Quality Technology &
Quantitative Management
Vol. 8, No. 2, pp. 75-85, 2011

QTQM

The Economic Design of Multivariate *MSE* Control Chart

Smiley W. Cheng[1] and Hong Mao[2]

[1]Department of Statistics, University of Manitoba, Winnipeg, Manitoba, Canada
[2]Economics and Management School, Shanghai Second Polytechnic University, Shanghai, China
(*Received* January 2009, *accepted* December 2009)

Abstract: In this paper, the economic design of *MSE* control chart is extended to the multivariate case. The important feature of this control chart is that it uses the target value instead of the process mean. According to Taguchi's viewpoint, any deviation from the target value represents a kind of loss. Therefore, we construct the model of economic design by considering not only the control costs occurred in the production process but also the loss resulted to the customer because the quality characteristics shifted from the target value. The expected loss of multivariate squared error is presented and used in the formulated cost model. A True Basic program is used to find the optimum parameters of the sample size, n; the sample interval, h and the width, E, of the control limits of the multivariate *MSE* chart. Finally, an example is used to illustrate the application of the proposed economic design of the multivariate *MSE* control chart.

Keywords: MSE chart, multivariate squared error.

1. Introduction

The multivariate control chart techniques have received considerable attention in the last couple decades. Ghute and Shirke [1] presented a multivariate synthetic control chart consisted of two sub charts: a T^2 sub-chart and a *CRL* (conforming run length) sub-chart. They pointed out that the substantial improvement in the reduction of ARL should justify its application. Aparisi and Deuna [2] developed the synthetic T^2 control chart, the results of comparison to other control charts showed that it performed better than the T^2 chart consistently, and given moderate and large shifts, the synthetic-T^2 chart is superior to that of the MEWMA or VSS-T^2 charts. Champ and Aparisi [3] proposed two double sampling (DS) Hotelling's T^2 charts. The results showed that they significantly increase the ability of detecting various changes in the process. Gonzales and Sanchez [4] proposed a methodology that helps to predict the main mean shifts, denoted as principal alarms, in a non-normal multivariate process by using independent component analysis. Makis [5] developed a multivariate Bayesian control chart for monitoring process mean and found an optimal stopping rule by minimizing the long-run expected average cost per unit time for giving sample size and sampling interval. Niaki and Fallah Nezhad [6] proposed a new methodology to monitor the overall mean shift and to classify the states of a multivariate quality control system by applying both the sequential analysis concept and Bayesian rule. Spiring and Cheng [7] proposed a new type of multivariate control chart, MSE_p chart. A special feature of this technique is that it uses the vector of target values instead of the normal means. It reflects Taguchi's view that quality control must focus on the proximity to the target and the variability and not simply on "conforming to specifications". Another development was that it uses one chart to simultaneously monitor the shift of mean vector and the change of covariance structure. In this article, we will discuss the economic design of this control chart.

There have been a lot of researches on the economic design. Duncan [8, 9] was the first to propose economic models for determining the three test parameters for the X-bar control chart that minimizes the average cost when a single out-of-control state exits. Montgomery [10] gave a thorough review of the literature of the economic designs of various control charts. Ho and Case [11] also gave a literature review of economic design of control charts. Alexander *et al.* [12] combined Duncan's cost model with the Taguchi loss function to develop a loss model for determining the three test parameters. There have been a few papers discussing the economic design of Multivariate control charts. Montgomery [13] presented a cost model of T^2 Control Charts to determine the optimal design parameters. Chou et al. [14] discussed the economic-statistical design of multivariate control charts using quality loss function. Chen [15] proposed the economic design of an adaptive T^2 control chart and then Chen [16] extended it to the case of variable sampling interval. Wu and Makis [17] built a cost model for both the economic and economic-statistical design of a χ^2 chart for a maintenance application. By considering the specific properties of CBM problem, a novel three state Morkov model is considered. In this paper, we use the economic design proposed by Alexander *et al.* [12] and construct the loss function of MSE_p control chart. The design combines the cost incurred in the production process and the loss due to the process variability.

The construction of multivariate MSE_p chart is briefly described below.

