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The L-Chart for Non-Normal Processes

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Abstract: This paper presents a proposal to construct control charts for non-normal processes and an application. This control chart is based on the least-square L-estimator, which replaces the average and the standard deviation usually calculated for Shewhart charts. This estimator has a minimum variance for estimation of the process position and scattering whatever the data distribution. We focus our attention on "multi-generators" processes, like screw-machines or multi-die holder for injection molding, these processes are non-normally distributed.

Keywords: Control chart, injection press, L-statistics, non-normal process, statistical process control.

1. Introduction

The purpose of Statistical Process Control (SPC) is to assess statistical stability, that is to say eliminate special causes that produce non stationarity of the process. In opposition, common causes are difficult to reduce. When special causes have been eliminated, the characteristics of the process are supposed to be normally distributed. The important number of common causes whose effects are of an equivalent size order justifies this hypothesis. Shewhart control charts based on the average are familiar and well understood by practitioners. But they only should be used with the quality characteristics that are normally distributed. And several quality characteristics in nowadays industry and business are far from being normally distributed. Approaches to handle non-normality are important. Different approaches are discussed by Burr [2], Janacek [9], Ramalhoto [15], Shilling [16], Chan *et al.* [4], Pappanastos and Adams [13].

Non-normality is not necessarily the result of an out of control process as defined by Shewhart (see Mortel and Runge [12]). Indeed, many processes are naturally non-normal whereas they are economically or technically suitable. We mention in this category multi die processes or multi-cavity presses. We propose in this paper a new control chart: the L-chart especially adapted for non-normal process. L-chart is found on a non parametric estimator of the average and the variance. These estimators are without bias and have a minimum variance. It is not the case of traditional Shewhart estimators (average and range). For non-normal distribution, the properties of L-chart (notably the Average Run Length) are improved with regard to the Shewhart control charts.

2. The Non-Normality

Two essential factors can cause non-normality of a manufacturing process:

- The first can be explained by the nature of the process. Many processes are indeed non symmetrical for physical or mechanical reasons. Let us take the example of the bottles filling proposed by Mortel and Runge [12]. This process is constituted of twenty four filling heads whose debit can be adjusted independently. During each cycle, twenty four bottles are filled simultaneously. We can conceive easily that all the twenty four ducts cannot have rigorously the same debit. Besides, ducts do not evolve necessarily the same way at the same time. That explains the non-normality of the process.

- The second depends on the type of data processed. Indeed, some physical magnitudes are either marked out or singularities: for example circularity is necessarily positive and pH equal 0 is a singularity.

In this article, we will focus our interest on multi-generator processes.

2.1. Is it Necessary to Have Normal Process?

The "multi-generator processes" appellation is used for all processes that have several independent "elementary machines" which manufacture the same characteristic. Each characteristic produced by an elementary machine is, most of the time, normally distributed. Thus, the resulting distribution of the production is a mix of probability distributions, also called stratification (see Figure 1). Although multi-generator processes are rarely studied in SPC applications, they represent an important part of production means.

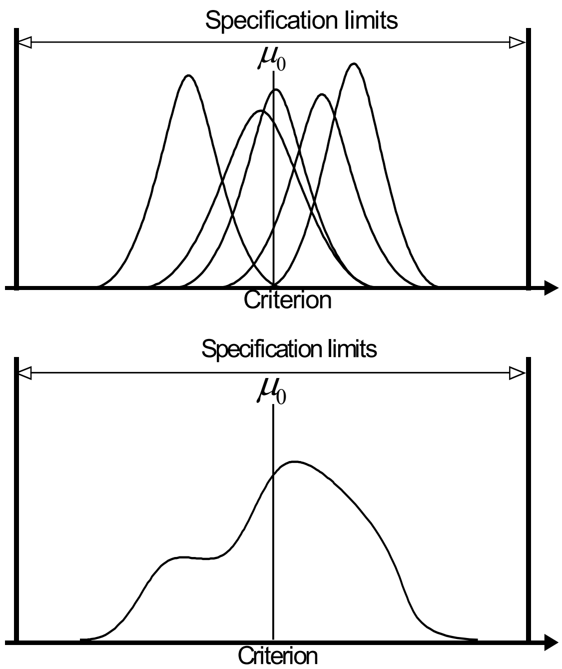


Figure 1. Mix of probability distributions.

The SPC goal is to reduce the special cause influence which is behind population non-normality. Figure 1 shows a non normal process which generates a very low proportion of scraps. So, is it necessary to improve the process capability by reducing the influence of the main cause of dispersion (differences between mold cavities?). In this case, we have shown that normality isn't necessary (see Pillet [14]) and that a special cause modifies the position of the distribution without changing its nature (example: a change in the rate of plastic humidity). Some special causes modify the nature of the distribution (example: a problem in one cavity).

Reducing dissimilarities between molds would consist in retouching them. This operation would be very expensive for a very hazardous result due to the tooling dispersion. It seems better, in this case, to accept that common causes generate a non-normal population and to maintain this distribution on the target value μ_0.

2.2. Repercussions of Non-Normality on Control Chart Performances

Many studies concern the effect of non-normality on control charts efficiency. Burr [2], for example, has shown that control charts could reasonably be used while the population is not strongly non-normal. One of the most important effects of non-normality on control charts performances is the modification of the α risk. Shilling and Nelson [16] consider that the application area of control charts is only valid for a risk close to 0.3%. Thus, two alternatives are foreseeable: the average is normally distributed so traditional control limits ($\mu_0 \pm 3\sigma_{\bar{X}}$) are used, or it is necessary to modify control limits. Anyway, the central limit theorem doesn't seem to be a good argument to establish control limits when the sample size is small. We will retain essentially two methods to choose control charts limits:

- The first one consists in keeping α risk close to 0.27% for the $\bar{X}$ chart, whatever the distribution of the population (see Yourstone [18]).
- Another approach, consists in placing systematically, 3 standard deviation of the estimator from the target value ($\mu_0 \pm 3\sigma_{estimator}$) whatever the control chart used. Although this kind of confidence interval is simplistic, it makes authority since most of control charts use this type of limits. Among these charts, we can mention standard deviation charts (*S*), range charts (*R*) attributes control charts (*p, np, c* and *u*). For each of these examples, the estimator isn't normally distributed whatever the distribution of the population (normal or not).

Another repercussion of non-normality concerns precision of estimations. Indeed, we can show that the average is not the optimal estimator in terms of variance when the population is non-normal. We can find an estimator without bias which provides better performances than the average.

So, in order to solve these problems, we propose in this article the use of a control chart (L-chart) built with a minimum variance estimator (see Duclos and Pillet [6]).

3. A Non Parametric Approach

3.1. