Quality Technology &
Quantitative Management
Vol. 9, No. 2, pp. 189-202, 2012

QTQM

An Enhanced Control Chart for Start-Up Processes and Short Runs

Giovanna Capizzi and Guido Masarotto

Department of Statistical Sciences, University of Padua, Italy
(*Received* June 2010, *accepted* October 2010)

Abstract: Classic charting procedures are usually designed assuming that process parameters are known or may be estimated using large Phase I samples gathered before a production run. However, in some manufacturing settings, such as during the process start-up, historical data cannot be collected to accurately estimate the in-control process parameters. In this article, we suggest a new self-starting control chart which uses consecutive observations to jointly update the parameter estimates and check for out-of-control conditions. In particular, we introduce a charting procedure, ACUSCORE, that updates the reference pattern of a type-CUSCORE chart using an adaptive EWMA. The proposed control chart seems to outperform traditional self-starting control charts which neglect the dynamic pattern of the mean change.

Keywords: Adaptive EWMA, CUSCORE, *Q* charts, quality control, recursive residuals.

1. Introduction

Traditional control charts are usually designed assuming that the in-control (IC) process parameters are exactly known. In most practical applications, however, the process parameters are unknown and are replaced with estimates from an IC Phase I sample. Previous research work (see Jensen *et al*. [11] for a systematic review) has demonstrated that parameter estimation substantially degrades the run-length performance of classical control charts. At least in the case of the Shewhart [18], EWMA [14] and CUSUM[15] control charts, using estimates in place of known parameters causes too many false alarms unless the Phase I samples are sufficiently large to accurately estimate the unknown parameters. However, gathering large calibration samples is usually costly and problematic and it may be even impossible in some settings such as during the start-up stages where data are not available before a production run.

In this context, three alternative approaches can be used to address the design issue:

i) modifying decision intervals: control limits of classical control charts are widened to account for the variability in the sampling distribution of the estimates; see for example Jones [13] and Capizzi and Masarotto [3] for independent and autocorrelated data, respectively;

ii) formulating a proper "change-point" model as suggested in Hawkins *et al.* [8], Hawkins and Zamba [9], Hawkins and Zamba [10]: a chart, based on the likelihood ratio test, is designed without assuming that the in-control process parameters are known;

iii) using "self-starting" control charts (Hawkins [5], Quesenberry [17], Quesenberry [19], Sullivan and Jones [24], Zantek [26], Hawkins and Maboudou-Tchao [6]).

Following the last two approaches, since the very beginning of the production run, the parameter estimates are updated with each new observation and out-of-control (OC) conditions are simultaneously checked.

It should be noted that the first approach has the disadvantage of yielding schemes with unpredictable run length properties. Indeed, the IC average run length, depending on the parameter estimates, is a random variable with its own distribution. On the other hand, the change-point formulation may be computationally burdensome.

The "self-starting" approach, conversely, seems to be able to identify shifts occurring in the early history of the process even maintaining the IC run length (RL) properties at the specified values. This approach sequentially transforms the consecutive readings from an unknown-parameter process to a sequence of independent Q statistics having a completely known distribution when the process is in-control. Thus, traditional control charts can be applied to these transformed values. Observe that, however, when the original process parameters shift to new stable levels (sustained changes), the effect on the transformed data does not last long. Since control charts that neglect this phenomenon may show a very poor out-control performance in detecting small shifts in the process mean, we here investigate an alternative self-starting chart explicitly accounting for dynamic patterns in the mean shift.

Practitioners should note that the new chart provides a relatively simple procedure which largely outperforms the previously suggested self-starting schemes. On the other hand, it exhibits some limitations. In particular,

- it only tries to detect a shift from an initial level. Incidentally, this advise also applies to the change-point approach;
- it is designed for detecting persistent step changes. Thus, performance might be unsatisfactory in other change points scenarios, such as in the presence of time varying changes and in particular when a gradual degradation occurs near the beginning of the monitoring phase.

Both these limitations, that are intrinsic to the self-starting methodology itself, arise due to a lack of any knowledge about the true IC process level when the monitoring process starts.

The remainder of the paper is organized as follows. In the case of a sustained change in the process mean of a normal distribution, Section 2 presents the corresponding Q statistics and some standard monitoring schemes based on these transformed values, such as the CUSUM and the EWMA. Section 3 introduces a new charting procedure that updates the reference pattern of a type-CUSCORE chart using an adaptive EWMA. The design of the proposed control chart is discussed in Section 4 and a practical example is presented in Section 5. Comparisons with other control charts of Q statistics are given in Section 6.

2. Q-Transformation and Related Control Charts

Assume that, under control, the process measurements $\{x_t\}, t = 1, 2, \ldots,$ are independent normal random variables with mean μ and variance σ^2, that is $N(\mu, \sigma^2)$. From some time period τ, the process mean changes from μ to $\mu + \delta\sigma$. Process parameters, time and magnitude of the mean shift are all unknown.

