an indicator or learning expectation (U.S. Department of Education, 2001). The California mathematics framework is predicated on the belief that proficiency in mathematics is a consequence of student effort and teacher instruction; standards are meant to scaffold one another so that the transition from grade level to grade level is smooth (CDE, 2000).

California mathematics standards are designed to prepare all children to study algebra by the eighth-grade (CDE, 2010). However, more children enter algebra classes unprepared than enter most other classes (Marquis, 1989). An increasing number of American students (not just California students) are failing or barely passing their first year of algebra (Miranda, 2008). Passing algebra requires a combination of procedural skills, conceptual understanding, and problem solving (NCTM, 1988). Students who do not possess all of these skills are most likely to fail an algebra class (NCTM, 1988; CDE, 2009). Students who do not understand basic procedural skills such as multiplication facts and rational numbers are not able to perform basic algebraic tasks (NMAP, 2010).

Multiplication Facts

Multiplication facts are often memorized in the early grades and forgotten at later grade levels (Baroody, 1984). According to current theories, efficient production of number combinations (basic addition, subtraction, division, and multiplication facts) is exclusively a reproductive process (Baroody, 1984). Multiplication facts are taught early in the California mathematics curriculum. A second-grade math standard (number sense 2.2) states that students should memorize to automaticity the multiplication table for numbers between one and 10 (CDE, 2000). Specific standards regarding multiplication occur 28 times in the California state mathematics standards before Algebra I, yet many students who enter Algebra I in eighth grade still do not recall their multiplication facts (Kent Willis, personal communication, November 11, 2009). Eighteen of 25 Algebra I standards specifically require memorization of multiplication facts (CDE, 2000).

There are various theories as to why students do not understand their basic multiplication facts. One theory is students have become so dependent on calculators and electronic devices to give them answers that they do not have to use basic facts anymore and thus forget them (Sheets & Wallace, 2007). Another theory notes that students are explicitly taught basic facts in grades 3 and 4 but not after that, implicitly deemphasizing their importance (Reese et al., 1996).

Rational Number Knowledge

Many students do not have a good understanding of rational numbers (Vamvakoussi & Vosniadou, 2004). A rational number is a number that can be expressed as the quotient (answer to a division problem) of two integers (positive and negative whole numbers), of which the denominator cannot be zero (CDE, 2000). A rational number can be written as a fraction (a÷b) (Reese, 1997).

Three important representations of rational numbers are decimals, fractions, and percents (CDE, 2000). Students begin to learn about fractions, decimals, and percents in second grade. A second-grade mathematics standard (number sense 3.0) states that students should understand the relationship among whole numbers, simple fractions, and decimals (CDE, 2000). Beyond this, multiplication facts and rational numbers are

repeated in the California state standards for mathematics 31 times prior to the Algebra I standards (CDE, 2000). Understanding fractions and rational numbers is heavily emphasized in these standards so that students will have ample skills to succeed in Algebra I (Flores & Roberts, 2008).

Rationale for Algebra I in Public Education

Passing algebra I is currently a requirement for obtaining a California high-school diploma (Flores & Roberts, 2008). Algebra I has traditionally been a gatekeeper course leading to further study of mathematics in secondary schools, to college entrance, and to jobs in technical fields (Wright, 2001). Nationwide, mathematics standards have changed in grades 2 to 7 to increase student success in algebra, with greater emphasis on such areas as rational numbers, ratios, and proportions (Wright, 2001).

Students' performance in Algebra I is often the primary criterion used by parents, teachers, and counselors to determine the readiness of eighth- and ninth-grade students for a sequence of college preparatory mathematics courses (Christmas & Fey, 1990). Beginning in eighth grade and continuing through high school, students must be expected to complete challenging academic coursework that includes the standards for Algebra I (NMAP, 2010).

The shift from an industrialized society to the information age has changed the mathematics that individuals need to learn (Sheets & Wallace, 2007). Over 75 percent of jobs require proficiency in fundamental algebraic concepts, either as a prerequisite for advanced training or as part of a licensure program (National Research Council, 1989).

Entry into many professional fields today requires knowledge of algebra, because employees must be able to use algebraic tools to translate problem situations involved in a given field to mathematical models that can be solved (Herscovics, 1989). Algebra is used in nearly every scientific discipline, and many science courses at the high-school level require successful completion of Algebra I as a prerequisite for enrollment (CDE, 2010).

The most common and familiar uses of algebra are the many formulas that relate to business, industry, science, technology, and daily life (Christmas & Fey, 1990). Examples of these include formulas for distance, rate, and time; perimeter, area, and volume; bank interest and installment loans; and servicing and pricing options for management information systems (CDE, 2009). Variables, functions, and relations are useful in analyzing situations involving costs, prices, rentals, and profits, both for the business manager and the intelligent consumer (Christmas & Fey, 1990). Algebraic expressions and equations serve as models for interpreting and making inferences about data (CDE, 2000). Algebraic reasoning and symbolic notations also serve as the basis for the design and use of computer-spreadsheet models (Christmas & Fey, 1989). As algebra becomes increasingly more important for employment, continued education, and daily living, all students must become capable in algebra, not just those who are the highest performers in mathematics (Christmas & Fey, 1990).

Politics of Algebra I in California

In 1992, the state of California started the debate on mandating passing Algebra I as a high school requirement for graduation. Now, the state is grappling with the notion of requiring it as an eighth grade requirement (EdSource, 2009). Nearly 45,000 more students have scored proficient and advanced on the Algebra I CST in 2008 than in 2003, however, too many students still struggle to get through the Algebra I gateway leading to more rigorous math and science courses in high school (EdSource, 2009).

Algebra I has been a hot political topic in California. On July 9, 2008, the California State Board of Education voted to implement Governor Schwarzenegger's proposal to require all eighth-grade students to be assessed in Algebra I within three years - that means that by the year 2011 all eighth-grade students will be required to take Algebra I for mathematics (CDE, 2009). Previously, eighth-grade students who did not receive Algebra I instruction took the General Mathematics STAR test instead of the Algebra I STAR test (CDE, 2009). The General Mathematics STAR test consists only of sixth- and seventh-grade mathematics standards, which the federal No Child Left Behind (NCLB) legislation strongly discourages (CDE, 2009). Students receive a deduction in their overall STAR test scores if they take the General Mathematics test instead of the Algebra I test (CDE, 2009). On the same day when the California State Board of Education released this mandate, California State Superintendent of Public Education Jack O'Connell said, "I have serious concerns with this proposal on its merits. I strongly disagree with the Governor's proposal to require all eighth-graders to take Algebra I within three years without also offering any of the support for our school districts and

schools to successfully make this major change" (Miranda, 2008). Superintendent O'Connell also told the governor that he would need an extra \$3.1 billion to add mandatory eighth-grade algebra instruction to the curriculum (CDE, 2009). O'Connell was quoted as saying, "It's instruction the governor pushed for, but can the state afford it ... short of that you'd be setting our kids up for failure" (CDE, 2009).

The CDE recommended that the California State Board of Education (CSBE) approve a revised mathematics blueprint forcing all eighth-grade students to take Algebra I in March, 2008 (CSBE, 2008). However, after tabling the issue to a later date to be voted on, the SBE concluded after much input that California could still meet the requirements of the Elementary and Secondary Education Act (ESEA) by creating a General Mathematics blueprint to be revised based on the standards for Algebra I , creating a curriculum called Algebra-Readiness (CSBE, 2008). Students who take a year-long Algebra Readiness course could take the General Mathematics CST which would primarily focus on sixth and seventh grade math content standards instead of the Algebra I CST which strictly contains Algebra I standards (CSBE, 2008).

Among the apparent obstacles to meeting this obligation of requiring all eighthgrade students to take Algebra I, California would need to recruit 3,300 new full-time math teachers, reduce class sizes, and create extra programs to help struggling students (Miranda, 2008). The California Mathematics Council stated that three years would be too short a time to get everyone up to speed without extra funding (Miranda, 2008). The pressure to pass Algebra I in high school was already a leading reason why students were dropping out, and the governor's proposal might thus further increase the dropout rate (Miranda, 2008).

In December 2008, Governor Schwarzenegger's Algebra I mandate for all eighthgrade students was overturned, after much deliberation and lawsuit threats by several education advocacy groups (Miranda, 2008). In a compromise between the governor and the California State Board of Education, Algebra Readiness curricula were adopted in place of the more traditional pre-algebra curricula of sixth and seventh grades (CDE, 2009; Miranda, 2008). Algebra Readiness is a hybrid of one-third algebra standards and two-thirds pre-algebra standards (Miranda, 2008). The creation of this curriculum has created a hope that more students will be ready for algebra after completing the Algebra Readiness course (Miranda, 2008). (See Table 2.1 for California's Algebra I content standards.)

Algebra and Children's Cognitive Development

Many educational experts and psychologists agree that not all eighth-grade students are ready to take algebra I (Lambert, 2002). Not all students are cognitively ready to digest the information they receive at school (Piaget, 1970). Piaget (1970) theorized four stages of cognitive development at which children's brains become ready to process certain kinds of information. During Piaget's sensorimotor stage (birth to age two), children learn about themselves and their environment through motor and reflex actions. During the preoperational stage (from when children start to talk to about age seven), they begin to personify objects and think about things that are not immediately present; their thinking is often influenced by fantasy. During the concrete operational stage (about first grade to early adolescence), children develop an ability to think abstractly and make rational judgments about concrete or observable phenomena. Finally, in the formal operations stage (adolescence), they no longer require concrete objects to make rational judgments, and they are capable of abstract reasoning, including such tasks as hypothetical and deductive reasoning (Piaget, 1970).

Algebra I requires that students be in stage four, or the formal operations stage (Lambert, 2002). This stage can begin in adolescence, but not all eighth-graders have reached that stage in development yet (Piaget, 1970). The age ranges for each of Piaget's stages vary by cognitive ability and not by precise age, because children develop mentally at different rates (Lambert, 2002). A major argument for not forcing all

students to take Algebra I in eighth grade is that many students simply are not cognitively ready for it (Miranda, 2008).

