GRADE 7 COURSE OUTLINE

CONTENT AREA	FORMATIVE ASSESSMENT LESSONS	# OF DAYS
7.0 Introduction to this course: Mathematical Investigations		5
7.1 Analyze proportional relationships and use them to solve real-world and mathematical problems 7.RP.1, 2, 3	Developing a Sense of Scale	10
7.2 Critical Area 1: 7.RP.3: Critical Area 1: Developing understanding of and applying proportional relationships		5
7.3 Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers 7.NS.1, 2, 3	Positive and Negative Numbers in Context	10
7.4 Complex problem solving: 7.RP.3 & 7.NS.3		10
7.5 Use properties of operations to generate equivalent expressions 7.EE.1, 2, 3		10
7.6 Solve real-life and mathematical problems using numerical and algebraic equations 7.EE.3, 4	Steps to Solving Equations	10
7.7 Critical Areas 1 & 2: 7.RP.3; 7.NS.3; 7.EE.3,4 Critical Area 2: Developing understanding of operations with rational numbers and working with expressions and linear equations	Increasing and Decreasing Quantities by a Percent	14
7.8 Draw, construct, and describe geometrical figures and describe the relationships between them 7.G.1, 2, 3		10
7.9 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume 7.G.4, 5, 6	Using Dimensions: Designing a Sports Bag Maximizing Area: Golden Rectangles	10
7.10 Critical Areas 1, 2, & 3: 7.RP.3; 7.NS.3; 7.EE.3, 4; 7.G.6 Critical Area 3: Solving problems involving scale drawing and informal geometric constructions, and working with two-and three-dimensional shapes to solve problems involving area, surface area, and volume	Applying Angle Theorems Estimating and Approximating: The Money Munchers	17
7.11 Use random sampling to draw inferences about a population 7.SP.1, 2 7.SP Draw informal comparative inferences about two populations 7.SP.3, 4		5
7.12 Investigate chance processes and develop, use, and evaluate probability models 7.SP.5, 6, 7, 8		5
7.13 Critical Areas 1, 2, 3, & 4: 7.RP.3; 7.NS.3; 7.EE.3 & 4; 7.G.6; 7.SP Critical Area 4:Drawing inferences about samples based on samples	Estimating: Counting Trees Evaluating Statements about Probability	23
Total Days		144

MIDDLE GRADES OVERVIEW

Green indicates a Smarter Balance Assessment Consortium *major* cluster, and yellow indicates an *additional or supporting* cluster.

GRADE 6	GRADE 7	GRADE 8
6.RP Understand ratio concepts and use ratio reasoning to solve problems.	7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.	8.EE Understand the connections between proportional relationships, lines, and linear equations.
6.EE Represent and analyze quantitative relationships between dependent and independent variables.		8.F Define, evaluate, and compare functions. 8.F Use functions to model relationships between quantities.
6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.	7.EE Use properties of operations to generate equivalent expressions.	8.EE Work with radicals and integer exponents.
6.EE Reason about and solve one-variable equations and inequalities.	7.EE Solve real-life and mathematical problems using numerical and algebraic equations.	8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.
6. NS Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	7. NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	8.NS Know that there are numbers that are not rational, and approximate them by rational numbers.
6.NS Compute fluently with multi-digit numbers and find common factors and multiples.		
6.G Solve real-world and mathematical problems involving area, surface area, and volume.	7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G Draw, construct, and describe geometrical figures and describe the relationships between them.	8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
		8.G Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G Understand and apply the Pythagorean theorem.
6.SP Develop understanding of statistical variability. 6.SP Summarize and describe distributions.	7.SP Use random sampling to draw inferences about a population. 7.SP Draw informal comparative inferences about two populations. 7.SP Investigate chance processes and develop, use, and evaluate probability models.	8.SP Investigate patterns of association in bivariate data.
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

CCSS TOOLS: PROGRESSIONS

Source: http://ime.math.arizona.edu/progressions/

GRADE 6	GRADE 7	GRADE 8
6.RP Understand ratio concepts and use ratio reasoning to solve problems.	7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.	8.EE Understand the connections between proportional relationships, lines, and linear equations.
6.EE Represent and analyze quantitative relationships between dependent and independent variables.		8.F Define, evaluate, and compare functions. 8.F Use functions to model relationships between quantities.
6.EE Apply and extend previous understandings of arithmetic to algebraic expressions. 6.EE Reason about and solve one-variable equations and inequalities.	7.EE Use properties of operations to generate equivalent expressions. 7.EE Solve real-life and mathematical problems using numerical and algebraic equations.	8.EE Work with radicals and integer exponents. 8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.
6.NS Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions. 6.NS Compute fluently with multi-digit numbers and find common factors and multiples.	7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	8. NS Know that there are numbers that are not rational, and approximate them by rational numbers.
6.G Solve real-world and mathematical problems involving area, surface area, and volume.	7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 8.G Draw, construct, and describe geometrical figures and describe the relationships between them.	8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 8.G Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G Understand and apply the Pythagorean theorem.
6.SP Develop understanding of statistical variability. 6.SP Summarize and describe distributions.	7.SP Use random sampling to draw inferences about a population. 7.SP Draw informal comparative inferences about two populations. 7.SP Investigate chance processes and develop, use, and evaluate probability models.	8.SP Investigate patterns of association in bivariate data.

7.0 MATHEMATICAL INVESTIGATIONS
5 DAYS

This introductory unit is designed to provide students with an opportunity to engage in worthwhile mathematical investigations; it has a number of related purposes. The most important of these is to let students draw heavily on the eight Mathematical Practices and establish their importance to this entire course. Through working with the mathematical practices, students will come to understanding the importance of seeing, doing, re-constructing, and supposing, in learning mathematics, and hopefully also realize that mathematics is not just facts to be memorized.

Examples of the kinds of mathematical investigations that students will work on during this unit can be found at Henri Piciotto's Mathematics Education Page (http://www.mathedpage.org/).

One of these investigations is called "Area on Graph Paper"; it explores the least and greatest perimeters that can be made with polyominoes of a fixed area. This calls on students to make and test hypotheses, be systematic, and draw conclusions. It will encourage students to deploy MP.1 (*Make sense of problems and persevere in solving them*) and MP.7 (*Look for and make use of structure*). (http://www.MathEdPage.org/new-algebra/new-algebra.html#graph)

The McNuggets Problem, another type of mathematical investigation, which can also be found on Henri's website is particularly valuable in an introductory unit such as this. Students are invited to consider the number of chicken nuggets that might be bought if a fast food store sells them in quantities of 6, 9, and 20. Students are asked to determine how many nuggets can and cannot be bought. Students are also invited to determine what is the largest number of nuggets that cannot be bought, and then prove that every number greater than this largest number can be bought. (http://www.mathedpage.org/early-math/early.html)

Other interesting investigations can be found at Henri's Website. For example, in his paper *Operation Sense, Tool-Based Pedagogy Curricular Breath: A Proposal* he describes investigations such as *Perimeter Patterns* and *Angles* which can be enacted using Pattern Blocks
(http://www.mathedpage.org/early-math/early.html#Fun-Math)

Another purpose of this introductory unit is to establish the norms for doing mathematics, and set the scene for the rest of the course. Teachers will be encouraged to enact these investigations in a way that enables students to take responsibility for their own learning and act as instructional resources for each other. The teacher will provide feedback that moves the learning forward and engineer effective discussion so as to facilitate learning. The intent of each mathematical investigation will be made explicit so as to provide a purpose for students.

The genius of this type of investigation is that, in a heterogeneous class of students, it is highly likely that every student will be able to do something, but also highly likely that no student will be able to do everything.