Let m_t and s_t^2 respectively denote the sample mean and sample variance of the first t observations. Their values can be computed, as each value x_t, for $t \geq 3$, is gathered, using the following updating formulas

$$\begin{cases} a_t = x_t - m_{t-1}, \\ m_t = m_{t-1} + \dfrac{1}{t} a_t, \\ s_t^2 = s_{t-1}^2 + \dfrac{1}{t}\left[a_t^2 - \dfrac{t}{t-1} s_{t-1}^2 \right], \end{cases} \tag{1}$$

with $m_2 = (x_1 + x_2)/2$ and $s_2^2 = (x_1 - x_2)^2 / 2.$

The statistics

$$T_t = \sqrt{\frac{t-1}{t}} \frac{a_t}{s_{t-1}}, \tag{2}$$

for testing the null hypothesis that the two samples x_t and $(x_1, \ldots, x_{t-1})$ come from distributions with the same unknown mean μ, are statistically independent and follow a Student t distribution with $t-2$ degrees of freedom. Given the number of degrees of freedom is varying from one observation to the next, the T_t are not identically distributed. However, let $G_\nu(\cdot)$ and $\phi^{-1}(\cdot)$ denote the cumulative distribution function (cdf) of a Student t random variable, with ν degrees of freedom, and the inverse standard normal cdf, respectively. Then, Hawkins [5] and Quesenberry [17] use the following transformation

$$Q_t = \phi^{-1}[G_{t-2}(T_t)], \quad t \geq 3. \tag{3}$$

Since, under control, the Q_t statistics are independent and identically distributed random variables from a $N(0,1)$ distribution, traditional control charts based on Q_t can be used for detecting a shift in the process mean.

In particular, a self-starting CUSUM (Hawkins [5], Hawkins and Olwell [7], Zantek [26]) of Q_t is defined, for $t \geq 3$, by

$$\begin{cases} C_t^L = \min\{0, C_{t-1}^L + Q_t + k\}, \\ C_t^U = \max\{0, C_{t-1}^U + Q_t - k\}, \end{cases} \tag{4}$$

with $C_2^L = C_2^U = 0$ and k denoting the CUSUM reference value. Given a suitable value of the control limit h, the control chart (4) signals when either $C_t^L < -h$ or $C_t^U > h.$

Correspondingly, a self-starting EWMA is given by

$$Z_t = Z_{t-1} + \lambda(Q_t - Z_{t-1}), \tag{5}$$

where $Z_2 = 0$ and $0 < \lambda \leq 1$. The EWMA of Q_t signals when $|Z_t| > h\sqrt{\lambda/(2-\lambda)}$.

3. An Adaptive CUSCORE Control Chart

Observe that traditional control schemes, such as the CUSUM and EWMA charts, are designed assuming that the step mean shift in the original series $\{x_t\}$ results in a sustained mean shift in the transformed series Q_t. However, after the change point, the mean of Q_t is a time dependent function. In particular, it is possible to show that the pattern of the mean change is substantially affected by both the magnitude δ and the time τ of the shift. For example, Figure 1 shows that, when a shift in the process mean occurs, there is an immediate and rapid increase in the mean value of Q_t; this expected value, however, gets closer to zero as more shifted observations are used to update the sample mean estimate, i.e. as t increases. Thus, a control chart which fails to signal within a few observations following the shift, that is inside a ''window of opportunity'', can result in poor out-of-control run length performances.

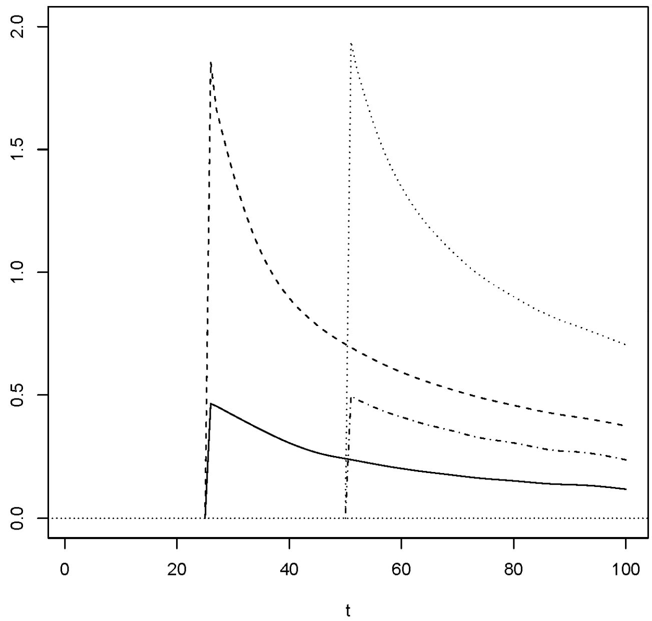

Figure 1. Mean values of Q_t when a mean shift of size $\delta\sigma$ occurs at τ. The four curves correspond to: (i) $\tau = 26, \delta = 0.5$ (solid); (ii) $\tau = 26, \delta = 2$ (dashed); (iii) $\tau = 26, \delta = 0.5$ (dotted and dashed); (iv) $\tau = 26, \delta = 0.5$ (dotted).

Thus, it would be desirable to design a self-starting control chart using not only the Q_t's level but also the fault signature of the mean shift, and in particular the information contained in the correlation between Q_t and its expected value.

When the fault signature is known, this issue may be addressed using a CUSCORE control chart (Box and Ramirez [1]). Unfortunately, the expected value of Q_t depends on the unknown magnitude and time of the change. Thus, starting from $t = 3$, we here suggest to

(a) estimate the current mean level of Q_t using the following adaptive EWMA (AEWMA)

$$f_1 = (1 - w_1) f_{t-1} + w_t Q_t, \tag{6}$$

where $f_2 = 0$ and w_t is the weight function given by

$$w_1 = \begin{cases} \lambda, & if\ |Q_t - f_{t-1}| \le \gamma, \\ 1 - (1-\lambda)\dfrac{\gamma}{|Q_t - f_{t-1}|}, & if\ |Q_t - f_{t-1}| \le \gamma, \end{cases} \tag{7}$$

with $\gamma \ge 0$ and $0 < \lambda \le 1$.