Some experts find evidence that California schools are meeting high expectations in mathematics, while others point to results that suggest growing numbers of students are being placed in algebra courses for which they are not prepared (EdSource, 2009). While the number of students taking Algebra I in eighth-grade is increasing, the number of students who are failing it is also increasing (EdSource, 2009).

Table 2.1 below offers an overview of the California Algebra I Content Standards.

Methods of Teaching Algebra

There are many different approaches to teaching algebra. Often math teachers are classified as either constructivist or behaviorist (Lambert, 2002). Constructivist and behaviorist models of teaching differ drastically from each other; however, a hybrid of the two models exists (Lambert, 2002). The next section offers an overview of both models as well as the hybrid that combines the two.

According to constructivism, knowledge and beliefs are formed within the learner (Lambert, 2002). Rather than considering learners as empty vessels, constructivist learning theory assumes that learners bring experience and understanding to the classroom (Lambert, 2002). Learners personally enjoy experiences with meaning; students should be allowed to suggest or interpret their

Table 2.1

California Algebra I Content Standards

5	
4	Students simply expressions and equations with one variable
5	Students can solve and justify multi-step problems and equations
6	Students can graph linear equations and inequalities and know x and y-intercepts
7	Students can derive linear equations and find points on a line
8	Students understand slope, parallel lines, and perpendicular lines
9	Students are able to solve and graph systems of equations
10	Students can add, subtract, multiply, and divide monomials and polynomials
11	Students can use and apply a variety of factoring techniques
12	Students can simplify fractions with polynomials
13	Students can add, subtract, multiply, and divide rational expressions
14	Students can solve quadratics using complete the square
15	Students can solve work, rate, and mixture problems
16	Students understand relations and functions
17	Students can determine the domain and range of functions and graphs
18	Students can determine if something is a function
19	Students know the quadratic formula and its derivation
20	Students know how to use the quadratic formula
21	Students graph quadratic functions and know the roots are x-intercepts
22	Students use multiple measures to determine a quadratics' intercepts
23	Students quadratic functions to solve physical problems with gravity
24	Students know the aspects of logical arguments
24.1	Students know inductive and deductive reasoning
24.2	Students know the hypothesis and conclusion in a logical deduction
24.3	Students can use couterexamples to prove a statement false
25	Students use number properties to judge the validity of results
25.1	Students know number properties to prove or to make a counterexample
25.2	Students judge validity of an argument using number properties
25.3	Students can determine if an equation or expression is true, sometimes, always, or never

own meanings rather than have a teacher telling them exactly how or what to think (Glaserfield, 1984; Lambert, 2002). Learning activities should cause students to gain access to their experiences, knowledge, and beliefs. The constructivist approach allows learners to use what they know to interpret new information and construct new knowledge (Lambert, 2002).

Constructivist theory also emphasizes that culture, race, and economic status affect student learning individually and collectively (Glaserfield, 1984). Who students are and where they come from affect students' experiences both in and out of school (Glaserfield, 1984). Learning is a social activity that is enhanced by shared inquiry, and group activity is preferred over individual activity because students can learn more when they collaborate with each other (Glaserfield, 1984; Lambert, 2002). The constructivist model also says that students should be able to assess their own learning, and that the outcomes of the learning process are varied and often unpredictable (Lambert, 2002). Constructivist theory says that students should direct their own learning because they are able to generate more understanding and meaning (Glaserfield, 1984).

The behaviorist approach to teaching is drastically different from the constructivist model (Lambert, 2002). The behaviorist teaching model is often referred to as direct instruction (Lambert, 2002). Direct instruction emerged as a style of teaching especially appropriate to basic skill acquisition (Roschelle, 1995). According to this approach, learning is broken down into small pieces, expectations are made very clear,

and approximations of desired behavior are rewarded (Lambert, 2002). The mission of school is generally focused on basic skill acquisition as measured by achievement tests (Lambert, 2002). When basic skills are mastered, the student then has the ability to do multi-step problems that involve complex and critical thinking (Lambert, 2002) The behaviorist model of teaching suggests that all children are capable of learning and that, when teachers hold high expectations for student achievement and press for academic performance, students tend to meet those expectations (Lambert, 2002).

The constructivist and behaviorist approaches to teaching are, in many ways, different from one another. However, many teachers find common ground between the two theories (Glaserfield, 1984), combining direct instruction with constructivist group discovery activities (Lambert, 2002).

The behaviorist approach to algebra is commonly referred to as traditional algebra teaching or direct instruction (Leinenbach & Raymond, 1996). The constructivist approach to teaching algebra is commonly referred to as hands-on algebra or manipulative-based algebra (Leinenbach & Raymond, 1996; Roschelle, 1995).

The behaviorist approach to learning Algebra I would have students use a traditional textbook as their guide to instruction; students would be taught through direct instruction, with a teacher pacing each of the lessons as the book would pace them (Leinenbach & Raymond, 1996). A traditional approach to algebra paces itself through particular units as a marking of time, because it is expected that a class will get through all 25 algebra I standards before the end of the school year (Leinenbach & Raymond,

1996; NCTM, 1989). The behaviorist approach to teaching emphasizes mathematics basics such as fractions, decimals, percents, and multiplication facts through drill-based activities before more complex ideas are introduced (Lambert, 2002).

The behaviorist model for teaching Algebra I emphasizes frequent assessment through quizzes and tests, much as California students are assessed by the STAR tests (Lambert, 2002). In essence, the results from these assessments tell whether or not the students are learning the material (Flores & Roberts, 2008).

The constructivist model to teaching Algebra I uses a more hands-on approach (Glaserfield, 1984; Lambert, 2002), involving, for example, the use of manipulatives and algebra tiles for student comprehension of algebraic concepts (Leinenbach & Raymond, 1996). Algebra tiles are small plastic (or paper) manipulatives that students use to help them learn (Sharp, 1995).

The constructivist argument for teaching hands-on algebra with manipulatives is that students are not fully understanding the algebraic concepts when they are taught in the behaviorist way (Leinenbach & Raymond, 1996). Students in traditional algebra classrooms do not know why they are learning algebra because they are not shown any real-world applications (Sharp, 1995). Students are also just learning the motions of algebra, seeing how to solve a problem but not really understanding what they did to solve it; they have just memorized the steps taken to do so (Sharp, 1995).

Another major argument for teaching algebra with a constructivist model is that mathematical meanings can be developed when individuals construct translations between algebra symbol systems (algebra tiles) and physical systems that represent one another (Sharp, 1995). According to Sharp (1995), few students connect whole number manipulations to algebraic manipulations. Students who encounter algebraic ideas through manipulating physical models gain a better conceptual knowledge of algebra (Sharp, 1995).

The behaviorist model to teaching algebra is characterized by strict linear teaching and assessment through a textbook (Leinenbach & Raymond, 1996; Sharp, 1995). The constructivist model to teaching algebra is characterized by a more studentpaced, hands-on, less textbook-oriented model (Sharp, 1995).

However, many teachers use a hybrid approach with both behaviorist and constructivist teaching (Leinenbach & Raymond, 1996). Some teachers do a combination of both group work and individual work I their classrooms (Sharp, 1995). Combining multiple methods of teaching styles opens up to students' multiple learning styles because not all children learn mathematics the same way, and it would benefit a greater number of students to combine group work with individual work as seen fit (Sharp, 1995).

STAR Test Scores and Assessment

Assessment is necessary in order to determine whether schools and students are doing their jobs (Kaput, 2000). The state of California uses the STAR program for this purpose. STAR test results are produced at the end of each school year, several months after public school students have taken their tests (CDE, 2009). Consistent with the federal No Child Left Behind (NCLB) legislation, California STAR tests are a measure of state and federal accountability in the public education system (Flores & Roberts, 2008). STAR test results can also be a measure of how well material is being taught or learned (CDE, 2000).

There are two forms of assessment: formal and informal (Lambert, 2002). Informal assessment is a less prevalent form of giving letter grades in education; one example of informal assessment is a verbal response to written work, in which the teacher looks at students' work line by line to identify their work procedures (Lambert, 2002). Another example of informal assessment is having students raise their hands to identify answers or explain verbally how to solve a problem (K. Willis, personal interview, May 10, 2011).

Formal assessments include written homework, quizzes, and tests; they usually involve written work that can be scored and given a letter grade (Lambert, 2002). Each state in the U.S. has standardized testing to assess student performance at the end of each school year (NCES, 1997). As a result, many teachers in California try to replicate STAR-like assessments to help students prepare for the actual STAR test (CDE, 2000). California's public education system is designed around the standardized testing model (Reys et al., 2003).

On the whole, STAR test scores have improved in most curricular areas over recent years in California (CDE, 2009). In 2009, the percentage of students scoring at the proficient level and above in mathematics showed a one-year increase of approximately 3

percentage points (CDE, 2009). From 2003 to 2009, the overall percentage of students scoring at the proficient level and above increased by 11 points (CDE, 2009). However, this trend has not held true for Algebra I STAR test scores (CDE, 2009). "When many students get to Algebra I, their STAR test scores tend to drop for the remainder of their school careers, and I found this prevalent at my last school in a big city where it was a contributing factor to the already higher dropout rate"(P. Meyers, personal interview, May 3, 2011). Algebra I is the beginning of a more complex type of mathematics that requires a vast array of basic skills and abilities (CDE, 2000). There is a very high Algebra I failure rate, as well as a high student dropout rate among students in grades seven to nine in California (Marquis, 1989).

The proficiency rates for Algebra I STAR test scores are lower than those for any other math test in California, with the exception of the Integrated Mathematics Level I test (CDE, 2009). Integrated Mathematics Level I is required only by certain high schools that force their lower-achieving students who barely pass Algebra I into taking this course for their third year of high-school math credit (Miranda, 2008).

In 2009, 758,139 students took the Algebra I STAR test, making it the most taken STAR test subsection in California (CDE, 2009). The least taken math STAR test was the Integrated Mathematics Level I test with 9,962 students.

There is a downward trend in proficiency rates in math STAR test scores as students get older (CDE, 2009). The STAR proficiency and advanced rates in mathematics for 2009 were as follows: grade 2 - 63%; grade 3 - 64%; grade 4 - 66%;

grade 5 – 57%; grade 6 – 49%; grade 7 – 43%; General Mathematics – 28%; Algebra I – 28%; Geometry – 28%; Algebra II – 28%; Integrated Mathematics I – 11%. The trend drastically jumps downward as soon as students enter Algebra I.