7.1 RATIO AND PROPORTIONAL RELATIONSHIPS
10 DAYS

In the *CCSS* the treatment of *Ratio and Proportional Relationships* represents a fundamental departure from traditional seventh grade standards. By design, there is no mention of *solving a proportion* or of *cross-multiplication.* Instead, the emphasis is on identifying the specific nature of the *relationship* between varying quantities. The *CCSS* calls on students to learn to recognize proportional relationships, identify the constant of proportionality, represent proportional relationships with tables, graphs, and formulas, and understand the unit rates contained in proportional relationships.

Ratios and Proportional Relationships 7.RP
Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities.

2a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

2b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

2c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

2d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.

3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

This broader treatment forges a conceptual link among ratio, proportional relationships, and functions. Students will learn that in any proportional relationship the ratio between the varying quantities is constant. When such a ratio is constant, the quantities are in direct proportion to each other. In a proportional relationship, when one quantity is multiplied by a certain factor, the other quantity will be multiplied by that same factor. The constant, which appears in division as the constant ratio and in multiplication as the constant multiple, is called a constant of proportionality. A useful form for a constant of proportionality is the unit rate.

This understanding of proportional relationships lays a firm foundation for learning about functions, particularly linear functions. Students will learn that proportional relationships can be represented by $y=kx$, where k is a constant of proportionality or unit rate. In a graph this constant shows up as the slope. Students will recognize that graphs of proportional relationships are linear and pass through the origin.

FORMATIVE ASSESSMENT LESSONS

Developing a Sense of Scale

This is a lesson about proportional reasoning. Students need to scale up or down quantities in different contexts: amounts of ingredients in recipes, lengths of enlargements of pictures, and price of cans of paint of different sizes. Although students can set up proportions to solve the problems, this method is inefficient, and the objective is to understand the multiplicative nature of these situations and the meaning and usefulness of the unit rate or constant of proportionality in each context. Students must especially distinguish between multiplicative and additive relationships in one of the given sample solutions they are asked to analyze. They must also realize that multiplying by a number with absolute value less than 1 results in a smaller (absolute value) number.

Students need to work in small groups, compare and improve solutions, explain and critique each other's work, and analyze given sample solutions.

TEACHING AND ASSESSMENT RESOURCES

H. Hironaka & Y. Sugiyama. *Mathematics for Elementary School, 6A*. Tokyo: Tokyo Shoseki, 2006. (ISBN 4-487-46620-2; distributed by Global Education Resources, www.globaledresources.com.)
- Chapter 7: Proportional Relationships
  - Section 1, pp. 70–71: Identifying the Relationship Between Varying Quantities
  - Section 2, pp. 72–74: Properties of Proportional Relationships
  - Section 3, pp. 75–78: Graphs of Proportional Relationships
  - Practice: p. 79
- (Chapter 8 of the same text deals with inversely proportional relationships; some teachers may find this chapter useful and appropriate.)

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)
- Chapter 5: Functions and Proportional Relationships
  - Section 1, pp. 102–104: Proportional Relationships
  - Section 3, pp. 108–110: Coordinates
  - Section 4, pp. 111–114: Graphing Proportional Relationships
  - Chapter exercises: pp. 115–116

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-865-0)
- Unit 2: Equations, Inequalities, and Linear Relationships
  - Activity 2.3, pp. 109–118: Slope / Rate of Change / Equivalent Ratios / Multiple Representations
  - Activity 2.6, pp. 133–140: Direct Variation
- Unit 3: Two-Dimensional Geometry and Similarity
  - Activity 3.6, pp. 193–198: Ratio / Rate / Unit Rate / Proportions

Novice Tasks	Apprentice Tasks	Expert Tasks
N01: Short Tasks—Ratios and Proportional Relationships	A01: 25% sale A08: Buses	

7.2 CRITICAL AREA 1
5 DAYS

The purpose of this phase of the mathematics course is to encourage the consolidation of the mathematics in 7.RP. In the seventh-grade *CCSS*, Critical Area 1 is characterized entirely by the mathematics of 7.RP, ratio and proportional relationships are therefore a priority for deep learning in the seventh grade.