Assume that there are p quality characteristics for a product. Let $\underset{\sim}{X}$ be the vector of the value of the characteristics and $T\sim$ the vector of target values, i.e.,

$$\underset{\sim}{X} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_p \end{bmatrix} \quad \text{and} \quad \underset{\sim}{T} = \begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ \cdot \\ T_p \end{bmatrix}.$$

When the process is in-control, $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu_0}, \Sigma_0)$.

$$\text{Let } MSE_p = \frac{1}{n-1}\sum_{i=1}^{n}(\underset{\sim}{X_i} - \underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{X_i} - \underset{\sim}{T})$$
$$= (\bar{\underset{\sim}{X}} - \underset{\sim}{T})'\Sigma_0^{-1}(\bar{\underset{\sim}{X}} - \underset{\sim}{T}) + \frac{1}{n-1}\sum_{i=1}^{n}(\underset{\sim}{X_i} - \bar{\underset{\sim}{X}})'\Sigma_0^{-1}(\underset{\sim}{X_i} - \bar{\underset{\sim}{X}}),$$

where X_i' is the vector of observed values of the sample. X_i' are i.i.d. and $X_i' \sim N(\mu_0, \Sigma_0)$ when the process is in-control.

MSE_p is the overall measure of both the variability and the proximity to the target vector and $n \cdot MSE_p \sim \lambda_{np,\delta_0}$, the non-central Chi-squared distribution with np degrees of freedom and the noncentrality parameter 0, when the process is in-control. The upper control limit of the MSE_p chart is

$$\mathrm{UCL}_{MSE_p} = \frac{1}{n-1}\chi^2_{np,\,\delta_0,\,(1-\alpha)} = E,$$

which equals the width of the control limits since the lower limit is 0, and

$$\delta_0 = n(\underset{\sim}{\mu}_0 - \underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{\mu}_0 - \underset{\sim}{T}).$$

In order to distinguish whether the out-of-control signals are caused by the change of the variability or the shift of the proximity to the target vector, we need to draw a chart.

$$\text{Let } \tilde{T}_p = (\bar{\underset{\sim}{X}} - \underset{\sim}{T})'\Sigma_0^{-1}(\bar{\underset{\sim}{X}} - \underset{\sim}{T}).$$

$\tilde{T}_p$ measures the proximity to the target vector. The upper control limit of the $\tilde{T}_p$ chart is

$$ULC_{\tilde{T}_p} = \frac{1}{n}x^2_{p,\,\delta_0,\,(1-a)} = F,$$

which equals the width of the control limits since the lower control limit is 0.

In this paper, the economic design of the MSE chart [18] is extended to the multivariate case. The formulated cost model includes both the production cost and the customer's loss due to an out-of-control process. The optimum values of the parameters: the sample size, $\underset{\sim}{n}$; the sample interval, h; and the width of control limits of the MSE_p chart, E and the T_p chart, F can be obtained by minimizing the function of the total expected cost.

2. The Model of Economic Design

2.1. Assumptions

(i) Assume that the process is $\underset{\sim}{X} \sim N(\mu_0, \Sigma_0)$ when the process is in-control. The process may be affected by types of assignable causes and the noncentrality parameter would change from 0 to i when the process is out-of-control as a result of the occurrence of the i^{th} assignable cause, where

$$\delta_0 = n(\underset{\sim}{\mu}_0 - \underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{\mu}_0 - \underset{\sim}{T}) \text{ and } \delta_i = n(\underset{\sim}{\mu}_i - \underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{\mu}_i - \underset{\sim}{T}).$$

When the vector of means changes but the covariance structure keeps constant, then

$$\underset{\sim}{\mu}_i \neq \underset{\sim}{\mu}_0 \text{ but } \Sigma_i = \Sigma_0.$$

When the covariance change but the vector of means keeps constant, then

$$\Sigma_i \neq \Sigma_0 \text{ but } \underset{\sim}{\mu}_i = \underset{\sim}{\mu}_0.$$

When both means vector and covariance change, then

$$\underset{\sim}{\mu}_i \neq \mu_0 \text{ and } \Sigma_i \neq \Sigma_0.$$

While the process is affected by the i^{th} assignable cause, other assignable causes are assumed not to disturb the process.

(ii) Assume that assignable causes occur during a time interval according to a Poisson process. That is, the occurrence times of the assignable causes are independent exponential random variables with means $1/\lambda_i$, $i = 1, 2, \ldots, v$.

(iii) It is necessary to determine whether to shut down the process during the search for the assignable causes. In this paper, only one model is formulated. The model is suitable to both cases of shutting the process down and keeping the process operative during the search for the assignable causes.

2.2. List of Notations

$D =$ the expected search time for real or false alarms, α the false alarm rate,

$g =$ the average time taken in inspecting and charting per unit sample,

$a_1 =$ the average cost per unit time of searching for the assignable cause,

$a_2(i) =$ the average cost per unit time of adjusting and repairing associated with the i^{th} assignable cause,

$d =$ the cost of down time per unit time,

$a_3 =$ the manufacturing cost per unit of defective,

$BG =$ the fix cost of sampling and charting,

$CG =$ the variable cost of sampling and charting,

$r =$ production rate per unit time,

$A_0 =$ the cost of each item borne by customer or next station of process if the item whose quality characteristic vector excesses the tolerance zone is mistaken as a conforming product and transferred to the customer or to the next station of process.

2.3. The Models of the Cycle of the Process and the Costs

The cycle of the process control consists of four periods. The expected time of these four periods are:

(1) T_1, the expected time of in-control

$$T_1 = \frac{1}{\lambda} + (1 - W)D \cdot \frac{\alpha}{e^{\lambda h} - 1},$$

$$W = \begin{cases} 1 & \text{if production continues during searches,} \\ 0 & \text{if production ceases during searches.} \end{cases}$$

and $\alpha / e^{\lambda h} - 1$ is the times of false alarming in one cycle of the process control.

(2) T_2, the expected time that the process is out-of-control before the search for the assignable cause is instituted.

$$T_2 = \sum_{i=1}^{v} (\frac{\lambda_i h}{p_i} - \lambda_i T_i) / \lambda + gn,$$

$p_i = P(MSE_p^i \ge UCL_{MSE_p})$, the power of finding out the i^{th} assignable cause.

$$(n-1)MSE_p^i \sim \chi^2_{(np,\,\delta_i)} \text{ and } \delta_i = n(\underset{\sim}{\mu_i} - \underset{\sim}{T})'\Sigma_i^{-1}(\underset{\sim}{\mu_i} - \underset{\sim}{T}).$$

T_i is the average time of occurrence of the i^{th} assignable cause within a sampling interval h if the i^{th} assignable cause occurs between the q^{th} and $(q+1)^{\text{th}}$ interval, and

$$T_i = \frac{\int_{qh}^{(q+1)h} (t-qh)\lambda_i e^{-\lambda_i t}\,\mathrm{d}t}{\int_{qh}^{(q+1)h} \lambda_i e^{-\lambda_i t}\,\mathrm{d}t} = \frac{1}{\lambda_i} - \frac{h}{\exp(\lambda_i h) - 1}.$$

(3) T_3, the expected search time for assignable causes.

$$T_3 = D.$$

(4) T_4, the expected time of shutdown and repairing

$$T_4 = \sum_{i=1}^{v} (\frac{\lambda_i}{\lambda}) t_i.$$

t_i is the time of shutting down and repairing when the i^{th} assignable cause occurs.

Therefore, the total expected time of control cycle

$$T_e = T_1 + T_2 + T_3 + T_4.$$

The components of the cost function per cycle are:

(1) $E(C_1)$ – the cost of inspecting and charting

$$E(C_1) = (BG + CG\cdot n)\frac{\left\{\frac{1}{\lambda} + \frac{\sum_{i=1}^{v}\left[\frac{\lambda_i h}{p_i} - \tau_i\lambda\right]}{\lambda} + gn + W\cdot D\right\}}{h}.$$

(2) $E(C_2)$ – the cost of search due to false alarm

$$E(C_2) = D[a_1 + d(1-W)]\cdot\frac{\alpha}{e^{\text{th}} - 1}.$$

(3) $E(C_3)$ – the cost of investigating true alarm, downtime and repairing.

$$E(C_3) = D[a_1 + d(1-W)] + \sum_{i=1}^{v}\frac{(a_2(i) + d)t_i\lambda_i}{\lambda}.$$

(4) $E(C_4)$ – the expected cost incurred due to a higher rate of defectives when the process is out-of-control.

$$E(C_4) = a_3 r\left[\sum_{i=1}^{v}(M_i - M_0)\frac{B_i\lambda_i}{\lambda}\right],$$

where $B_i = h/P_i - \tau_i + gn + W\cdot D.$

M_i is average rate of defectives when the process is affected by the i^{th} assignable cause and M_0 is average rate of defectives when the process is in-control.

M_0, M_i are determined according to the following methods:

Let the tolerance zone be $(\underset{\sim}{X}-\underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{X}-\underset{\sim}{T}) \le C^2, C^2$ satisfying the following equation: $C^2 = (\underset{\sim}{T}_U-\underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{T}_U-\underset{\sim}{T})$ and $\underset{\sim}{T}_U$ is the upper or lower limit vector of engineering specification.

Then,

$$M_0 = P(\chi_0^2 > C^2) \text{ and } M_i = P(\chi_i^2 > C^2),$$

where $x_0^2 \sim x_{p,\delta_0}^2, x_i^2 \sim x_{p,\delta_i}^2$,

$$\delta_0 = n(\underset{\sim}{U}_0-\underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{U}_0-\underset{\sim}{T}) \text{ and } \delta_i = n(\underset{\sim}{U}_i-\underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{U}_i-\underset{\sim}{T}),$$

U_0 = the vector of means of quality characteristics when the process is in control, and U_i = the vector of means of quality characteristics when the process is out of control due to i^{th} assignable cause.

(5) $E(C_5)$ – the expected loss borne by customers because of the greater shift of mean vector and the change of covariance due to the out-of-control. When the process is out-of-control, the direct loss borne by an enterprise is the cost due to a higher rate of defectives. However, even if what customer received were conforming products, a change of the mean vector and covariance could also cause indirect loss to customers. The loss is proportional to the square error.

Assume that the loss of square error is expressed as

$$Y = l\left|E(\underset{\sim}{X}-\underset{\sim}{T})(\underset{\sim}{X}-\underset{\sim}{T})'\right|, \text{ where } |\ | \text{ is the sign of determinant.}$$

Y can be rewritten as

$$Y = l|\Sigma_0| + (\underset{\sim}{U}_0-\underset{\sim}{T})'(\underset{\sim}{U}_0-\underset{\sim}{T}),$$

where l is determined by the following method.

Let the economical tolerance zone be $(\underset{\sim}{X}-\underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{X}-\underset{\sim}{T}) \le C^2$.

When $(\underset{\sim}{X}-\underset{\sim}{T})'\Sigma_0^{-1}(\underset{\sim}{X}-\underset{\sim}{T}) = C^2$ or $\underset{\sim}{X} = \underset{\sim}{T}_X$, the loss of square error would satisfy the following equation:

$$l(\underset{\sim}{T}_x-\underset{\sim}{T})(\underset{\sim}{T}_x-\underset{\sim}{T})' = A_0.$$

From the equation, l can be found by

$$l = \frac{A_0}{(\underset{\sim}{T}_x-\underset{\sim}{T})(\underset{\sim}{T}_x-\underset{\sim}{T})'}.$$

Therefore,

$$E(C_5) = l\sum_{i=1}^{v}\{r(1-M_i)(\lambda_i/\lambda)[(\Sigma_i(+(\underset{\sim}{U}_i-\underset{\sim}{T})(\underset{\sim}{U}_i-\underset{\sim}{T})' - (\Sigma_0(-(\underset{\sim}{U}_0-\underset{\sim}{T})(\underset{\sim}{U}_0-\underset{\sim}{T})']\}B_i.$$

The total expected cost per cycle is:

$$E(C) = E(C_1) + E(C_2) + E(C_3) + E(C_4) + E(C_5).$$

Let

$$L = \frac{E(C)}{T_e}.$$

The optimal parameters n^*, h^* and k^* of multivariate MSE control chart are obtained by minimizing L.

The W value in the function of $E(C)$ and T_e is determined by the following rule. Let

$$N_1 = \sum_{i=1}^{v}\{r(1-M_i)\lambda_i/\lambda.T - rM_i,\lambda_i/\lambda.a_3\},$$

then

$$W = \begin{cases} 1, & \text{when } N_1 > 0, \\ 0, & \text{otherwise.} \end{cases}$$

3. An Example and Relevant Analysis

A spring manufacturer produces a kind of spring used in spiracles. The engineering specifications of two quality characteristics of inner diameter and free height are 28.30±0.10 mm. and 46.0±0.5 mm. respectively. The target value vector $T = (28.30, 46.0)$. According to the historical data, the normal mean vector is $U_0 = (28.29, 45.85)$ and covariance

$$\Sigma_0 = \begin{pmatrix} 0.0035 & -0.0046 \\ -0.0046 & 0.0226 \end{pmatrix}.$$

The process may be affected by one of the two types of assignable causes. The input values of different states are listed in Table 1. $\underset{\sim}{T}_x = (28.40, 46.5)$ $\underset{\sim}{T} = (28.30, 46.0)$ Table 2 lists the result of optimization.

Table 1. The input parameter value of economic design of MSE_p chart.

State \ Par	$\underset{\sim}{U}_i$	Σ_i	λ_i	t_i	$a_2(i)$	g	D	BG	CG	r	A_0	a_1	a_3	d
0	(28.29, 45.85)	$\begin{pmatrix} 0.0035 & -0.0046 \\ -0.0046 & 0.0226 \end{pmatrix}$	0	–	–									
1	(28.15, 45.55)	$\begin{pmatrix} 0.0065 & -0.0045 \\ -0.0045 & 0.0423 \end{pmatrix}$	0.00037	36	171	0.003	1	0.03	0.01	1000	0.9	0.2	0.8	100
2	(28.20, 45.79)	$\begin{pmatrix} 0.0035 & -0.0046 \\ -0.0046 & 0.0226 \end{pmatrix}$	0.1667	0	0									

Table 2. The output optimum parameter values.

n	Optimum E	Optimum h	Minimum Cost	α
2	10.0111	0.171	5.1145	0.0158
3	9.2447	0.221	4.9872	0.0097
4	8.6522	0.251	4.9449	0.0072
5	**8.3136**	**0.275**	**4.9357**	**0.0051**
6	8.0442	0.297	4.9432	0.0038
7	7.8526	0.316	4.9602	0.0028
8	7.7238	0.333	4.9827	0.0020
9	7.6327	0.348	5.0084	0.0014
10	7.5501	0.377	5.0359	0.0010

Optimum $n^* = 5$, Optimum $k^* = 8.31$, Optimum $h^* = 0.275$, Optimum Cost $= 4.9357$, $\alpha^* = 0.0051$.

It can be seen from Table 2 that the optimal parameters n^*, h^* and E^* are 5, 0.275, 8.3136, respectively. The minimal cost L^* is 4.9357. Using $F = (1/n)X^2_{p}, \delta_0, (1-\alpha_1^*)$ and $\alpha_1^* = 1-\sqrt{1-\alpha^*}$ and $\alpha^* = 0.0026$, the optimum limit of T_p chart is obtained, that is $F =$ 6.6733. The parameters on time and cost in Table 1 and Table 2 are scaled with hours and yuan. Table 3 shows the value of power $p(i)$. Tables 4-9 list the effects of the change of the cost parameters on the optimal design.

Table 3. The value of power $p(i)$.

The state of out of control	$p(i)$
1	0.9569
2	0.8961

Table 4. The effect of the fixed cost of taking a sample (BG) on the optimal design.