Methods to Improve Test Scores

California's mathematics STAR test scores are in need of improvement (CDE, 2000; CDE, 2009). The Final Report of the National Mathematics Advisory Panel (2008) directly links proficiency in fractions with success in algebra. The most important foundational skill not presently developed appears to be proficiency in fractions (including decimals, percents, and negative fractions) (Vamvakoussi & Vosniadou, 2004). "Because a significant portion of middle school mathematics deals with ideas in this conceptual field, solid understanding of multiplication in elementary school is essential for a student's success in middle school" (Wantanabe, 2003, p. 111).

California has since developed a program called the Algebra I Success Initiative to help students become more successful in Algebra I courses (CDE, 2010). The program calls for an additional \$3.1 billion in funding in the following areas: (1) student support; (2) professional development and instructional materials; and (3) recruitment, retention, and preservice teacher development (CDE, 2010).

Details of student support mainly focus on giving students more time for mathematics in the school day:

If we want to make sure all students are fully prepared for Algebra I in eighth grade, we are going to need to give all teachers at all levels more time to instruct

their students in mathematics. Therefore, in order to honor this commitment without lessening our commitment to English or doing away with other subjects such as science, history, physical education, and art, we must provide the resources to extend instruction in combination with the additional comprehensive support structure recommended in this proposal. ... Results show that our mathematics proficiency rates are highest in kindergarten through grade three where we have class sizes no larger than 20:1. While class size is not the sole factor in the issue, we do believe it contributes to early success. According to data collected by the CDE, the average class size for Algebra I in California is more than 26 students per class. We simply must do better. (CDE, 2010)

On the idea of professional support, California has recognized a lack in mathematical knowledge among its teachers. The Algebra Success Initiative has recognized that students in California will only have the same number of eighth grade students succeed in Algebra I today as they did three years ago, and to ensure students meet state and federal standards, their teachers must get the training necessary to develop the knowledge and skills to effectively teach Algebra I. Unfortunately, teacher professional development programs have repeatedly been cut in recent years (CDE, 2010).

The third piece of the California Algebra I Success Initiative is recruitment, retention, and preservice teacher development. Recruiting, training, and retaining math teachers in California has been one of the biggest issues with math education (CDE, 2010). According to the Center for the Future of Teaching and Learning, teachers are projected to retire by the tens of thousands over the next decade. Nearly one in five teachers in projected to retire within five years, and about 100,000 teachers, or one-third of the workforce, are expected to retire by 2017 (CDE, 2010). Knowing this, California is expected to invest money into college-age students in the recruitment process of turning them into mathematics teachers. Money will be spent to keep math teachers adequately prepared to teach math and to stay in math teaching rather than changing professions (CDE, 2010).

Another approach to improving student success in mathematics is the Response to Intervention (RtI) model. The RtI model is a three-tier approach where tier one is to screen all students and to identify struggling students, tier two provides additional remediation to those struggling students in areas where they need it, and tier three provides more intensive remediation to students who are still struggling (Gersten et al., 2009).

The RtI model gives eight recommendations to mathematics teachers to improve student success in mathematics. Recommendation two states that students in tier two should be receiving intervention focused on in-depth treatment of whole numbers in Kindergarten through fifth grades and rational numbers in grades four through eight (Gersten et al., 2009). This position was reinforced by the National Mathematics Advisory Panel (NMAP) in its 2008 report, which provided detailed benchmarks and again emphasized in-depth coverage of key topics involving whole numbers and rational numbers as crucial for all students (Gersten et al., 2009).

Recommendation six states that interventions at all grade levels should devote ten minutes in each session to building fluent retrieval of basic arithmetic facts. Studies have shown a series of small but positive effects on measures of fact fluency and procedural knowledge for diverse student populations in elementary grades. Fact fluency has been proven beneficial for students in elementary school and middle school (Gersten et al., 2009).

Summary

The literature suggests that students taking Algebra I that lack basic skills like multiplication facts and rational number knowledge will struggle to succeed in class; and they therefore need remediation in those areas. The literature also suggests that additional study of the relationship between knowledge of basic math facts, particularly multiplication and rational number facts, and learning Algebra I. This hypothesized relationship leads to the central research question of this thesis: What is the impact of practicing multiplication facts and fractions twice a week with a selected group of eighthgrade students on problems similar to those on the eighth-grade Algebra I STAR test? The next chapter provides detailed explanation of the methods used to conduct this study.

CHAPTER THREE

METHODOLOGY

Setting and Participants

Three eighth-grade classes participated in this study of whether weekly practicing of fractions and multiplication facts improves eighth-grade Algebra I STAR test scores. These three classes were located at two schools in northern California.

At Fort Apache Middle School (a sixth- to eighth-grade middle school in Northern California), I used two Algebra I classes for the study. One class had 27 students enrolled and the other class had 28. Both classes were taught by the same instructor, Clark. This was Clark's 40th year of teaching and his 38th year of teaching Algebra I to eighth-grade students.

At Fern Canyon Elementary School (a kindergarten-eighth grade elementary school in Northern California), I used one Algebra I class, taught by me and containing 19 students. This was my fifth year of teaching overall, and my fourth year of teaching Algebra I to eighth-grade students. (See Table 3.1 for a fuller description of the two teachers and their responsibilities.)

Even though Researcher 2 was an experienced teacher of 40 years and Researcher 1 was a relatively young teacher, we had very similar teaching styles. We used the same Algebra I textbook, Prentice-Hall's *Algebra I* (2009). We each taught the curriculum at the same pace, because we used the same pacing guides. We even had similar classroom management styles, partly because I had done my student teaching under Clark's supervision six years earlier. It would have been hard to find two more similar teachers for this study. During the study Clark and I maintained regular contact by e-mail, telephone, and occasional face-to-face meetings; after I had collected the data we met to review the results together.

Table 3.1

	Name	School	Number of Years Taught	Students per Class	Classroom Structure
1	Researcher 1	Fern Canyon Elementary School	5 total years; 1 year of 6th grade; 3 years of 7th and 8th grade; 1 year of 8th-12th grades	19	Researcher 1 teaches 7th and 8th grade mathematics in 50-minute periods, 5 days per week. Researcher 1 also teaches 2 periods of 7th and 8th grade physical education to most of the same students.
2	Researcher 2	Fort Apache Middle School	40 total years; 2 years of 7th grade; 38 years of 8th grade	27 and 28	Researcher 2 teaches 8th- grade mathematics and science in 100-minute blocks, 5 days per week. Researcher 2 also teaches one 8th grade elective period of art class or study hall.

Demographics of Research Facilitator Participants

Procedures

This study took place over the course of 11 academic weeks, beginning on Monday, November 15, 2010, and ending on Friday, February 25, 2011. The three classes of students were broken up into two separate groups: an experimental group and a control group. One class at Fort Apache Middle School was the control group, and the other class from Fort Apache Middle School and the class from Fern Canyon Elementary School were combined to form the experimental group.

On the first day of the study, both the experimental and control groups took an Algebra I pretest (see Appendix B), drawn from the California Department of Education's online list of previously released STAR test questions (CDE, 2009). Students did 48 questions and took approximately one hour for most classes to finish. The test was 96 questions long, but students only did the even-numbered problems making a total of 48 questions. We chose to only do half of the problems to minimize time away from regular instruction. Students can finish 48 problems in one math period, but it would take them two math periods to complete 96 problems. We feared students would soon become disinterested and not put forth as much effort if the test was too long in length. As teachers, we did not want to lose any additional work days to taking this pretest and posttest than necessary. Both the control group and the experimental group were not given grades based on their performance of the pretest and posttest.

The pretest and posttest (which is actually the same test) was structured in such a way that when the odd -numbered problems were eliminated from the test, it still

contained an adequate variety of problems from all content categories. The test was split into four categories of questions, and, conveniently, all questions in the same categories were grouped together chronologically. This was a big factor in deciding to assign just even-numbered problems instead of all problems, because it helped ensure the validity of the results.

For the next 10 weeks, the control group did not do any additional practice with multiplication facts and fractions outside their normal Algebra I instruction. On the other hand, the experimental group practiced multiplication facts and fractions twice a week in the form of a timed (two and a half minutes), 10-question quiz in addition to their regular Algebra I instruction. Students were given homework grades based on their quiz performance. Researchers 1 and 2 assigned grades for this part of the study because we thought it would create an incentive for students to want to learn their multiplication facts and fractions better. Researcher 1 created the quizzes (see Appendix A for Multiplication and Fraction Quizzes). After each quiz, Researcher 1 and Researcher 2 did each problem from the quiz step-by-step on the overhead projector so students could see them done. Students were allowed to ask questions during this process if they were unsure how to do any of the problems.

Researcher 1 and Researcher 2 have very similar grading systems, and the way in which we give homework scores is nearly identical. So ensuring a similar grading component to this study was not a problem, because we made absolutely sure that we gave the same weight to these quiz grades for each student at both sites. After the 10 weeks, both the experimental and control groups were given a posttest, containing the identical problems to the pretest. Results were then analyzed to determine which group achieved a larger gain in test scores. An independent samples T-test was used with a computer program called Mini-Tabs. I also broke down the test into the four categories identified by the California Department of Education, according to what specific mathematics standard each questions addressed, and conducted independent samples T-tests for each of these four categories to identify the degree of correlation in each standard area. The four categories are Numbering Properties, Operations, and Linear Equations; Graphing and Linear Systems of Equations; Quadratics and Polynomials; and Functions and Rational Expressions.

The only procedural difference in this was study was that it was conducted at two different school sites. Fort Apache Middle School is a sixth- to eighth-grade school that operates on a block period schedule. A block is a 100-minute period where two classes are taught by the same teacher in the same classroom. Researcher 2 had the convenience of teaching in block periods because he could use some of his Science teaching time for math if he felt that he needed more time. At Researcher 1's school Fern Canyon Elementary School, there are 50-minute periods. One teacher teaches one class to one group of kids in a 50 minute period, and when the 50 minutes is done the students must go to the next class. So, when Researcher 1 felt there was not adequate time for something in math, the lesson had to be cut short rather than extending it into another

class period. I believe that this was only a minor issue that did not affect the study results.

CHAPTER FOUR

RESULTS

In this chapter I will present the results of my pretests and posttests of both the experimental and control groups, using questions drawn from previous years of the California Algebra I STAR test.

As noted above, the control group did not receive any additional remediation on fractions and multiplication facts, beyond the standard Algebra I instruction given in the class, between the pretest and posttest. The control group consisted of 27 students. Their results are summarized below and also presented more fully in Tables 4.1 and 4.2.

Control Group Results

On the pretest, the control group answered 224 questions correctly out of a possible 1,296 questions (17.2%). On the posttest, the group answered 338 questions correctly (26.1%).

In category I (Numbering Properties, Operations, and Linear Equations) of the pretest, the control group provided 109 correct answers of a possible 324 (33.6%). On the posttest the group answered 128 questions correctly (39.5%).

In category II (Graphing and Linear Systems of Equations) of the pretest, the control group answered 38 of 297 questions correctly (12.8%); on the posttest this group answered 86 questions correctly (29.0%).

Table 4.1Control Group—Test Scores by Category

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1	12 10 7	9 9
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	40	11
1	10	12
	3	4
1	11	16
11	4	14
11	6	12
	2	9
	2	4
	1	8
1	12	11
	3	10
	1	7
	3	4
	2	4
	2	3
	2	8
	4	2
	9	9
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Table 4.2

Control Group Percentage Scores on Pretest and Posttest

Control Group Percentage Scores on Pretest and Posttest			
	8		
Section(s)	Pretest	Posttest	
Category I	33.60%	39.50%	
Category II	12.80%	29.00%	
Category III	11.40%	18.80%	
	11.500/	17.000/	
Category IV	11.50%	17.80%	
Querell (Cote genies L BA	17 200/	26 100/	
Overall (Categories I-IV)	17.20%	26.10%	

In category III (Quadratics and Polynomials) of the pretest, the control group gave 46 of a possible 405 correct answers (11.4%); on the posttest it answered 76 questions correctly (18.8%).

In category IV (Functions and Rational Expressions) of the pretest, the control group answered 31 questions correctly out of a possible 270 (11.5%); on the posttest the group answered 48 questions correctly (17.8%). (See Tables 4.1 and 4.2 for control group raw scores and percentage scores by category.)

Experimental Group Results

The experimental group took the same pretest and posttest on the same dates. Unlike the control group, the experimental group received multiplication facts and fraction remediation twice a week in addition to regular Algebra I instruction. The experimental group consisted of 47 students. Their results are as follows.

On the pretest overall, the experimental group gave 461 of a possible 2,256 correct answers (20.4%); on the posttest, this group answered 875 questions correctly (38.8%).

In category I (Numbering Properties, Operations, and Linear Equations) of the pretest, the experimental group answered 249 questions correctly out of a possible 564 (44.1%); on the posttest the group gave 292 of 564 correct answers (51.8%).

In category II (Graphing and Linear Systems of Equations) of the pretest, the experimental group answered 79 questions correctly out of a possible 517 (15.2%). On the posttest the group answered 245 questions correctly (47.3%).

In category III (Quadratics and Polynomials) of the pretest, the experimental group answered 65 questions correctly out of a possible 705 (9.2%); on the posttest the group answered 204 questions correctly (28.9%).

In category IV (Functions and Rational Expressions) of the pretest, the experimental group answered 38 questions correctly out of a possible 470 (8.1%); on the posttest the group answered 133 questions correctly (28.3%). (See Tables 4.3 and 4.4 for experimental group raw scores and percentage scores by category.)

Table 4.3

Experimental Group—Test Scores by Category

Question	Category	Number Correct - Pretest	Number Correct - Posttest
2	I	30	34
4	I	35	37
6	I	27	24
8	I	3	8
10	I	13	16
10	I	29	29
12	I	19	28
16	I	19	28
18	I	21	27
20	I	15	16
20	I	12	16
24	I	26	30
24	I	11	29
28	II	11	29
30	II	3	28
30	II	9	11
32	II	9	24
36	п	16	35
38		9	18
40	II	3	14
42	II	6	17
44	II	3	20
46	II	5	21
48	III	6	25
50	III	5	9
52	III	15	18
54	III	5	8
56	III	3	19
58	III	8	13
60	III	2	11
62	III	2	9
64	III	2	6
66	III	4	11
68	III	2	12
70	III	2	30
72	III	4	17
74	III	1	7
76	III	4	9
78	IV	1	8
80	IV	2	9
82	IV	0	7
84	IV	1	16
86	IV	1	9
88	IV	7	16
90	IV	5	7
92	IV	5	13
94	IV	11	29

Table 4.4

Experimental Group Percentage Scores on Pretest and Posttest

Experimental Gr	roup Percentage Scores on Pre	test and Posttest
Section(s)	Pretest	Posttest
Section(s)	incust	TÖSTUST
Category I	44.10%	51.80%
Category II	15.20%	47.30%
	15.2070	47.5070
Category III	9.20%	28.90%
Category IV	8.10%	28.30%
	0.1070	20.5070
Overall (Categories I-IV)	20.40%	38.80%

CHAPTER FIVE

ANALYSIS

This study was designed to examine whether practicing multiplication facts and fractions twice a week is likely to help eighth-grade Algebra I students improve their STAR test scores. Results of both the control group and the experimental group, overall and in the four categories of questions, were analyzed.

As much bias as possible was removed from this study. Research participants from Fern Canyon Elementary School and Fort Apache Middle School were similar in a variety of ways.. Both schools are located in Northern California and are approximately eight miles apart from each other. Both schools are geographically in the same county. The economies of both towns are somewhat similar. Both towns are classified as rural.

Researcher 1 and Researcher 2 are very similar teachers. Their teaching styles and classroom management styles are virtually identical because Researcher 1 did his student teaching with Researcher 2 six years prior to this study. Researcher 1 and Researcher 2 use the same math textbook and they also use the same pacing guide to go through the textbook. Much effort was made to make sure they were studying the same lessons on the same weeks.

In category I questions (Number Properties, Operations, and Linear Functions), the results of the pretests and posttests of the experimental group and control group show relative balance between the groups. On average, a member of the control group was likely to answer 0.70 more category I questions correctly on the posttest than on the pretest. In comparison, a member of the experimental group was, on average, likely to get 0.91 more correct answers. Thus the experimental group had a 30% higher gain in category I scores between the pretest and posttest than the control group. An independent samples T-test obtained a p-value of .12, meaning that the difference is not statistically significant to the 95% confidence level.

In category II questions (Graphing and Linear Systems of Equations), the results are more unbalanced. On average, a member of the control group was likely to get 1.78 more correct answers on the category II posttest questions than in the pretest. In comparison, a member of the experimental group was, on average, likely to get 3.53 more correct answers. The experimental group thus had a 98% higher gain on category II questions. An independent samples T-test of the increase in scores for the two groups obtained a p-value of .0001. This is statistically significant to the 95% confidence level, indicating that practicing multiplication facts and fractions twice a week improves eighth-grade Algebra I STAR test scores for category II questions.

In category III questions (Quadratics and Polynomials), the results of the pretests and posttests showed unbalanced gains again. On average, a member of the control group was likely to get 1.11 more correct category III answers on the posttest than on the pretest, while a member of the experimental group was, on average, likely to answer 2.96 more questions correctly. These numbers mean that the experimental group had a 166% higher gain in scores between the pretest and posttest than the control group on category III questions. An independent samples T-test of the increases obtained a p-value of .001, statistically significant to the 95% confidence level. The result suggests that practicing multiplication facts and fractions twice a week improves eighth-grade STAR test scores for category III questions.

In category IV questions (Functions and Rational Expressions), on average, a member of the control group was likely to answer 0.63 more questions correctly on the posttest than on the pretest. In comparison, a member of the experimental group was, on average, likely to answer 2.02 more questions correctly on the posttest than the pretest in category IV. The experimental group thus had a 221% higher gain in scores on category IV questions. An independent samples T-test of the increase in scores obtained a p-value of .0002. This result was again statistically significant to the 95% confidence level, indicating that practicing multiplication facts and fractions twice a week improves eighth-grade STAR test scores for category IV questions. (See Figure 5.1 for further clarification.)

Considering the two groups' test results as a whole obtains similar results. On average, a member of the control group was likely to answer approximately 4.7 more questions correctly on the posttest than on the pretest. In comparison, a member of the experimental group was, on average, likely to answer 9.4 more questions correctly. On average, therefore, the experimental group had twice the increase in score as the control group (see Figure 5.2). An independent samples T-test of the two groups' overall increases obtained a p-value of .006, which is statistically significant to the 95% confidence level. The results suggest strongly that practicing multiplication facts and fractions twice a week improves eighth-grade STAR test scores for all categories combined. Category I was the only area where the results could not be considered statistically significant. The timing of the study and the content of category I questions may explain this result. First, I did not start my study until November 15, 2010. Both Fern Canyon Elementary School and Fort Apache Middle School started school on August 27, 2010. By the time when the study began, one-fourth of the school year had already been completed. Both the experimental group and control group had already been taught the concepts in category I (Number Properties, Operations, and Linear Equations) before taking the pretest. This prior

knowledge may have affected the extent to which remediation on multiplication tables and fractions could improve student performance on questions in this category.

Second, the questions in category I do not require knowledge of multiplication facts and fractions as frequently as do the questions in categories II, III, and IV. Category I contains questions about math vocabulary and properties, two question types that would not require any math operations in order to solve them. For example, a question in category I may ask, "Which of the following problems is an example of the commutative property of multiplication?" This question, along with several others in category I, requires no mathematical operations to solve it. These types of questions can be categorized as "recall

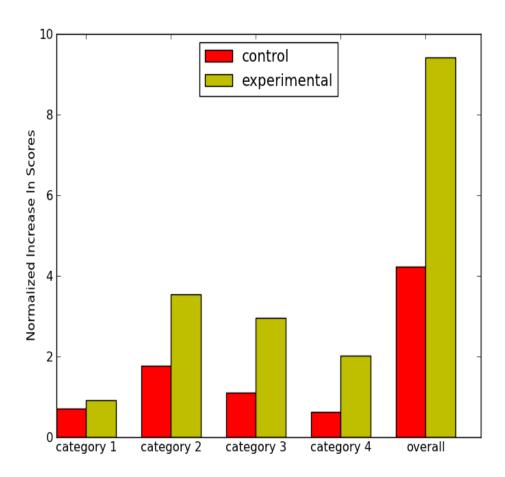


Figure 5.1. Increase in Raw Scores by Groups

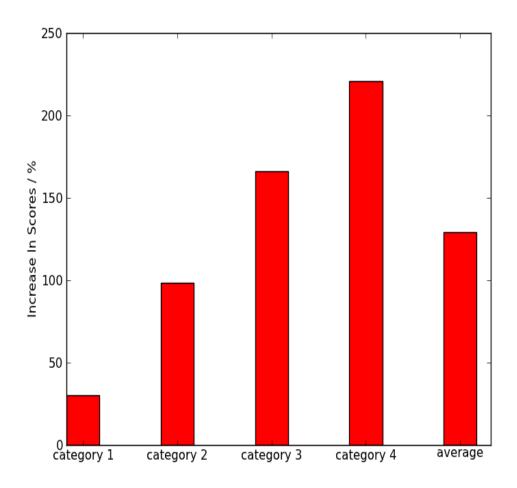


Figure 5.2. Percentage Increase of Experimental Versus Control Group

Note: Percentages indicate the extent to which the experimental group achieved a greater increase in correct answers than the control group between the pretest and posttest. For example, on Category I questions the experimental group's increase in number of correct answers was 30 percent greater than that of the control group . questions, which require only rote memorization of facts.

On the whole, the data show evidence that the experimental group had larger score increases from pretest to posttest relative to the control group in categories II through IV. The experimental group scored higher in category I as well, but not enough to be considered statistically significant.

CHAPTER SIX

CONCLUSION

The purpose of this study was to investigate the impact of practicing multiplication facts and fractions twice a week on eighth-grade Algebra I STAR test scores. The results suggest that providing additional time in the eighth-grade Algebra I curriculum for remediation of multiplication facts and fractions will enable students to achieve higher Algebra I STAR test scores at the conclusion of the school year. This conclusion gels in conjunction with research-based literature on the topic. Research shows that students will succeed in Algebra I if they follow certain steps, one of them being that all students need daily practice to build fluent retrieval of basic arithmetic facts (Gersten et al., 2009).

In conducting this study, both research facilitators identified some instructional challenges. For example, both researchers experienced difficulty in finding time for additional remediation while trying to teach all of the standard Algebra I curriculum. This was a bigger problem with my (Researcher 1's) schedule of 50-minute class periods each day than with the block schedule within which Researcher 2 teaches. Researcher 2 could allot a small amount of math quiz time more readily with the block schedule by taking away time from the other subject (science) being taught within the same block. However, in the period schedule no extra time could be allotted, because students had to leave class and go to another class at the end of the period.

The results of this study carry implications for both school site administrators and mathematics teachers. One of the school site administrator's responsibilities is to get the highest STAR test scores as possible. With math scores being low statewide, school administrators are constantly brainstorming and implementing better ways for their mathematics teachers to teach and improve their STAR test scores. As noted earlier in this study, if a school repeatedly does not make adequate yearly progress on their STAR test scores both teachers and administrators can lose their jobs.

In public education there is a market for remediation concepts in all areas of curriculum. Classroom teachers are constantly looking for ways to fix gaps in student knowledge. Currently, many students in California are not scoring proficient or higher on the eighth-grade Algebra I STAR test. Many educators and education experts agree that a large number of students entering middle school do not have a firm grasp of basic math concepts, and that this weakness prevents them from excelling in higher-order math classes like Algebra I. Many remediation programs cost money and time. Book publishers often sell remediation materials for large profits. Some of these curricula not only cost substantial money but also take a long time to institute. In contrast, the intervention proposed in this study, practicing multiplication facts and fractions two times per week, is simple and virtually cost-free to institute. The results of this study offer evidence that eighth-grade students will do better on problems similar to those on the eighth-grade Algebra I STAR test if they take part in this remediation.

There are additional reflections that arise from the completion of this study. My first thought is contemplating if the test I used for this study translates to the same levels of performance as the actual STAR test. This is very important because the results of this study are based from this test. I don't think the actual STAR test would have yielded the exact same results as the test I used for this study. However, considering the logistics of the study, I do not believe I could have used a better test unless I used the actual STAR test as a means of evaluation at the end of the school year.

Another important question that arose from my study is if this remediation is as effective on high-achieving students as it is on low-achieving students. I cannot make a conclusion based on the scope of this study. The socioeconomic factors and previous academic profiles of students were not investigated thoroughly beforehand to make a generalization of that magnitude.

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APPENDIX A

MULTIPLICATION AND FRACTION QUIZZES

Name:_____ Class:_____

Quiz Week 1 (#1)

1) $2_{1/3} + 1_{1/2}$ 2) 4/5 - 5/9 3) $3_{4/11} \times 2/7$

4) $7_{7/9} \div 1/2$ 5) 6×12 6) 11×4

7) 3×9 8) 2×10 9) 5×3

10) Convert to an improper fraction... $6_{7/8}$

		Name: Class:	
	Quiz Week 1 (#2)		
1) 3/4 + 2/3	2) 1_1/2 - 4/5	3) 3/11 × 3/7	
4) 8_1/3 ÷ 1/9	5) 7 × 7	6) 2 × 9	
7) 8 × 2	8) 3 × 6	9) 5 × 11	

10) Convert to an improper fraction... $4_{1/5}$

		Name: Class:	
	Quiz Week 2 (#3)		
1) 3_1/3 + 1/4	2) 6/7 - 1/5	3) 7_1/2 × 4/15	
4) 5/12 ÷ 1_1/5	5) 4 × 7	6) 12 × 11	
7) 6 × 6	8) 11 × 10	9) 3 × 0	

10) Convert to a mixed fraction... 14/11

		Name: Class:	
	Quiz Week 2 (#4)		
1) 12/7 + 2/9	2) 1_4/5 - 1/8	3) 2_1/2 × 3_9/10	
4) 1_1/4 ÷ 1/4	5) 2 × 3	6) 9 × 6	
7) 1 × 5	8) 11 × 3	9) 3 × 12	

10) Convert to a mixed fraction... 20/3

		Name: Class:	
	Quiz Week 3 (#5)		
1) 4_9/10 + 2_1/3	2) 3/10 - 1/6	3) 2_1/2 × 4/11	
4) 2/3 ÷ 2/3	5) 1 × 9	6) 9 × 1	
7) 12 × 9	8) 7 × 3	9) 10 × 11	

10) Convert to an improper fraction... $1_{1/10}$

		Name: Class:	
	Quiz Week 3 (#6)		
1) 11/5 + 2/1	2) 3_1/7 - 2/5	3) 3/11 × 4/9	
4) 3_1/11 ÷ 6/1	5) 2 × 12	6) 3 × 2	
7) 8 × 12	8) 5 × 2	9) 11 × 11	

10) Convert to a mixed fraction... 12/5

		Name: Class:	-
	Quiz Week	4 (#7)	
1) 2_1/3 + 4/11	2) 7/8 - 7/9	3) 16/5 × 3/4	
4) 1/2 ÷ 4/9	5) 3 × 12	6) 4 × 1	
7) 8 × 11	8) 10 × 12	9) 2 × 11	

10) Convert to an improper fraction... $1_2/12$

		Name: Class:
	Quiz Week 4 (#8)
1) 3_1/4 + 2/10	2) 6/15 - 2/10	3) 3/11 × 1
4) 4 ÷ 1/9	5) 2 × 0	6) 2 × 10
7) 7 × 9	8) 11 × 4	9) 12 × 6

10) Convert to an improper fraction... $3_9/12$

		Name: Class:
	Quiz Week 5 (#9)
1) 3/4 + 3/4	2) 4/3 - 10/9	3) 4_1/2 × 2_1/9
4) 4_4/5 ÷ 5_1/11	5) 3 × 7	6) 1 × 8
7) 4 × 10	8) 12 × 12	9) 11 × 11

10) Convert to a mixed fraction... 13/2

		Name: Class:
	Quiz Week 5 (#10))
1) 9/10 + 7/10	2) 2/7 - 1/6	3) 10_1/8 × 1/6
4) 3_1/8 ÷ 1/6	5) 2 × 10	6) 11 × 4
7) 8 × 5	8) 9 × 9	9) 10 × 10

10) Convert to a mixed fraction... 30/4

		Name: Class:
	Quiz Week 6 (#11))
1) 2_1/9 + 9/10	2) 1/10 - 1/12	3) 3/15 × 1/3
4) 4_1/8 ÷ 9/11	5) 4 × 1	6) 7 × 10
7) 3 × 9	8) 6 × 4	9) 12 × 10

10) Convert to an improper fraction... 3_8/9

		Name: Class:
	Quiz Week 6 (#12)
1) 1_2/9 + 2	2) 2 - 3/4	3) 5_1/2 × 5_1/3
4) 6_1/11 ÷ 3/5	5) 12 × 5	6) 11 × 2
7) 9 × 7	8) 7 × 8	9) 9 × 3

10) Convert to an improper fraction... $9_{1/2}$

		Name: Class:
	Quiz Week 7 (#13))
1) 1_1/4 + 3_1/3	2) 19/9 - 1/9	3) 6/7 × 2/11
4) 3/5 ÷ 2/5	5) 4 × 12	6) 1 × 8
7) 11 × 10	8) 4 × 3	9) 8 × 5

10) Convert to a mixed fraction... 17/3

		Name: Class:
	Quiz Week 7 (#14)
1) 3_2/3 + 2_1/8	2) 6_1/2 - 3_1/2	3) 4/3 × 10/11
4) 10/12 ÷ 2/3	5) 7 × 2	6) 3 × 11
7) 4 × 9	8) 9 × 10	9) 2 × 3

10) Convert to a mixed fraction... 15/7

		Name: Class:
	Quiz Week 8 (#15))
1) 1/2 + 5/8	2) 5/8 - 1/2	3) 9/11 × 11/9
4) 1/7 ÷ 7/1	5) 7 × 12	6) 10 × 10
7) 2 × 9	8) 0 × 3	9) 3 × 3

10) Convert to an improper fraction... $4_4/7$

		Name: Class:
	Quiz Week 8 (#16))
1) 3/11 + 2_1/6	2) 4/5 - 1/10	3) 7/9 × 2/3
4) 7/9 ÷ 2_1/7	5) 6 × 1	6) 12 × 12
7) 2 × 4	8) 9 × 8	9) 3 × 10

10) Convert to an improper fraction... $5_{1/9}$

		Name: Class:
	Quiz Week 9 (#17)
1) 8_5/6 + 1_1/6	2) 5_1/2 - 3/2	3) 2_1/2 × 3/5
4) 8/9 ÷ 9/18	5) 2 × 6	6) 9 × 5
7) 4 × 11	8) 11 × 11	9) 7 × 12

10) Convert to a mixed fraction... 20/7

		Name: Class:
	Quiz Week 9 (#18))
1) 4/7 + 4/5	2) 9_1/2 - 4_1/3	3) 3/7 × 2/7
4) 3/5 ÷ 1_1/2	5) 3 × 9	6) 11 × 10
7) 4 × 5	8) 8 × 3	9) 2 × 7

10) Convert to a mixed fraction... 13/3

		Name: Class:
	Quiz Week 10 (#1	9)
1) 3/5 + 2/3	2) 10_1/4 - 2_1/3	3) 3_3/4 × 2/3
4) 8_1/5 ÷ 4/5	5) 6 × 7	6) 2 × 8
7) 9 × 3	8) 5 × 8	9) 11 × 1

10) Convert to an improper fraction... $3_{1/12}$

		Name: Class:
	Quiz Week 10 (#20))
1) 1_3/4 + 3_5/8	2) 5_1/2 - 3/2	3) 6/7 × 9/10
4) 9/12 ÷ 1/8	5) 4 × 5	6) 11 × 9
7) 2 × 12	8) 5 × 12	9) 11 × 0

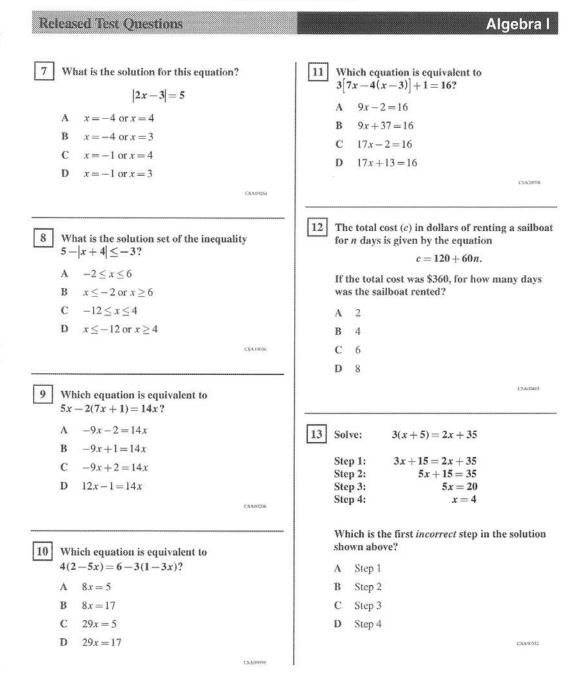
10) Convert to an improper fraction... $1_{3/9}$

APPENDIX B

PRETEST AND POSTTEST

CALIFORNIA STANDARDS TEST Algebra I **Released Test Questions** 1 Is the equation 3(2x-4) = -18 equivalent to 4 Which expression is equivalent to x^6x^2 ? 6x - 12 = -18?Λ $x^{4}x^{3}$ Yes, the equations are equivalent by the $x^{5}x^{3}$ A B Associative Property of Multiplication. $x^{7}x^{3}$ C B Yes, the equations are equivalent by the D $x^{9}x^{3}$ Commutative Property of Multiplication. CSA20167 С Yes, the equations are equivalent by the Distributive Property of Multiplication over Addition. 5 Which number does not have a reciprocal? D No, the equations are not equivalent. CSA101-06 A --1 2 Which statement is false? **B** 0 The order in which two whole numbers are A subtracted does not affect the difference. 1 С 1000 The order in which two whole numbers are B added does not affect the sum. C The order in which two rational numbers are D 3 added does not affect the sum. C5A30452 D The order in which two rational numbers are multiplied does not affect the product. CNADED What is the multiplicative inverse of $\frac{1}{2}$? 6 $\sqrt{16} + \sqrt[3]{8} =$ 3 -2A 4 A B 6 B C 9 D 10 CSA00471 D 2 CSA10155

CALIFORNIA STANDARDS TEST



Algebra I	Released Test Question
 A 120-foot-long rope is cut into 3 pieces. The first piece of rope is twice as long as the second piece of rope. The third piece of rope is three times as long as the second piece of rope. What is the length of the longest piece of rope? A 20 feet B 40 feet C 60 feet D 80 feet 	17 The lengths of the sides of a triangle are y, y + 1, and 7 centimeters. If the perimeter is 56 centimeters, what is the value of y? A 24 B 25 C 31 D 32
15 The cost to rent a construction crane is \$750 per day plus \$250 per hour of use. What is the maximum number of hours the crane can be used each day if the rental cost is not to exceed \$2500 per day? A 2.5 B 3.7 C 7.0 D 13.0	18 Beth is two years older than Julio. Gerald is twice as old as Beth. Debra is twice as old as Gerald. The sum of their ages is 38. How old is Beth? A 3 B 5 C 6 D 8
16 What is the solution to the inequality x-5>14? A $x>9$ B $x>19$ C $x<9$ D $x<19$ CANNER	

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CALIFORNIA STANDARDS TEST Algebra I **Released Test Questions** 19 Which number serves as a counterexample to 22 The chart below shows an expression evaluated the statement below? for four different values of x. All positive integers are divisible by 2 or 3. $x^{2} + x + 5$ x 1 7 2 11 A 100 6 47 57 B 7 61 С 30 D 25 Josiah concluded that for all positive values of CN0333197 $x, x^2 + x + 5$ produces a prime number. Which value of x serves as a counterexample to prove Josiah's conclusion false? 20 What is the conclusion of the statement in the A 5 box below? B 11 If $x^2 = 4$, then x = -2 or x = 2. C 16 D 21 CSA26027 $x^2 = 4$ A B x = -223 John's solution to an equation is shown below. x = 2С $x^2 + 5x + 6 = 0$ Given: x = -2 or x = 2D (x+2)(x+3)=0Step 1: CSA30055 x+2=0 or x+3=0Step 2: x = -2 or x = -3Step 3: Which of the following is a valid conclusion to Which property of real numbers did John use 21 for Step 2? the statement "If a student is a high school band member, then the student is a good musician"? A multiplication property of equality A All good musicians are high school band zero product property of multiplication B members. commutative property of multiplication С A student is a high school band member. B D distributive property of multiplication over С All students are good musicians. addition All high school band members are good D CSA20134 musicians. CSADER

Algebra I

													165		

24 Stan's solution to an equation is shown below. 26 What is the y-intercept of the graph of 4x + 2y = 12?Given: n + 8(n + 20) = 110-4 A Step 1: n + 8n + 20 = 110--2 В 9n + 20 = 110Step 2: С Step 3: 9n = 110 - 206 9n = 90Step 4: D 12 <u>= 90</u> 9*n* Step 5: 155400239 9 9 n = 10Step 6: 27 Which inequality is shown on the graph below? Which statement about Stan's solution is true? Stan's solution is correct. A B Stan made a mistake in Step 1. С Stan made a mistake in Step 3. D Stan made a mistake in Step 5. CSA30033 25 When is this statement true? -2 -3 The opposite of a number is less than the original number. -5 -6 .7 A This statement is never true. B This statement is always true. $\mathbf{A} \quad y < \frac{1}{2}x - 1$ C This statement is true for positive numbers. This statement is true for negative numbers. D $\mathbf{B} \qquad y \le \frac{1}{2}x - 1$ CSA30147 $C \quad y > \frac{1}{2}x - 1$ $\mathbf{D} \qquad y \ge \frac{1}{2}x - 1$ CSA10130



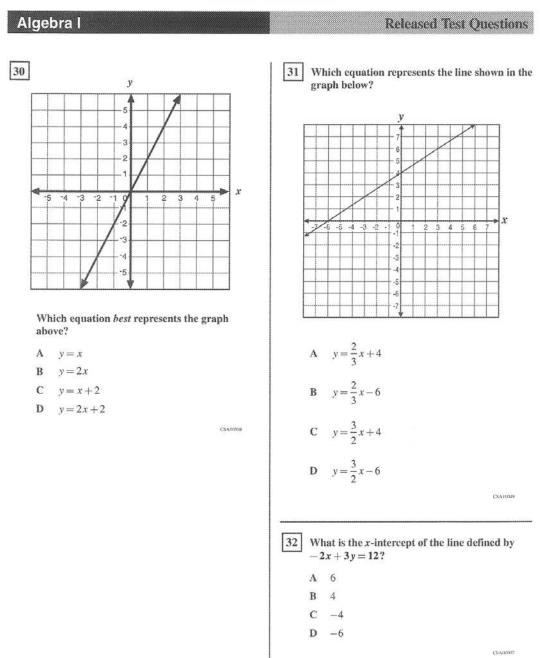
CALIFORNIA STANDARDS TEST Algebra I **Released Test Questions 28** Which *best* represents the graph of y = 2x - 2? Which inequality does the shaded region of the graph represent? 29 ÿ v 1 A С 3 A 5 6 v x $3x + y \leq 2$ A $3x + y \ge 2$ B B D CSAD(299 С $3x + y \leq -2$ $3x + y \ge -2$ D

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CALIFORNIA STANDARDS TEST

Algebra I

33	Which point lies on the line defined by	ŕ
	3x + 6y = 2?	



A (0, 2)



D
$$\left(1, -\frac{1}{3}\right)$$

34

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LADINOS

35 The data in the table show the cost of renting a bicycle by the hour, including a deposit.

Renting a	Bicycle

Hours (h)	Cost in dollars (c)
2	15
5	30
8	45

If hours, *h*, were graphed on the horizontal axis and cost, *c*, were graphed on the vertical axis, what would be the equation of a line that fits the data?

What is the equation of the line that has a slope of 4 and passes through the point
$$(3, -10)$$
?

A
$$y = 4x - 22$$

B $y = 4x + 22$
C $y = 4x - 43$

$$\mathbf{D} \qquad \mathbf{y} = 4x + 43$$

LSA10150

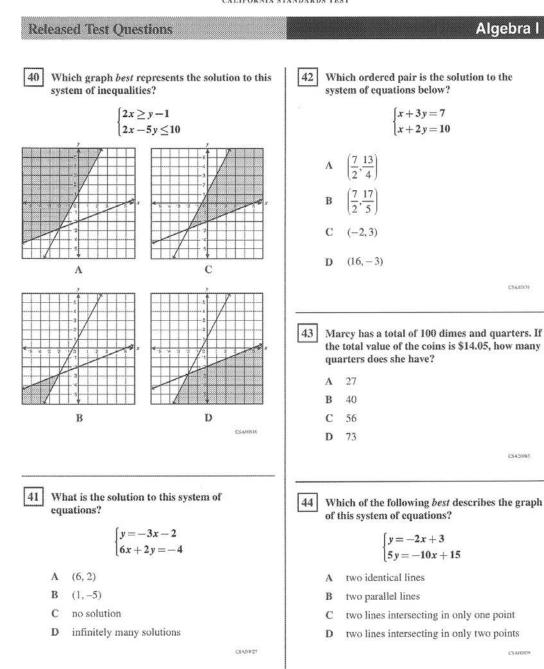
A c = 5hB $c = \frac{1}{5}h + 5$ C c = 5h + 5D c = 5h - 5

CSAH009

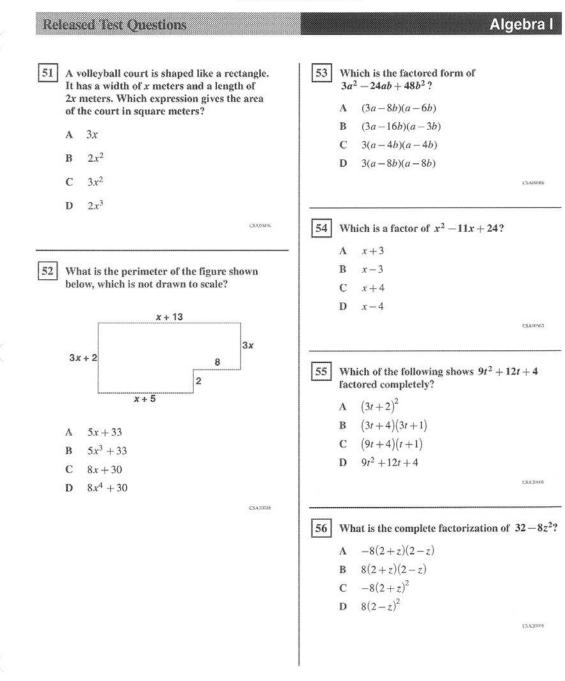
CALIFORNIA STANDARDS TEST Algebra I **Released Test Questions** 36 38 The equation of line l is 6x + 5y = 3, and the Some ordered pairs for a linear function of x are given in the table below. equation of line q is 5x - 6y = 0. Which statement about the two lines is true? A Lines l and q have the same y-intercept. x y 1 1 B Lines l and q are parallel. 3 7 C Lines l and q have the same x-intercept. 5 13 D Lines l and q are perpendicular. 7 19 CSA(8)341 Which of the following equations was used to generate the table above? 39 Which equation represents a line that is A y = 2x + 1parallel to $y = -\frac{5}{4}x + 2?$ B y = 2x - 1С y = 3x - 2 $\mathbf{A} \quad \mathbf{y} = -\frac{5}{4}\mathbf{x} + 1$ y = 4x - 3D CSAIDIN $\mathbf{B} \qquad y = -\frac{4}{5}x + 2$ C $y = \frac{4}{5}x + 3$ 37 Which point lies on the line represented by the equation below? 5x + 4y = 22 $\mathbf{D} \qquad y = \frac{5}{4}x + 4$ $\left(-2,\frac{11}{4}\right)$ \$5510112 Α $\left(-1, \frac{17}{4}\right)$ B (2, 3)С D (6, 2)CSA191-3

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CALIFORNIA STANDARDS TEST Algebra I Released Test Questions $48 \quad (4x^2 - 2x + 8) - (x^2 + 3x - 2) =$ 45 Members of a senior class held a car wash to raise funds for their senior prom. They charged A $3x^2 + x + 6$ \$3 to wash a car and \$5 to wash a pick-up truck or a sport utility vehicle. If they earned a total **B** $3x^2 + x + 10$ of \$275 by washing a total of 75 vehicles, how C $3x^2 - 5x + 6$ many cars did they wash? D $3x^2 - 5x + 10$ A 25 CSAMERS B 34 C 45 D 50 49 The sum of two binomials is $5x^2 - 6x$. If one of the binomials is $3x^2 - 2x$, what is the other CSA19167 binomial? A $2x^2 - 4x$ 46 At what point do the lines represented by the equations 2x + y + 1 = 0 and 4x + y - 3 = 0 $2x^2 - 8x$ B intersect? C $8x^2 + 4x$ A (2,5) D $8x^2 - 8x$ (2, -5)B CSA10160 (-1,1)C **D** (1, -1)CNA2806 50 Which of the following expressions is equal to (x+2)+(x-2)(2x+1)? $\frac{5x^3}{10x^7} =$ 47 A $2x^2 - 2x$ **B** $2x^2 - 4x$ A 2x4 C $2x^2 + x$ $\frac{1}{2x^4}$ B D $4x^2 + 2x$ CSAID191 $\frac{1}{5x^4}$ С D $\frac{x^4}{5}$ 1.5400303



Algebra IReleased Test Questions
$$[57]$$
If x^2 is added to x , the sum is 42. Which of the
following could be the value of x^2 $x^2 + 4x = 6$ by completing the square.
 $x^2 + 4x = 6$ by completing the square.
Which equation is part of her solution? $A - 7$
 $B - 6$
 $C 14$
 $D - 42$ $A (x + 2)^2 = 8$
 $B (x + 2)^2 = 10$
 $C (x + 4)^2 = 10$
 $D (x + 4)^2 = 22$ $\mathbb{S8}$ What quantity should be added to both sides of
this equation to complete the square? $x^2 - 8x = 5$ $A 4$
 $B - 4$
 $C 16$
 $D - 16$ $\mathbb{S9}$ What are the solutions for the quadratic
equation $x^2 + 6x = 16$? $A -2, -8$
 $B -2, 8$
 $C 2, 2, -3$ \mathbb{Course} $[59]$ What are the solutions for the quadratic
equation $x^2 + 6x = 16$? $A -2, -8$
 $B -2, 8$
 $C 2, 2, -3$ \mathbb{Course} $[52]$ What are the solutions for the quadratic
equation $x^2 - 8x = 9$? $A 3$
 $B -3, 3$
 $C 1, -9$ $D -1, 9$ \mathbb{Course}

Released Test Questions

Algebra I

63 Toni is solving this equation by completing the square.

 $ax^2 + bx + c = 0$ (where $a \ge 0$)

Step 1: $ax^2 + bx = -c$ Step 2: $x^2 + \frac{b}{a}x = -\frac{c}{a}$ Step 3:?

Which should be Step 3 in the solution?

- $\mathbf{A} \quad x^2 = -\frac{c}{b} \frac{b}{a}x$
- $\mathbf{B} \qquad x + \frac{b}{a} = -\frac{c}{ax}$
- $C \qquad x^2 + \frac{b}{a}x + \frac{b}{2a} = -\frac{c}{a} + \frac{b}{2a}$

$$\mathbf{D} \qquad x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

64 Four steps to derive the quadratic formula are shown below.

$$1 \qquad x^{2} + \frac{bx}{a} = \frac{-c}{a}$$

$$II \qquad \left(x + \frac{b}{2a}\right)^{2} = \frac{b^{2} - 4ac}{4a^{2}}$$

$$III \qquad x = \pm \sqrt{\frac{b^{2} - 4ac}{4a^{2}} - \frac{b}{2a}}$$

$$IV \qquad x^{2} + \frac{bx}{a} + \left(\frac{b}{2a}\right)^{2} = \frac{-c}{a} + \left(\frac{b}{2a}\right)^{2}$$

What is the correct order for these steps?

- A I, IV, II, III
- B I, III, IV, II
- C II, IV, I, III
- D II, III, I, IV

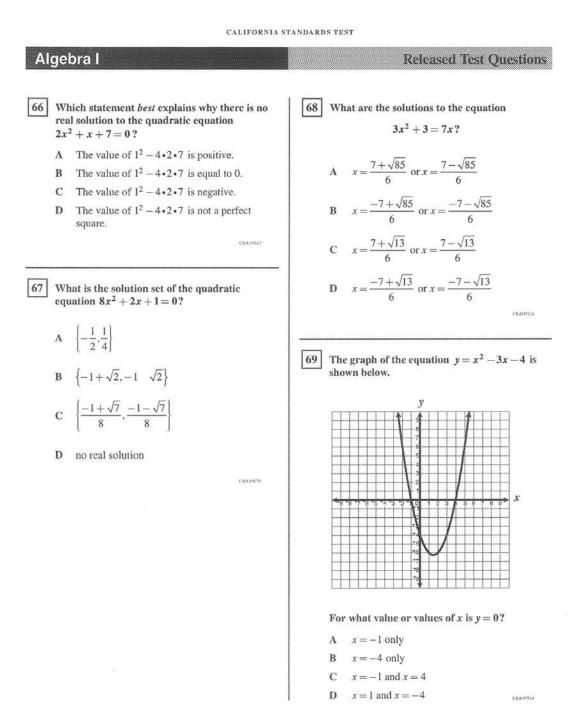
CS40872

CSA20062

65 Which is one of the solutions to the equation $2x^2 - x - 4 = 0$?

$$A \quad \frac{1}{4} - \sqrt{33}$$
$$B \quad -\frac{1}{4} + \sqrt{33}$$
$$C \quad \frac{1 + \sqrt{33}}{4}$$
$$D \quad \frac{-1 - \sqrt{33}}{4}$$