Critical Area 1
Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

TEACHING AND ASSESSMENT RESOURCES

H. Hironaka & Y. Sugiyama. *Mathematics for Elementary School, 6A*. Tokyo: Tokyo Shoseki, 2006. (ISBN 4-487-46620-2; distributed by Global Education Resources, www.globaledresources.com.)
Chapter 7: Proportional Relationships
Section 4, pp. 80–82: Using Proportional Relationships

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Review Exercises, p. 174

SpringBoard Middle School Math 1. College Board, 2010. (ISBN 0-87447-879-2)

Unit 4: Ratio and Rates

Activity 4.5, pp. 223–232: Sales Tax / Discount / Tips / Simple Interest

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)

Unit 1: Integers and Rational Numbers
Activity 1.6, pp. 53–58: Percent Increase / Percent Decrease /Discount / Markup / Commission
Unit 3: Two-Dimensional Geometry and Similarity
Activity 3.7, pp. 199–206: Scale Drawings
Activity 3.8, pp. 207–216: Similar Figures / Scaling Problems

Novice Tasks	Apprentice Tasks	Expert Tasks
		E05: Ice Cream

7.3 THE NUMBER SYSTEM
10 DAYS

In seventh grade students consolidate their understanding of operations on signed numbers. They add, subtract, multiply, and divide rational numbers, gaining fluency in one of the parts of the middle school curriculum that is most difficult to conceptualize.

The Number System 7.NS
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

1a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

1b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

1c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

1d. Apply properties of operations as strategies to add and subtract rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

2a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

2b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real- world contexts.

2c. Apply properties of operations as strategies to multiply and divide rational numbers.

2d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

[1]Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Operations on signed numbers is particularly difficult for students because of a conflation between the symbol indicating the operation of subtraction and the symbol indicating that a number is negative. Another aspect that leads to difficulty in learning about operations on signed numbers is the proliferation of rules that students are expected to remember, too often without recourse to methods for reasoning out how the numbers behave. Collections of rules that students memorize without understanding are likely to break down under the pressure of difficult problems or after a period of disuse.

FORMATIVE ASSESSMENT LESSONS

Using Positive and Negative Numbers in Context

This lesson is about adding and subtracting integers, both positive and negative, and about translating back and forth between these arithmetic operations and the context in which they arise. The context is that of an initial temperature, a temperature change, and a final temperature. Students are given two of these and they need to find the third. They are also given a sum or difference of integers and asked to match this calculation with one of several possible stories. Finally, they are given one set of cards with names of cities, and some of them are labeled with given temperatures, and another set with "change of temperature arrows". They need to connect cities with these arrows in order to obtain triples of cards where the initial temperature plus the temperature change results in the final temperature.

Students need to be able to add and subtract integers, reason about these operations in context, and understand that the temperature change equals the final temperature minus the initial temperature (and equivalent formulations of this statement). They are asked to work in small groups, explain their reasoning, and critique each other's work.

TEACHING AND ASSESSMENT RESOURCES

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Chapter 2, pp. 27–53: Positive and Negative Numbers

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)

Unit 1: Integers and Rational Numbers

Activity 1.2, pp. 11–22: Addition/Subtraction of Integers

Activity 1.3, pp. 23–28: Multiplication/Division of Integers

Activity 1.5, pp. 41–52: Fraction/Decimal Operations

SpringBoard Middle School Math 3. College Board, 2010. (ISBN 0-87447-881-2)

Unit 1: Patterns and Numerical Relationships

Activity 1.4, pp. 31–40: Fraction Operations

Novice Tasks	Apprentice Tasks	Expert Tasks
	A11: Division	E03: A Day Out
		E11: Taxi Cabs

7.4 COMPLEX PROBLEM SOLVING
10 DAYS

Students have been working to understand proportional relationships and make sense of how such relationships behave. They have also continued to build their number sense and fluency with fractions by applying the four mathematical operations to rational numbers. Both of these concepts are vital to middle and high school mathematics and need to be strengthened through application. Students are given the opportunity to apply the understanding they have built by engaging in rich problem solving tasks that not only address the appropriate mathematical content but require development of the mathematical practices as well.

Ratios and Proportional Relationships 7.RP
Analyze proportional relationships and use them to solve real-world and mathematical problems.
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*.

The Number System 7.NS
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

[1]Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHING AND ASSESSMENT RESOURCES

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

7.5 EXPRESSIONS
10 DAYS

Learning about algebraic expressions provides an excellent opportunity for students to **put to work** all of their learning about rational numbers. For example, in kindergarten students learn that adding means **putting together**. They also learn that 3 apples can be added to 5 oranges to give 8 fruits, but that if 3 paper clips were added to 5 nails no sensible quantity will result. Thus, kindergarten students learn that only quantities with the same unit can be added. It is exactly this learning that is required when students start to add and subtract linear expressions (7.EE.1). For example, if students add 3a + 3b + 5a, it will make sense from Kindergarten learning that they can put 3a and 5a together to give 8a and that the result is 8a+ 3b. The work of kindergarten can be extended to the adding and subtracting of monomials. Unfortunately, this link with previous learning is rarely invoked. Instead, when it comes to adding and subtracting monomials students are usually taught to *combine or collect like terms*. Although we all know that when we ask students to combine or collect like terms we are asking them to add or subtract monomials, our students do not. Instead of connecting with previous learning, learning to combine like terms creates the opportunity for mistakes and misconceptions. For example, many students imagine that you cannot multiply 3a by 5b because 3a and 3b are not "like terms." We recommend that students are not taught to combine or collect like terms but instead are taught to add or subtract monomials.

Expressions and Equations 7.EE
Use properties of operations to generate equivalent expressions.
1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions.
3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

TEACHING AND ASSESSMENT RESOURCES

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Chapter 3, pp. 66–74: Operations on Algebraic Expressions

Novice Tasks	Apprentice Tasks	Expert Tasks
	A12: Fencing	

7.6 EQUATIONS
10 DAYS

In working with equations, students will learn that an equation consists of two expressions linked by an equal sign and will be able to identify examples and non-examples of equations. Students must understand that solving an equation in one unknown means finding a value of the letter that makes the left side equal to the right side. In working with equations we recommend not introducing the term *variable*. Instead, we recommend just reminding students that in mathematics letters are used to stand for numbers. Students' work with equations provides a site for deepening their learning of operations on rational numbers. For example, before the end of their work on equations students will have had the opportunity to handle equations whose coefficients are simple decimals, fractions, or negative numbers. This will provide students with the opportunity to see the relevance of their earlier work with rational numbers.

A final critical idea here is that students should know how set up and solve a linear equation in order to solve a word problem.

Expressions and Equations 7.EE
Solve real-life and mathematical problems using numerical and algebraic equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

4a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

4b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

FORMATIVE ASSESSMENT LESSONS

Steps to Solving Equations

This is a lesson about linear equations in one variable. Students need to write down equations that can be used to solve given word problems in a variety of contexts. They also need to match given equations with stories that can be modeled by them. They have to solve these equations, displaying the steps and reasons for each step in logical order, and they have to look for alternative methods of solution. The main facts they need to use are the distributive property, combining like terms, and performing the same operations to both sides of an equation. They need to understand why each step they choose makes the equation simpler. Students are

asked to work in small collaborative groups, explain their reasoning, and critique each other's work.

TEACHING AND ASSESSMENT RESOURCES

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Chapter 4: Pages 76-94: Equations

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)

Unit 2: Equations, Inequalities, and Linear Relationships

Activity 2.2, pp. 93–106: Writing and Solving Linear Equations

7.7 CRITICAL AREAS 1 AND 2
14 DAYS

Unified understanding number and expressions and equations is at the core of Critical Area 2. Approached in this way, *Expressions and Equations* becomes a site for deep learning and for application of operations on rational numbers.

> Critical Area 2
> Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

FORMATIVE ASSESSMENT LESSONS

Increasing and Decreasing Quantities by a Percent

The goal of this lesson is for students to understand increase and decrease of a quantity by a percent, and how the new amount can be obtained from the original amount by multiplying by a single number. Students are led to explore and understand the relationship between this number and the percent change. They also need to go back and forth fluently between percents, decimals, and fractions. Fractions are very useful for understanding the relationship between two consecutive percent changes (one an increase and the other a decrease) that leave the original quantity unchanged. They are led to understand that the two numbers used in the multiplications that reflect these percent changes are reciprocals of each other. They will also discover that an increase by a percent followed by a decrease (of the resulting quantity) by the same percent (or vice-versa: first a decrease and then an increase by the same percent) always results in a quantity that is smaller than the original quantity.

TEACHING AND ASSESSMENT RESOURCES

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Review Exercises, pp. 171–173

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)

Unit 2: Equations, Inequalities, and Linear Relationships
Embedded Assessment 2, pp. 125–126

7.8 GEOMETRIC FIGURES
10 DAYS

Students draw on their work with ratio and proportional relationships to solve problems involving scale drawings of geometric figures. They prepare for a study of similarity and congruence in eighth grade by focusing on constructing triangles from three measures of angle or side. Finally, they explore two-dimensional figures by analyzing the cross-sections of three-dimensional ones.

Geometry 7.G
Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three- dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

TEACHING AND ASSESSMENT RESOURCES

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)

Unit 3: Two-Dimensional Geometry and Similarity

Activity 3.7, pp. 199–206: Scale Drawings

Novice Tasks	Apprentice Tasks	Expert Tasks
	A22: Photographs	

7.9 GEOMETRIC MEASUREMENT
10 DAYS

In grade seven students extend their work on area to include finding surface area of three three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Rather than introduce formulas for surface area, we recommend that students use nets (two-dimensional representations of three-dimensional figures) to calculate surface area. Through the use of nets, eventually students can derive the various formulas for surface area and be able to reconstruct them when needed.

Geometry 7.G
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

FORMATIVE ASSESSMENT LESSONS

Using Dimensions: Designing a Sports Bag

In this lesson, students create scale models of two-dimensional figures that must be sewn together in order to produce a cylindrical sports bag with prescribed dimensions. They need to realize that they need a rectangle and two circles. They need to allow for required seam allowances or hems around each piece of fabric, and they need to realize that the length of the rectangle (before hem is added) must match the prescribed circumference of the circular end pieces (before hem is added). In order to achieve this task, students need to know the formula for the circumference of a circle, they need to use proportional reasoning when making a scale drawing, they need accuracy in measuring lengths, and they need to make reasonable rounding off of numbers, especially making sure that they round up when rounding down would lead to a problem, like the pieces not fitting well together. Students are asked to explain and critique each other's solutions as well as three given sample solutions with different flaws and strengths. Finally, another question asks for the optimal arrangement of the three necessary pieces on a strip of fabric 1 yard wide in order to be able to buy the shortest possible piece. This problem requires perseverance, reasoning, and explanation of one's arguments.

Maximizing Area: Golden Rectangles

In this lesson, students are given a scenario where they need to obtain as big a plot of land as possible by using a rope of fixed length and four stakes to make a rectangular plot. They need to realize that this problem asks for finding the rectangle of maximum area among all rectangles with a fixed perimeter. Students can find that the rectangle that maximizes the area is a square by considering several rectangles and organizing their data in a table, or they can actually prove this using the fact that $(a + b) \times (a - b)$ is less than $a \times a$ if b is positive (although neither the teacher notes nor the sample work shown mention this method). Students are then asked a second question, where they need to decide if two or more people can get more land per person

by joining their ropes together and making one big square. The answer is yes, and students can again find this result by checking a few cases, or they can realize that when you multiply the perimeter of a square by a constant, the area is multiplied by the square of that constant. Therefore, if *n* people join ropes together, the area is multiplied by the square of *n*, leaving each person with *n* times as much area as they would get on their own. This is an issue of similar figures and the behavior of length and area under scaling.

Students need to work collaboratively to create posters with their solutions. They need to analyze and critique each other's work as well as a few samples of student work.

TEACHING AND ASSESSMENT RESOURCES

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)

Unit 3: Two-Dimensional Geometry and Similarity

Activity 3.1, pp. 151–158: Angle Relationships

Activity 3.2, pp. 159–164: Area of Two-Dimensional Figures

Unit 4: Three-Dimensional Geometry

Activity 4.1, pp. 241–251: Nets / Surface Area of Three Dimensional Figures

Novice Tasks	Apprentice Tasks	Expert Tasks
	A13: Historic Bicycle	E07: Roman Mosaic
	A21: Octagon Tile	

7.10 CRITICAL AREAS 1, 2, AND 3
17 DAYS

Critical Area 3 provides an opportunity for students to consolidate their work in geometry, and make connections among the various geometry standards. Through this special focus, students can be given the opportunity to forge connections with Critical Areas 1 and 2.

Critical Area 3
Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three- dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two- dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

FORMATIVE ASSESSMENT LESSONS

Applying Angle Theorems

In this lesson, students are given problems about finding some interior or exterior angles in polygons. They need to solve these problems, and they need to think carefully about sample solutions, and criticize them. In these sample solutions, students need to recognize whether false assumptions are being made, or if there are computational errors. Students are given a sheet with relevant required terminology and facts about angles formed by lines, angles formed by a transversal to parallel lines, interior angle sum of a triangle, and interior and exterior angle sums of polygons. In order to criticize some solutions, or to come up with their own, students need to be familiar with triangle congruence theorems and basic facts about parallelograms. They also may need to add auxiliary lines in a given figure.

Estimating and Approximating: The Money Munchers

In this lesson, students need to estimate the height of 24,400 one-dollar bills stored under a bed, and also whether the bills would fit in a 14'' by 19'' by 7'' suitcase. They need to make reasonable estimates for the three dimensions of a dollar bill and for the dimensions of a bed, and use these estimates productively as they reason about the ways of stacking the money under a bed or in the given suitcase. They need to explain their chain of reasoning, which involves drawing some stacking models and proportional reasoning.

They also need to carefully examine and criticize some given sample solutions.

TEACHING AND ASSESSMENT RESOURCES

SpringBoard Middle School Math 2. College Board, 2010. (ISBN 0-87447-880-4)
Unit 4: Three-Dimensional Geometry
Activity 4.2, pp. 251–258: Relationships between Nets and Three-Dimensional Figures
Activity 4.3, pp. 259–264: Volume of Three-Dimensional Figures

7.11 STATISTICS
5 DAYS

It is important in this unit for students to begin to think **statistically** about the world around them. They need to understand what is meant by the word *random* in mathematics. Many will define this word as "unexpected" or "out of nowhere" as it is used in their everyday vocabulary. The teacher should spend time developing the mathematical concept of *random* as well as the idea of **random sampling,** emphasizing that a large enough random sample from a population will produce a **representative** sample. They can then use that sample to make inferences about the world around them. They should then be able to informally compare two data sets. The key word is **informally**. Do not get bogged down in arithmetic procedures, but rather focus on conceptual ideas of *center*, *shape*, *spread*, and *unusual features*. For example, "This class was more consistent on the test, because their grades are less spread out. Therefore, it will have the lower mean absolute deviation". However, do not neglect the arithmetic, as it is a valuable skill in the student's toolbox.

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

TEACHING AND ASSESSMENT RESOURCES

Activity-Based Statistics

"Random Rectangles"

A Random Sampling Applet from W.H. Freeman:
http://bcs.whfreeman.com/scc/content/cat_040/spt/randomsample/randomsample.html

A Comparing Samples activity for the TI-Inspire:
http://education.ti.com/calculators/timathnspired/US/Activities/Detail?sa=1008&t=1135&id=17255

Novice Tasks	Apprentice Tasks	Expert Tasks
	A26: Temperatures	E12: Candy Bars

7.12 PROBABILITY
5 DAYS

In order for students to be successful, it is critical that they build a strong foundation of conceptual understanding. Students need to develop an understanding of probability as the long-term frequency of successes by performing simulations. People often misunderstand the idea of long-term frequency, assuming that if one event happens many times in succession, the other event is "due" to occur. Supplement a strong conceptual understanding with the rules of probability, and the understanding of how the likelihood of an event occurring grows as the probability approaches 1.

Statistics and Probability 7.SP
Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

7b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

8a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

8b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

8c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Students should also be able to recognize that probability can be expressed in many forms, including fraction, decimal, and percentage. It is important that students develop the probability models, especially with events that are not equally likely. Students should also be able to model probability problems by using diagrams and charts. By creating tree diagrams, students are making a visual representation that paves the way for the multiplication rule for probability.

TEACHING AND ASSESSMENT RESOURCES

A long-run probability simulator for flipping coins from W.H. Freeman:
http://bcs.whfreeman.com/ips4e/cat_010/applets/Probability.html

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Unit 5: Data and Probability

Activity 5.1, pp. 321–328 Probability Experiment / Likelihood

Activity 5.2: pp. 329–334 Probability of Compound Events

Novice Tasks	Apprentice Tasks	Expert Tasks
	A10: Card Game	E06: Lottery
		E09: Spinner Bingo

7.13 CRITICAL AREAS 1, 2, 3, AND 4
23 DAYS

Applying their work in sampling thus far is a valuable experience for students as they grow in their statistical thinking. Taking samples, comparing data, and making inferences is at the very heart of statistics theory and practice. By mastering these skills, students become more mathematically literate and prepared to when evaluating the vast quantities of statistical evidence that they will encounter the real world.

> Critical Area 4
> Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Students solve mathematics problems that cut across Critical Areas 1, 2, 3, & 4, and make cross-cutting connections among the four critical area. This section of the course serves many purposes. Its most important function is to give students the opportunity to internalize or to make their own the mathematics of seventh grade. Another function of this section is to enable teachers to engage in further formative assessment of their students and determine how their students are understanding or internalizing the mathematics of the seventh-grade course, and in particular that of the four critical areas.

FORMATIVE ASSESSMENT LESSONS

Estimating: Counting Trees

In this lesson, students are given a diagram of a tree farm having old and new trees. They need to estimate the number of each type. The tree farm has the shape of a square, with 50 rows and 50 columns of trees, where there are also some gaps. The number of old trees is significantly larger than the number of new trees. In order to solve this problem, students must be able to select a reasonable sampling method and to use proportional reasoning to scale up their sample to make estimates about the entire tree farm. For the sampling, they need to realize that they should choose at least two areas of equal size and preferably not too close together. They should also take into account the gaps with no trees. They have to understand that their computations yield estimates, and not the exact number of trees. So, for example, it would be advisable to round answers to the nearest ten. They are also asked to explain their assumptions, choice of sampling method, and computations involving proportional reasoning. They are asked to interpret and critique each other's solutions and three given sample solutions, paying attention to any flaws in the assumptions made (for example, assuming an equal number of each type of tree), or in the sampling method.

Evaluating Statements About Probability

This is a lesson about probability of events with a finite number of possible outcomes. Students are asked to evaluate many different statements about the probability of certain events, and they need to compute a few probabilities. For the latter, students need to be able to count and

organize all possible outcomes, count the favorable outcomes, decide whether or not the outcomes all have equal probability, and form the correct fraction or use proportional reasoning to find the probability.

The lesson specifically targets several typical misconceptions about the probability of events. Some of these misconceptions, which students need to unmask and refute, are that previous events somehow can affect later events in contexts where the events are in fact independent; that all possible outcomes are equally likely in contexts where they are not; that some events that seem special are less likely than others that appear more representative in contexts where all outcomes are equally likely; or that probabilities give the proportion of outcomes that must actually occur in an experiment.

TEACHING AND ASSESSMENT RESOURCES

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/