Introduction to Game Theory

Lecture Note 1: Strategic-Form Games and Nash Equilibrium (1)

HUANG Haifeng

University of California, Merced

Rational choice and preference relations

- Game theory studies rational players' behavior when they engage in strategic interactions.
- Rational choice: the action chosen by a decision maker is better or at least as good as every other available action, according to her preferences.
- Preferences (偏好) are rational if they satisfy
  - **Completeness** (完备性): between any x and y in a set, $x \succ y$ (x is preferred to y), $y \succ x$, or $x \sim y$ (indifferent)
  - **Transitivity** (传递性): $x \succeq y$ and $y \succeq z \Rightarrow x \succeq z$ ($\succeq$ means $\succ$ or $\sim$)
    - $\Rightarrow$ Say apple $\succ$ banana, and banana $\succ$ orange, then apple $\succ$ orange

Preferences and payoff functions (utility functions)

- No other restrictions on preferences. Preferences can be altruistic.
  - But individual rationality does not necessarily mean collective rationality.
- Payoff function/utility function (支付函数/效用函数): $u(x) \geq u(y)$ iff $x \succeq y$
- For now we only deal with ordinal (as opposed to cardinal) preferences, so you can use many different utility functions to represent the same preference relation.
  - Any strictly increasing transformation of the same utility function will do.
  - Say $x \succ y \succ z$. Then $u(x) = 3$, $u(y) = 2$, $u(z) = 1$ represents the same preferences as $u(x) = 100$, $u(y) = 10$, $u(z) = 2$.

Types of games

- Games with complete information
  - Static games
  - Dynamic games
- Games with incomplete information
  - Static games (Bayesian games)
  - Dynamic games (dynamic Bayesian games)

Static games of complete information

- Static games: simultaneous-move, single-shot games
- Complete information (完全信息): a player knows other players' utility functions (and other characteristics that affect their decision making)
- We use the strategic form/normal form (策略型/正规型) to represent a static game of complete information.
- Definition: A strategic-form game consists of
    1. a set of players
    2. for each player, a set of actions (i.e., strategies)
    3. for each player, preferences over the set of action/strategy profiles

Static games of complete information

- **Strategy profile** (策略组合): a list of all the player's strategies
  - E.g, my strategies: left or right; your strategies: up or down
  - Strategy/action profiles: (left, up), (left, down), any other?
- Preferences are over strategy profiles rather than one's own actions/strategies.
- In single-shot games, actions are equivalent to strategies.

Illustration: Prisoner's dilemma (囚徒困境)

- Players: two suspects, 1 and 2
- Actions: {stay silent, confess}
- Preferences:
  - ▷ $u_1(\text{confess, silent}) > u_1(\text{silent, silent}) > u_1(\text{confess, confess}) > u_1(\text{silent, confess})$
  - ▷ $u_2(\text{silent, confess}) > u_2(\text{silent, silent}) > u_2(\text{confess, confess}) > u_2(\text{confess, silent})$
- Game representation

Suspect 2

		silent	confess
Suspect 1	silent	$0, 0$	$-2, 1$
	confess	$1, -2$	$-1, -1$

Nash equilibrium

- Definition: A strategy profile a^* is a **Nash equilibrium** (纳什均衡) if, for every player i and every strategy a_i of player i, a^* is at least as good for player i as the strategy profile (a_i, a^*_{-i}) in which player i chooses a_i while every other player j chooses a^*_j.
- In other words: $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ for every strategy a_i of every player i.
- In plain English: no one can do better by unilaterally deviating from the strategy profile.
- A Nash equilibrium is a **steady state**. It embodies a stable "social norm": if everyone else sticks to it, no one has incentive to deviate from it.

Prisoner's dilemma (囚徒困境)

- What's the Nash equilibrium in PD?

Suspect 2

Suspect 1		silent	confess
	silent	$0, 0$	$-2, 1$
	confess	$1, -2$	$-1, -1$

- Only the strategy profile (confess, confess) is a NE.
- In PD each player has an **dominant strategy** (恒优策略): a strategy that is better for a player regardless of what other players do.

Prisoner's dilemma cont.

- Tragedy of the PD game: there is an outcome that is better for both players, but they just cannot achieve it.
- Would communication between the two players help them?
  - Watch a real game: http://www.youtube.com/watch?v=p3Uos2fzIJ0&feature=player_embedded
- Applications: tragedy of commons; arms race

Battle of sexes (两性之战)

- He wants to watch soccer, she wants to watch ballet, but they would rather be together than separate.

She

He		soccer	ballet
	soccer	2, 1	0, 0
	ballet	0, 0	1, 2

- What are the Nash equilibria?
- 2 Nash equilibria: (soccer, soccer); (ballet, ballet)
- BoS models situations in which two parties want to cooperate but differ on which point to cooperate.

Matching pennies (翻硬币)

- A purely conflictual game (PD and BoS have elements of cooperation)

Player 2

Player 1		head	tail
	head	$1, -1$	$-1, 1$
	tail	$-1, 1$	$1, -1$

- Player 1 wants to take the same action as player 2, but player 2 wants to take the opposite action.
- Any (pure-strategy) Nash equilibrium?

  $\Rightarrow$ No.

Stag hunt (猎鹿博弈)

- Two hunters can succeed in catching a stag if they all exert efforts, but each can catch a hare alone.

Hunter 2

Hunter 1		stag	hare
	stag	2, 2	0, 1
	hare	1, 0	1, 1

- What are the Nash equilibria?
  - $\Rightarrow$ (stag, stag) and (hare, hare)
- Application: cooperative project; security dilemma

The chicken game (hawk-dove) (斗鸡博弈)

- Two drivers drive towards each other on a single lane. If neither swerves, they crash and may die; if one swerves while the other does not, the one who swerves loses face while the other gains respect.

		Driver 2	
		straight	swerve
Driver 1	straight	$-10, -10$	$1, -1$
	swerve	$-1, 1$	$0, 0$

- Application: brinkmanship
- Reducing options in a chicken game: throwing away the steering wheel? Burning the bridge after crossing the river?

Coordination and the focal point

- A coordination game: choosing a restaurant

She

He		Italian	Japanese
	Italian	1, 1	0, 0
	Japanese	0, 0	1, 1

- NE: (Italian, Italian); (Japanese, Japanese)
- **Focal point**: in some real-life situations players may be able to coordinate on a particular equilibrium in a multiple equilibria game, by using information that is abstracted away from the strategic form.
  - ▷ Schelling's experiment about meeting in New York

Public good provision

- Osborne (2004) exercise 33.1: Each of n people chooses whether to contribute a fixed amount toward the provision of a public good. The good is provided iff at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contribution are not refunded. Each person ranks outcomes from best to worst as follows: (a) any outcome in which the good is provided and she does not contribute; (b) any outcome in which the good is provided and she contributes; (c) any outcome in which the good is not provided and she does not contribute; (d) any outcome in which the good is not provided and she contributes. Formulate this situation as a strategic game and find the NE.

Public good provision: strategic form

- Players: the n people
- Actions: each player's set of action is contribute, not contribute
- Preferences: $U_i(a) > U_i(b) > U_i(c) > U_i(d)$

Public good provision: NE

- Is there a NE in which more than k people contribute? One in which k people contribute? One in which fewer than k contribute?
- NE: k people contribute; none contributes

Strict and non-strict equilibria

- If an action profile a^* is a NE, then $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ for every action a_i of every player i.
- An equilibrium is **strict** if each player's equilibrium action is **better** than all her other actions. Or, $u_i(a^*) > u_i(a_i, a^*_{-i})$ for every action $a_i \neq a^*_i$ of player i.
- A variant of the prisoner's dilemma game

Player 2

Player 1		split	steal
	split	5, 5	0, 10
	steal	10, 0	0, 0

- How many Nash equilibria? Any strict NE?

  $\Rightarrow$ 3 and 0.