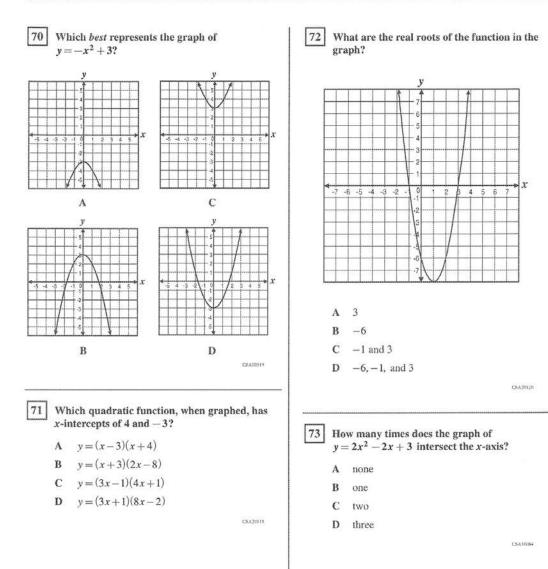
CNA00314



CALIFORNIA STANDARDS TEST

Released Test Questions





77 What is $\frac{x^2 - 4xy + 4y^2}{3xy - 6y^2}$ reduced to lowest An object that is projected straight downward with initial velocity v feet per second travels a terms? distance $s = vt + 16t^2$, where t =time in seconds. If Ramón is standing on a balcony 84 feet above A $\frac{x-2y}{3}$ the ground and throws a penny straight down with an initial velocity of 10 feet per second, in how many seconds will it reach the ground? $\mathbf{B} = \frac{x - 2y}{3y}$ 2 seconds 3 seconds $\frac{x+2y}{3}$ С 6 seconds 8 seconds $\frac{x+2y}{3y}$ D CSA0015 The height of a triangle is 4 inches greater Simplify $\frac{6x^2 + 21x + 9}{4x^2 - 1}$ to lowest terms. than twice its base. The area of the triangle is 78 $\frac{3(x+1)}{2x-1}$ A $\frac{3(x+3)}{2x-1}$ B 14 in. $\frac{3(2x+3)}{4(x-1)}$ С CRAUDIN $\frac{3(x+3)}{2x+1}$ D

168 square inches. What is the base of the triangle? 7 in. Α B 8 in. С 12 in.

D

Algebra I

Α

B

С D

75

74

76 A rectangle has a diagonal that measures 10 centimeters and a length that is 2 centimeters longer than the width. What is the width of the rectangle in centimeters?

A 5

B 6

- С 8
- D 12

CSA40200

CSA00403

CSA10025

Released Test Questions

Released Test Omertions

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Released Test Questions	Algebra I
79 What is $\frac{x^2 - 4x + 4}{x^2 - 3x + 2}$ reduced to lowest terms? A $\frac{x-2}{x-1}$ B $\frac{x-2}{x+1}$ C $\frac{x+2}{x-1}$ D $\frac{x+2}{x+1}$	81 What is the simplest form of the fraction $\frac{x^2 - 1}{x^2 + x - 2}$? A $\frac{-1}{x - 2}$ B $\frac{x - 1}{x - 2}$ C $\frac{x - 1}{x + 2}$ D $\frac{x + 1}{x + 2}$
CSAIBLSy	x+2 (55.20127
80 What is $\frac{12a^3 - 20a^2}{16a^2 + 8a}$ reduced to lowest terms?	$\boxed{\begin{array}{c} \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ $
A $\frac{a}{2}$	A $\frac{7(z-2)}{4(z+1)}$
$\mathbf{B} = \frac{3a-5}{2a+1}$	$\mathbf{B} = \frac{7(z+2)}{4(z-1)}$
$\mathbf{C} = -\frac{2a}{4+2a}$	$C = \frac{7z(z+1)}{4(z+2)}$
D $\frac{a(3a-5)}{2(2a+1)}$	$D = \frac{7z(z-1)}{4(z+2)}$

		ALIFORNIA STANDARDS TEST	
7	Algebra I	Re	leased Test Questions
C	83 Which fraction equals the product $\left(\frac{x+5}{3x+2}\right)\left(\frac{2x-3}{x-5}\right)?$		quivalent to $\frac{\frac{3x}{5}}{\frac{x}{4} + \frac{x}{2}}$?
	$A \frac{2x-3}{3x+2}$	$\Lambda = \frac{x^2}{5}$	
	$\mathbf{B} = \frac{3x+2}{4x-3}$	$\mathbf{B} = \frac{9x^2}{20}$	
	C $\frac{x^2 - 25}{6x^2 - 5x - 6}$ D $\frac{2x^2 + 7x - 15}{3x^2 - 13x - 10}$	C $\frac{4}{5}$	
	$3x^2 - 13x - 10$	$\mathbf{D} = \frac{9}{5}$	
			CSA10141
Ć	$\frac{x^2 + 8x + 16}{x + 3} \div \frac{2x + 8}{x^2 - 9} =$ A $\frac{2(x + 4)^2}{(x - 3)(x + 3)^2}$	with some 15%-sal 100 mL of a 12%-s	d some 10%-saline solution ine solution to obtain aline solution. How much of ation did the pharmacist use
		A 60 mL B 45 mL	
	$\mathbf{B} = \frac{2(x+3)(x-3)}{x+4}$	C 40 mL D 25 mL	
	$C \frac{(x+4)(x-3)}{2}$		CSA9033
	$\mathbf{D} = \frac{(x+4)(x-3)^2}{2(x+3)}$	was 45 miles per ho he drove 40 miles p	ving speed for a 4-hour trip our. During the first 3 hours er hour. What was his he last hour of his trip?
		A 50 miles per ho	ur
		B 60 miles per ho	ur
		C 65 miles per ho	ur
C		D 70 miles per ho	UF CSA98576

91

Released Test Questions

Algebra I

88	One pipe can fill a tank in 20 minutes, while
	another takes 30 minutes to fill the same tank.
	How long would it take the two pipes together
	to fill the tank?

- A 50 min
- B 25 min
- C 15 min
- **D** 12 min

CSA00161

Jena's Vacation							
Miles Traveled	600	450	300	960			
Gallons of Gasoline	20	15	10	x			

Jena's car averaged 3θ miles per gallon of gasoline on her trip. What is the value of x in gallons of gasoline?

A	32	
B	41	
С	55	
D	80	
		CNA10064

89 Two airplanes left the same airport traveling in opposite directions. If one airplane averages 400 miles per hour and the other airplane averages 250 miles per hour, in how many hours will the distance between the two planes be 1625 miles?

A 2.5

B 4

C 5

D 10.8

6.8A1(8)55

90 Lisa will make punch that is 25% fruit juice by adding pure fruit juice to a 2-liter mixture that is 10% pure fruit juice. How many liters of pure fruit juice does she need to add?

- A 0.4 liter
- B 0.5 liter
- C 2 liters
- D 8 liters

CSA,191386

92 Which relation is a function?

- $\mathbf{A} \quad \{(-1, 3), (-2, 6), (0, 0), (-2, -2)\}$
- $\mathbf{B} = \{(-2, -2), (0, 0), (1, 1), (2, 2)\}$
- $\mathbb{C} \quad \{(4,0), (4,1), (4,2), (4,3)\}$
- $\mathbf{D} = \{(7, 4), (8, 8), (10, 8), (10, 10)\}$

C5A1000

CALIFORNIA STANDARDS TEST

Algebra I

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93	Which relation is a function?
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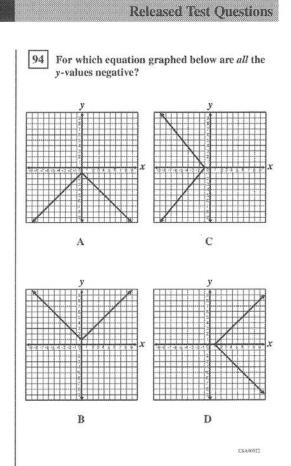
	Input	Output
	1	2
A	2	2
	3	3
	4	3

	Input	Output
	2	6
В	2	5
	6	4
	6	3

Input	Output
1	2
2	4
4	6
4	8

С

	Input	Output
	0	1
D	0	2
	1	3
	1	4



CSA10071

Released Test Questions

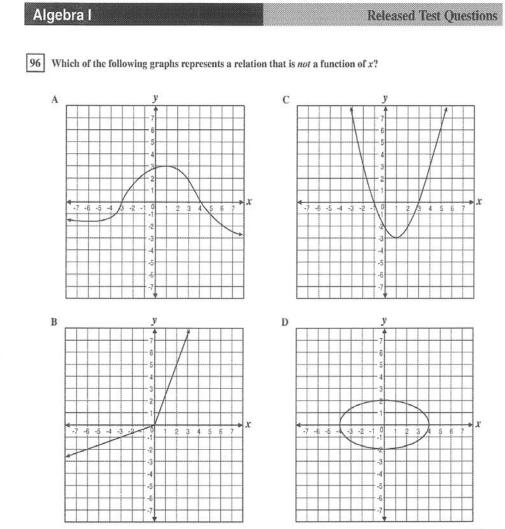
Algebra I

95 What is the domain of the function shown on the graph below?

	4	TTT	11	
	- 7			+
	6			+++
				+++
	4			
	2			
	1			
			1.1	
-7 -6 -5 -4	-3 -2 -1 0	1 2 3	4 5	6 7
-7 -6 -5 -4	-1-	1 2 3	4 5	6 7
-7 -6 -5 -4	-1	1 2 3	4 5	6 7
-7 -6 -5 -4	-1-	1 2 3	4 5	<u>6</u> 7
-7 -6 -5 -4	-1	1 2 3	4 5	6 7
-7 -6 -5 -4	-1 -2 -3 -4	•	4 5	6 7 ·
-7 -6 -5 -4	-1 -2 -3	1 2 3	4 5	6 7

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CSA105772



CSA30002