Such an estimate of the current process mean level has been introduced by Yashchin [25] and it has been used in the SPC framework to implement an adaptive version of both the EWMA and CUSUM control chart (see Capizzi and Masarotto [2], Shu [21] and Jiang *et al.* [12]). For a better understanding of the weight function (7) readers can refer to these references and, in particular, to the related plot in Capizzi and Masarotto [2].

(b) monitor the process using the following CUSCORE-type control chart

$$\begin{cases} AC_t^L = \min\{0, AC_{t-1}^L + |f_t| \cdot (Q_t + |f_t|/2)\}, \\ AC_t^U = \max\{0, AC_{t-1}^U + |f_t| \cdot (Q_t - |f_t|/2)\}, \end{cases} \tag{8}$$

with $AC_2^L = AC_2^U = 0$ and f_t given by (6).

The control chart (8) signals when either $AC_t^L < -h$ or $AC_t^U > h.$ In the following, we will refer to (8) as the Adaptive CUSCORE (ACUSCORE) control chart.

As soon as two observations are available, the self-starting scheme (8) makes use of each new observation to calculate the statistics T_t and Q_t via equations (2) and (3); if the ACUSCORE does not signal, the current observation is then used to update the sample mean m_t and the variance s_t^2 via equation (1). Sampling and updating of parameter estimates continue until the hypothesis that the process is in control is rejected.

The control statistics AC_t^L and AC_t^U are similar to a two sided CUSUM. The difference is that detection is based on the cumulative sum of the product between $|f_t|$ and x_t and not on the cumulative sum of x_t itself. Given this similarity, post-signal interpretation of the ACUSCORE control statistics can be done as in the CUSUM case. In particular,

1. the shift direction can be determined from which of the two control statistics AC_t^L and AC_t^U hit the control limit;
2. the time of the change-point can be estimated using the last time the control statistic, responsible for the signal, has been equal to zero.

Observe that, an adaptive approach has not been previously discussed for the self-starting charts. However, control chart (8) is far from being the first proposal for schemes adaptively updating the reference value of type-CUSCORE and CUSUM charts or accounting for a dynamic pattern of the mean change. A similar idea has been used, for example, by Sparks [23] and Jiang *et al.* [12] to implement an adaptive version of the CUSUM control chart (ACUSUM), by Han and Tsung [4] to design a Reference-Free CUSCORE (RFCUSCORE) and by Shu *et al.* [22] to develop a weighted CUSUM chart (WCUSUM). With regard to these proposals, it should be noted that

i) the ACUSUM has been suggested for detecting a persistent change in the process mean. Thus, it may result in poor performances when the mean of the monitoring stream has a dynamic pattern as in the considered framework (see Shu *et al.* [22]);

ii) the RFCUSCORE is a special case of ACUSCORE chart when $|f_t|$ in (8) is replaced with $|Q_t|$, i.e., when λ and γ are equal to zero in equation (7). However, given the behavior of the mean response in the self-starting situation, we found that using a smooth estimate of the fault signature can lead to an improved performance;

iii) the WCUSUM is quite similar to our proposal and, even if comparisons with this chart are not presented in this paper, its performance is essentially equivalent to that of the ACUSCORE chart when an adaptive EWMA, in place of a standard EWMA used in Shu *et al*. [22], is chosen to update the weighting function f_t.

4. Design of the ACUSCORE Control Chart

The design of the ACUSCORE chart involves the choice of three parameters λ, γ and h.

We performed an extensive simulation study to understand the effect of λ and γ on the ACUSCORE performance. In particular, for various values of λ, γ, δ and τ, we studied the expected delays in detecting a change of size δ occurring at observation τ, i.e.

$$d_\delta = E_{OC}(RL - \tau - 1 \mid RL \geq \tau). \tag{9}$$

Results can be summarized as follows.

i) When $\gamma < 6$, the best detection power is obtained for values of λ in the interval [0.05, 0.25]. Different choices of λ inside this interval seem to have a relatively small effect on the detection power. On the other end, larger values of λ reduce the performance against small shifts. See, for example, Table 1 which illustrates the effect of λ when $\gamma = 3$.

ii) For every value of λ, small γ 's perform better against large mean shifts while large γ 's are better against small shifts. A good compromise can be obtained choosing γ in the interval [2.5, 3.5] See, for example, Table 2.

Table 1. Expected delays for various values of $\lambda, \gamma = 3$ and $\tau = 51$. For each scheme, the critical limit has been computed so that $Pr(RL < \tau) = 0.1$. Each entries is based on 100000 Monte Carlo replications.

	λ								
δ	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60
0.5	35.77	35.06	35.78	37.16	38.77	40.69	45.26	51.45	59.52
1.0	13.78	13.14	13.13	13.28	13.43	13.57	13.73	13.82	13.87
1.5	8.22	7.64	7.51	7.53	7.59	7.66	7.78	7.82	7.79
2.0	5.70	5.27	5.12	5.08	5.08	5.10	5.15	5.17	5.16
3.0	3.01	2.93	2.88	2.86	2.84	2.84	2.83	2.82	2.81
4.0	1.70	1.77	1.81	1.83	1.85	1.86	1.86	1.86	1.85
5.0	1.18	1.24	1.28	1.32	1.34	1.36	1.38	1.38	1.38
6.0	1.03	1.05	1.07	1.08	1.10	1.11	1.12	1.13	1.12

Table 2. Expected delays for various values of $\gamma, \lambda = 0.15$ and $\tau = 51$. For each scheme, the critical limit has been computed so that $\Pr(RL < \tau) = 0.1$. Each entries is based on 100000 Monte Carlo replications.

	γ								
δ	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.5	64.72	53.63	42.82	39.92	35.78	34.87	34.69	34.69	34.69
1.0	14.59	14.57	13.85	13.59	13.13	13.03	13.00	13.00	13.01
1.5	7.81	7.55	7.42	7.60	7.51	7.56	7.59	7.59	7.59
2.0	4.98	4.69	4.72	4.97	5.12	5.24	5.30	5.30	5.30
3.0	2.63	2.42	2.43	2.68	2.88	3.10	3.30	3.30	3.33
4.0	1.72	1.58	1.55	1.73	1.81	2.03	2.37	2.37	2.44
5.0	1.29	1.20	1.17	1.27	1.28	1.41	1.76	1.76	1.88
6.0	1.08	1.04	1.03	1.07	1.07	1.12	1.33	1.33	1.47

Given these results, we recommend to set $\lambda = 0.15$ and $\gamma = 3$ since this choice provides good performance in a variety of OC conditions.

Hence, ACUSCORE design is reduced to the relatively easier selection of the control limits h which will be discussed later. It is interesting to note that this practical advantage of the suggested scheme is due to the use of an AEWMA statistic to estimate the fault signature. Indeed, if the adaptive EWMA is replaced by a standard EWMA, i.e., if we estimate the fault signature using

$$f_t = \omega f_{t-1} + (1-\omega)Q_t, \tag{10}$$

with ω fixed, the performance of the resulting scheme crucially depends on the smoothing parameter ω. This point is illustrated in Figure 2 which shows the expected delays of three ACUSCORE schemes based on the standard EWMA (10). Note that smaller values of ω lead to a quicker detection of smaller shifts, while larger values of ω produce better performance for larger shifts. Figure 2 also lets us to compare the three standard EWMA-based schemes with the suggested AEWMA-based chart. Observe that the expected delay of the ACUSCORE based on an adaptive EWMA is either the shortest or near the shortest for every value of the mean shift. Hence, no standard EWMA-based scheme offers a comparable protection against both small and large shifts.

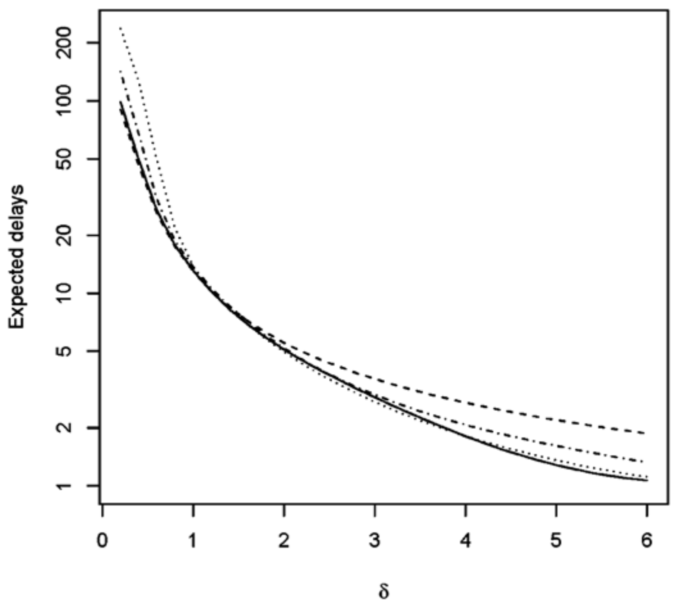

Figure 2. Expected delays when $\tau = 51$. The four curves correspond to: (i) the suggested ACUSCORE chart based on the AEWMA estimate of the fault signature ($\lambda = 0.15, \gamma = 3$; solid line); (ii) a CUSCORE based on a standard EWMA with $\omega = 0.1$ (dashed); (iii) a CUSCORE based on a standard EWMA with $\omega = 0.4$ (dotted and dashed); (iv) a CUSCORE based on a standard EWMA with $\omega = 0.7$ (dotted). Critical limits of the four schemes have been computed so that $\Pr(RL < \tau)$=0.1. Expected delays have been estimated using 100000 Monte Carlo replications.

In choosing the decision interval h, practitioners may need to balance the control chart performances in the in and out-of-control conditions. Table 3, in particular, shows the

ACUSCORE control limits able to achieve a desired value of the in-control ARL , ARL_0. Then, for each value of the control limit h, Table 3 also shows the probability of giving a false alarm within the first γ observations,

$$p_r = P_{IC}(RL \le r),$$

and the expected delay in detecting a change of size δ occurred at observation τ. Since the analytical computation of the run length distribution seems difficult, Table 3 has been obtained by simulations. We found that an efficient approach for the computation of h consists in using the Polyak-Ruppert stochastic approximation algorithm [16, 20]. In particular, our implementation is based on the recommendations given by Capizzi and Masarotto [3].

Table 3. Adaptive CUSCORE decision intervals and corresponding in-control and out-of-control ($\tau = 51$) run length characteristics. All quantities have been estimated using 100000 Monte Carlo replications.

	h					
	2.698	4.196	6.033	7.970	8.977	11.558
			In-control			
ARL_0	50.000	100.000	200.000	370.400	500.000	1000.000
p_{25}	0.241	0.058	0.009	0.002	0.000	0.000
p_{50}	0.614	0.256	0.071	0.015	0.006	0.001
p_{100}	0.917	0.617	0.287	0.107	0.061	0.012
p_{200}	0.996	0.908	0.628	0.345	0.241	0.086
			Out-of-control			
$d_{0.25}$	24.827	49.888	101.704	198.155	271.731	607.465
$d_{0.50}$	14.598	24.288	40.284	66.433	86.364	172.486
$d_{0.75}$	9.417	14.244	21.664	31.458	37.441	57.463
$d_{1.00}$	6.814	9.880	14.420	20.115	23.661	33.868
$d_{1.50}$	4.256	5.882	8.163	10.909	12.495	17.081
$d_{2.00}$	3.041	4.083	5.487	7.138	8.072	10.692
$d_{3.00}$	1.844	2.368	3.063	3.886	4.346	5.573
$d_{5.00}$	1.070	1.157	1.333	1.594	1.742	3.186

5. A Practical Example

In this section, the adaptive CUSCORE is illustrated using an example discussed in Hawkins [5]. Two laboratories carry out routine indirect (instrumental) assays for precious metals of batches of a feedstock. A portion of a reference material is taken and assayed alongside the unknown material coming from a new batch in order to detect changes in the level and/or variability of its assays.

In the following we illustrate the application of the ACUSCORE to data from the first laboratory.

The original, x_t and transformed, Q_t measurements are plotted in Figure 3. According to Hawkins [5], there is an upward mean change approximately after the 15th observations. To detect this shift, we use a ACUSCORE control chart setting, as recommended, $\lambda = 0.15$ and $\gamma = 3$. Further, we will assume that an in-control ARL equal 100 is desired; hence, using Table 3, we set $h = 4.196$. The control statistics AC_t^L and AC_t^U, along with the control limits, are plotted in Figure 4. Details of the computation are listed in Table 4. Note that starting from $t = 16$, AC_t^U increases and an alarm is signaled at $t = 33$. Since $t = 15$ is the last instant of time such that $AC_t^U = 0$, the scheme suggests that a mean shift occurs at or near $t = 16$.

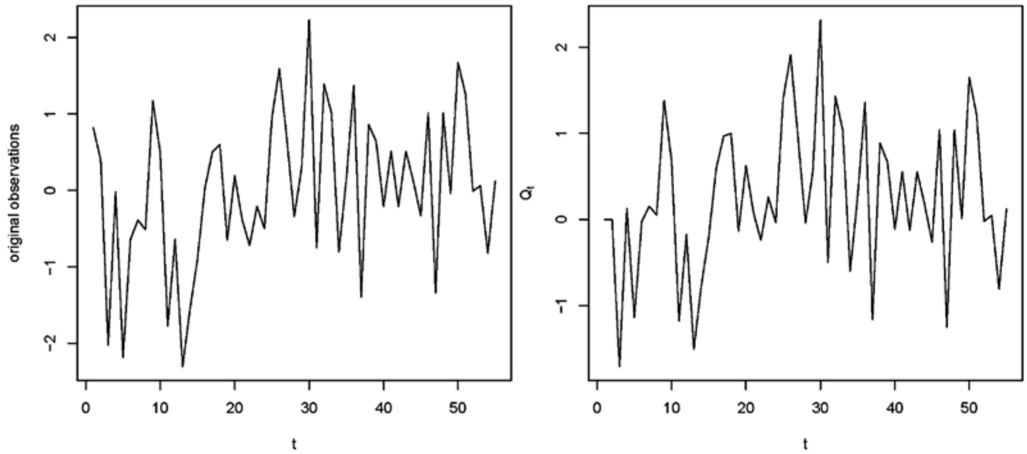

Figure 3. Laboratory data (left) and related Q_t statistics (right).

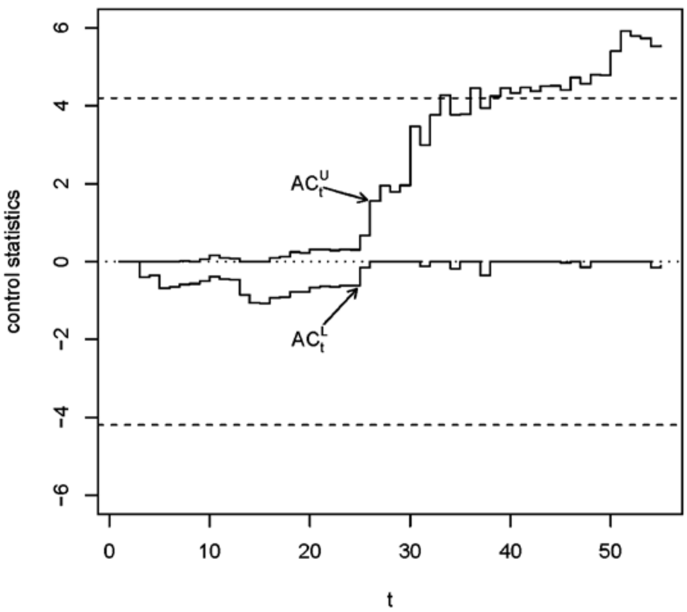

Figure 4. ACUSCORE control statistics for the laboratory data.

6. Comparison with Other Control Charts

To illustrate the advantages of the ACUSCORE control scheme, we compare its sensitivity to that of two traditional Q-charts which neglect the dynamic pattern of the mean change, such as the CUSUM and EWMA control charts of Q_t given by (4) and (5), respectively. Comparisons are shown for some of the most commonly used values of the design constants k and λ.