BG	n	E	h	L	α	$p(1)$	$p(2)$
0.03	5	8.3136	0.28	4.9357	0.0051	0.9569	0.8961
0.06	5	8.0983	0.33	5.0341	0.0067	0.9645	0.9115
0.3	7	7.3203	0.61	5.5343	0.0064	0.9946	0.9778

Table 5. The effect of the variable cost of taking a sample (CG) on the optimal design.

CG	n	E	h	L	α	$p(1)$	$p(2)$
0.01	5	8.3136	0.28	4.9357	0.0051	0.9569	0.8961
0.02	4	8.3186	0.32	5.0830	0.0101	0.9352	0.8659
0.1	3	7.6097	0.57	5.6667	0.0353	0.9355	0.8778

Table 6. The effect of the cost of investing assignable causes a_1 on the optimal design.

a_1	n	E	h	L	α	$p(1)$	$p(2)$
2	5	8.3136	0.28	4.9357	0.0051	0.9569	0.8961
4	6	8.3965	0.29	4.9954	0.0023	0.9680	0.9120
20	7	8.8942	0.31	5.3135	0.0005	0.9639	0.8959

Table 7. The effect of the cost of correcting the process $a_2(1)$ on the optimal design.

$a_2(1)$	n	E	h	L	α	$p(1)$	$p(2)$
171	5	8.3136	0.28	4.9357	0.0051	0.9569	0.8961
342	5	8.2821	0.28	7.1396	0.0053	0.9580	0.8983
1710	5	8.3136	0.30	24.7682	0.0051	0.9564	0.8961

Table 8. The effect of the manufacturing cost per unit of defective a_3 on the optimal design.

a_3	n	E	h	L	α	$p(1)$	$p(2)$
0.8	5	8.3136	0.28	4.9357	0.0051	0.9569	0.8961
1.6	5	8.2821	0.25	5.5016	0.0053	0.9580	0.8983
8.0	5	8.2977	0.15	10.4020	0.0052	0.9576	0.8976

Table 9. The effect of the quality loss A_0 on the optimal design.

A_0	n	E	h	L	α	$p(1)$	$p(2)$
0.9	5	8.3136	0.28	4.9357	0.0051	0.9569	0.8961
1.8	5	8.3136	0.19	7.2543	0.0051	0.9569	0.8961
9.0	4	8.6254	0.08	24.7629	0.0074	0.9198	0.8402

The results of Table 4 show that as the sample interval increases, the upper control limit decreases and the power of control chart increases respectively when BG increases. From Table 5, we can see that as the sample interval increases, and the sample size decreases when CG increases. In addition, increasing CG will decrease the upper control limit. The results of Table 6 show that the sample size, the upper control limit and the sample interval tend to increase as a_1 increases, but the sample interval is not sensitive to the change of a_1. From Table 7, we can see that the total cost of unit time greatly increases as $a_2(1)$ increases, which illustrates that the total cost of unit time is very sensitive to the change of the cost of correcting the process and it is important to estimate the value of $a_2(1)$ accurately. From Table 7, we can also see that the control limit, the sample size and the sampling interval are not sensitive to the change of $a_2(1)$. The results of Table 8 show that increasing a_3 will lead to the decrease of the sample interval and the increase of the total cost of unit time, which means that sampling should be conducted more frequently in order to avoid high cost of producing defective products. From Table 9, we note that as A_0 increases, the sampling interval decreases and the total cost of unit time increases greatly, which means that the quality loss to the customer is an important factor that must be estimated accurately. In addition, the results of analysis listed in Tables 4-9 show that the powers of finding out assignable causes are not sensitive to the changes of the parameters of costs.

4. Conclusions

In this article, we discussed the economic design of multivariate MSE_p control chart. We combined the cost of production process and the quality cost due to quality variation when establish loss function.

With the help of optimization technique, we found the optimized solutions of sampling interval, upper control limit and sample size. An example is provided to illustrate the application of multivariate MSE_p control chart and relevant analysis of the effects of cost parameters on the design is carried out.

Further study may be focus on economic design of multivariate MSE_p control chart with variable sampling interval and double sampling.

References

1 Ghute, V. B. and Shirke, D. T. (2008). A multivariate synthetic control chart for monitoring process mean vector. *Communications in Statistics - Theory and Methods*, 37, 2136-2148.

2 Aparisi, F and Deuna, M. A. (2009). The design and performance of the multivariate synthetic-T^2 control chart. *Communications in Statistics - Theory and Methods*, 38, 173-192.

3 Champ, C. W. and Aparisi, F. (2008). Double sampling Hotelling's T^2 charts. *Quality and Reliability Engineering International*, 24, 153-166.

4 Gonlalez, I and Sanchen, I. (2008). Principal alarms in multivariate statistical process control using independent component analysis. *International Journal of Production Research*, 46, 6345-6366.

5 Makis, V. (2008). Multivariate Bayesian control chart. *Operations Research*, 56, 487-496.

6 Niaki, S. T. A. and Fallah Nezhad, M. S. (2009). Decision making in detecting and diagnosing faults of multivariate statistical quality control. *International Journal of Advanced Manufacturing Technology*, 42, 713-724.

7 Spiring, F. A. and Cheng, S. W. (1998). An alternative variables control chart: The univariate and multivariate case. *Statistica Sinica*, 8(1), 273-287.

8 Duncan, A. J. (1956). The economic design of X-bar charts used to maintain current control of a process. *Journal of the American Statistical Association*, 52, 228-242.

9 Duncan, A. J. (1971). The economic design of X-bar chart when there is a multiplicity of assignable causes. *Journal of the American Statistical Association*, 66, 107-121.

10 Montgomery, D. C. (1980). The economic design of control charts: A review of literature survey. *Journal of Quality Technology*, 12, 75-87.

11 Ho, C. and Case, K.E. (1994). Economic design of control charts: A literature review for 1981-1991. *Journal of Quality Technology*, 26, 1-78.

12 Alexander, M. A., Dillman, M. A., Usher, J. S. and Damodaran, B. (1995). Economic design of control charts using Taguchi loss function. *Computers and Industrial Engineering*, 28, 671-679.

13 Montgomery, D. C. (1972). Economic design of T^2 control charts to maintain current control of a process. *Management Science*, 19, 76-89.

14 Chou, C. Y., Liu, H. R., Chen, C. H. and Huang, X. R. (2002). Economic-statistical design of multivariate control charts using quality loss function. *International Journal of Advance Manufacture Technology*, 20, 916-924.

15 Chen, Y. K. (2007). Economic design of an adaptive T^2 control chart. *Journal of the Operational Research Society*, 58, 337-345.

16 Chen, Y. K. (2007). Economic design of variable sampling interval T^2 control charts: A hybrid Markov chain approach with genetic algorithms. *Expert Systems with Applications*, 33, 683-689.

17 Wu, J. and Makis, V. (2008). Economic and economic-statistical design of a chi-square chart for CBM. *European Journal of Operational Research*, 188, 516-529.

18 Mao, H. and Cheng, S. W. (2010). The economic design of *MSE* control chart. *Communications in Statistics - Simulation and Computation*, 39, 17-27.

Authors' Biographies:

Smiley W. Cheng is Professor and Head of the Department of Statistics at the University of Manitoba. He has served as Associate Head and Acting Head of the Department in the past. He was the President of the International Chinese Statistical Association and the Managing Editor of *Statistica Sinica*. He is currently the Executive Editor of *Quality Technology and Quantitative Management*, Associate Editor or member of Editorial Board of five international statistics journals. Dr. Cheng is heavily involved in the research in statistical quality control (SQC), statistical inference, order statistics, and lottery. He is a *Fellow* of American Statistical Association; Senior Member of American Society for Quality, a member of Statistical Society of Canada, American Statistical Association; International Chinese Statistical Association; and an Elected Member of the International Statistical Institute.

Hong Mao is an Associate Professor of Economic and Management School at Shanghai Second Polytechnic University. During her teaching years, she got many honors including Distinguished Teacher of Shanghai Municipal, Distinguished Teacher of Shanghai Second Polytechnic University, and Distinguished Teacher Most Liked by Students. Her papers received First-class and Second-class Awards conferred by Shanghai Second Polytechnic University.