The three control schemes exhibit a different tendency to signal early false alarms. Thus, in order to make a fair comparison between the corresponding detection powers, decision intervals of (4), (5) and (8) are determined so that the probability to give a false alarm before τ is equal for the three control charts. In particular, the values of h are here obtained so that this probability is equal to that of a Shewhart chart with known μ and σ^2 and standard $\pm 3\sigma$ control limits. For two values of the time of the shift ($\tau = 26$ and 51), Table 5 lists the expected delays (9) for the compared charts. In general, it is found that the ACUSCORE performs much better than the other two charts for detecting small mean shifts whereas the behavior of the three control schemes is substantially equivalent for both intermediate and large shifts. It should also be noted that, although the expected delay in detecting a change decreases with the shift time τ, throughout the three charts, the out-of-control performances of the CUSUM and EWMA charts seem to be more strongly depending on the shift occurrence time than the ACUSCORE chart when shifts are both small and large. In addition, observe that results concerning the CUSUM chart are consistent with those shown by Zantek [26] who found that the best performance of the self-starting CUSUM may be obtained choosing the reference value in the interval [0.25, 0.35].

Table 4. Data and control statistics computation.

	x_t	Q_t	m_t	s_t^2	AC_t^L	AC_t^U
1	0.82	0.00	0.00	0.00	0.00	0.00
2	0.40	0.00	0.61	0.09	0.00	0.00
3	–2.02	–1.71	–0.27	2.35	–0.41	0.00
4	–0.02	0.12	–0.20	1.58	–0.36	0.00
5	–2.18	–1.14	–0.60	1.97	–0.69	0.00
6	–0.64	–0.02	–0.61	1.57	–0.65	0.00
7	–0.39	0.15	–0.58	1.32	–0.59	0.01
8	–0.51	0.05	–0.57	1.13	–0.57	0.00
9	1.17	1.38	–0.37	1.32	–0.50	0.07
10	0.49	0.68	–0.29	1.25	–0.39	0.16
11	–1.77	–1.18	–0.42	1.33	–0.45	0.09
12	–0.64	–0.18	–0.44	1.21	–0.46	0.08
13	–2.30	–1.50	–0.58	1.37	–0.85	0.00
14	–1.55	–0.77	–0.65	1.34	–1.06	0.00
15	–0.90	–0.20	–0.67	1.24	–1.07	0.00
16	0.03	0.59	–0.63	1.19	–0.94	0.10
17	0.50	0.97	–0.56	1.19	–0.92	0.12
18	0.60	1.00	–0.50	1.20	–0.78	0.24
19	–0.65	–0.14	–0.50	1.13	–0.78	0.22
20	0.19	0.62	–0.47	1.10	–0.66	0.32
21	–0.38	0.08	–0.46	1.04	–0.64	0.32
22	–0.72	–0.24	–0.48	0.99	–0.66	0.29
23	–0.21	0.26	–0.46	0.95	–0.62	0.31
24	–0.50	–0.04	–0.47	0.91	–0.62	0.30
25	0.95	1.41	–0.41	0.95	–0.16	0.67
26	1.59	1.91	–0.33	1.07	0.00	1.56
27	0.68	0.94	–0.29	1.07	0.00	1.94
28	–0.34	–0.04	–0.30	1.03	0.00	1.79
29	0.30	0.57	–0.28	1.00	0.00	1.95
30	2.23	2.32	–0.19	1.18	0.00	3.47
31	–0.75	–0.50	–0.21	1.15	–0.12	3.00
32	1.39	1.43	–0.16	1.19	0.00	3.77
33	1.01	1.04	–0.12	1.19	0.00	4.27

Table 5. Expected delays in detecting a mean shift occurring at observation τ. Estimates are based on 100000 Monte Carlo replications.

Parameters					Standardized shift (δ)									
τ	λ	γ	k	h	0.25	0.5	0.75	1	1.5	2	3	4	5	6
							Adaptive CUSCORE							
26	0.15	3		4.06	60.19	34.11	19.17	12.60	7.21	4.92	2.85	1.84	1.31	1.08
							CUSUM							
			0.30	6.16	171.52	103.05	44.06	17.31	6.70	4.62	3.06	2.40	2.03	1.82
			0.50	4.47	245.90	186.44	110.73	48.30	7.88	4.24	2.61	2.02	1.73	1.50
			1.00	2.45	320.31	304.06	279.07	239.86	121.95	30.41	2.21	1.50	1.18	1.04
							EWMA							
	0.05			2.21	149.78	7.59	32.33	15.69	7.99	5.73	3.91	3.12	2.67	2.38
	0.15			2.69	244.86	170.45	94.82	41.98	8.71	4.77	2.97	2.29	1.93	1.70
	0.40			2.91	310.41	282.45	244.76	195.11	101.95	38.35	3.38	1.69	1.35	1.17
							Adaptive CUSCORE							
51	0.15	3		5.24	75.39	32.58	18.12	12.32	7.13	4.87	2.76	1.74	1.25	1.06
							CUSUM							
			0.30	6.64	203.80	83.49	24.04	11.53	6.26	4.46	2.97	2.31	1.94	1.76
			0.50	4.63	268.99	157.18	57.72	17.40	5.79	3.86	2.46	1.91	1.62	1.35
			1.00	2.49	335.36	295.42	229.63	151.44	29.06	4.25	1.96	1.38	1.11	1.02
							EWMA							
	0.05			2.37	176.65	59.98	21.33	12.61	7.42	5.42	3.68	2.89	2.45	2.16
	0.15			2.75	259.06	133.78	46.36	16.13	6.23	4.21	2.70	2.08	1.75	1.54
	0.40			2.94	324.38	263.98	184.33	110.38	22.42	5.15	2.13	1.54	1.25	1.09

To better investigate the performance of the ACUSCORE over the other two charts, we also compared the ACUSCORE cumulative distribution function with those of the CUSUM and EWMA charts designed to have the best detection power, that is a CUSUM with $k = 0.30$ and an EWMA with $\lambda = 0.05$. Figures 5-7 confirm that, for a small value of the mean shift, the ACUSCORE exhibits a clear advantage over the other two charts in terms of detection power. On the other hand, the ACUSCORE reaction to a relatively larger value of the mean shift is at least comparable to that of the EWMA and CUSUM control charts.

7. Conclusions

An enhanced CUSCORE control chart is here proposed to monitor the mean of a sequence of data in those situations where either no observation or limited amounts of data are available prior to the start of process monitoring. In particular, we introduce and discuss a self-starting CUSCORE chart, based on the Q statistics, whose reference value is updated using an adaptive EWMA. Our results show that, in the presence of small mean shifts, the ACUSCORE is able to outperform those control charts designed to be optimal for detecting a constant mean shift. Future research will concern the generalization of the proposed procedure to the multivariate framework.

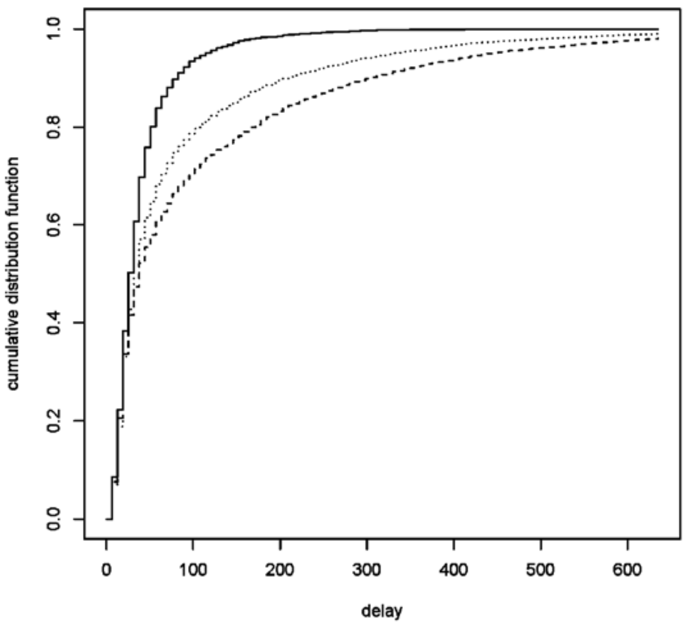

Figure 5. Cumulative distribution function of the run length when a mean shift of size 0.5σ occurs at $\tau = 26$. The three curves correspond to the following control charts: (i) ACUSCORE with $\lambda = 0.15$ and $\gamma = 3$ (solid line); (ii) CUSUM with $k = 0.30$ (dashed line); (iii) EWMA with $\lambda = 0.05$ (dotted line).

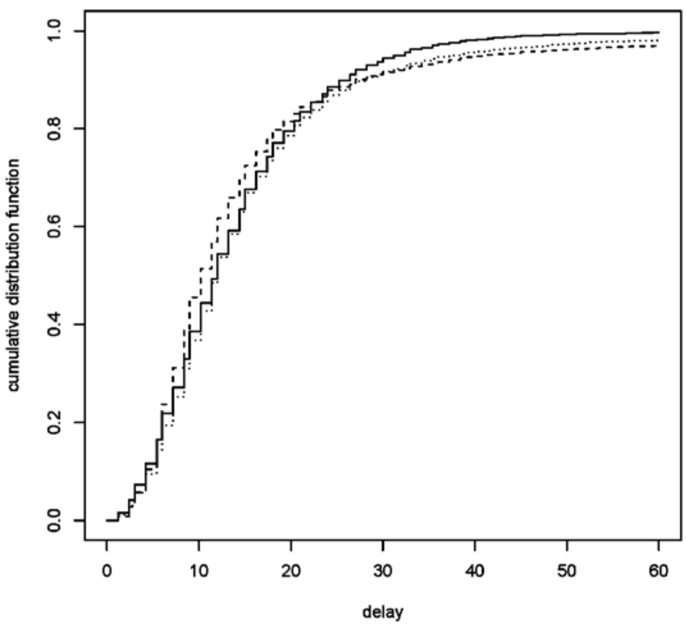

Figure 6. Cumulative distribution function of the run length when a mean shift of size 1σ occurs at $\tau = 26$. Legend: see Figure 5.

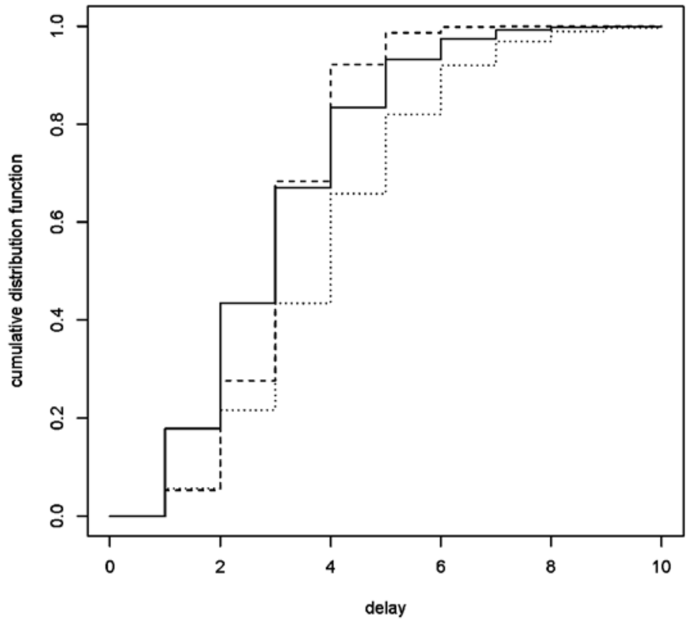

Figure 7. Cumulative distribution function of the run length when a mean shift of size 3σ occurs at $\tau = 26$. Legend: see Figure 5.

Acknowledgments

The authors thank the editor and two referees for their timely reviews and many helpful comments. This research was partially funded by Padua University PA2008 grants.

References

1. Box, G. E. P. and Ramirez, J. G. (1992). Cumulative score charts. *Quality and Reliability Engineering International*, 8, 17-27.
2. Capizzi, G. and Masarotto, G. (2003). An adaptive exponentially weighted moving average control chart. *Technometrics*, 45, 199-207.
3. Capizzi, G. and Masarotto, G. (2008). Practical design of generalized likelihood ratio control charts for autocor-related data. *Technometrics*, 50, 357-370.
4. Han, D. and Tsung, F. (2006). A reference-free CUSCORE chart for dynamic mean change detection and unified framework for charting performance comparison. *Journal of American Statistical Association*, 101, 368-386.
5. Hawkins, D. M. (1987). Self-Starting cusum Charts for Location and Scale. *The Statisti-cian*, 36, 299-315.
6. Hawkins, D. M. and Maboudou-Tchao, E. M. (2007). Self-starting multivariate exponen-tially weighted moving average control charting. *Technometrics*, 49, 199-209.
7. Hawkins, D. M. and Olwell, D. H. (1998). *Cumulative Sum Chartsand Charting for Quality Improvement.* New York: Springer-Verlag.
8. Hawkins, D. M., Qiu, P. and Kang, C. W. (2003). A change-point model for statistical process control. *Journal of Quality Technology*, 35, 355-366.
9. Hawkins, D. M. and Zamba, K. D. (2005). A change-point model for a shift in variance. *Journal of Quality Technology*, 37, 21-31.
10. Hawkins, D. M. and Zamba, K. D. (2005). Statistical process control for shifts in mean or variance using a change-point formulation. *Technometrics*, 47, 164-173.

11. Jensen, W. A., Jones-Farmer, A. L., Champ, C. W. and Woodall, W. H. (2006). Effects of parameter estimation on control charts properties: a literature review. *Journal of Quality Technology*, 38, 349-364.

12. Jiang, W., Shu, L. and Apley, D. W. (2008). Adaptive CUSUM procedures with EWMA-based shift estimators. *IIE Transactions*, 40, 992-1003.

13. Jones, A. L. (2002). The statistical design of EWMA control charts with estimated parameters. *Journal of Quality Technology*, 34, 277-288.

14. Jones, A. L., Champ, C. W. and Rigdon, S. E. (2001). The performance of exponentially weighted moving average charts with estimated parameters. *Technometrics*, 42, 156-167.

15. Jones, A. L., Champ, C. W. and Rigdon, S. E. (2004). The run length distribution of the CUSUM with estimated parameters. *Journal of Quality Technology*, 36, 95-108.

16. Polyak, B. T. and Juditsky, A. B. (1992). Acceleration of stochastic approximation by averaging. *SIAM Journal of Control Optmization*, 30, 838-855.

17. Quesenberry, C. P. (1991). SPC Q charts for start-up processes and short or long Runs. *Journal of Quality Technology*, 23, 213-224.

18. Quesenberry, C. P. (1993). The effect of sample size on estimated limits for X and X control charts. *Journal of Quality Technology*, 25, 237-247.

19. Quesenberry, C. P. (1995). On properties of Q charts for variables. *Journal of Quality Technology*, 27, 184-203.

20. Ruppert, D. (1991). Stochastic Approximation, In: *Handbook of Sequential Analysis*, eds. Ghosh, B. K. and Sen, P. K., New York: Marcel Dekker, 503-529.

21. Shu, L. (2008). An adaptive exponentially weighted moving average control chart for monitoring process variances. *Journal of Statistical Computation and Simulation*, 78, 367-384.

22. Shu, L., Jiang, W. and Tsui, K. L. (2008). A weighted CUSUM chart for detecting patterned mean shifts. *Journal of Quality Technology*, 40, 194-213.

23. Sparks, R. S. (2000). CUSUM charts for signaling varying location shifts. *Journal of Quality Technology*, 32, 157-171.

24. Sullivan, J. H. and Jones, A. L. (2002). A self-starting control chart for multivariate individual observations. *Technometrics*, 44, 24-33.

25. Yashchin, E. (1995). Estimating the current mean of a process subject to abrupt changes. *Technometrics*, 37, 311-323.

26. Zantek, P. F. (2006). Design of cumulative sum schemes for start-up processes and short runs. *Journal of Quality Technology*, 38, 365-375.

Authors' Biographies:

Giovanna Capizzi is an Associate Professor of Statistics at the Department of Statistical Sciences, Padua University, Italy. She holds a Ph.D. in Statistics from Padua University. Her publications have appeared in referred journals including Environmetrics, Technometrics, IIE Transactions and Journal of Quality Technology. She has been a referee for IIE Transactions, Technometrics, Quality Technology & Quantitative Management, Journal of Quality Technology and Journal of Statistical Computation and Simulation. Her current research is focused on change-point models, statistical process monitoring and quality control.

Guido Masarotto is a Professor of Statistics at the Department of Statistical Sciences, Padua University, Italy. His current research includes statistical process monitoring, time series analysis and statistical computing. His research has been published in various referred journals including Biometrika, Biostatistics, Journal of Time Series Analysis, Technometrics and Journal of Quality Technology.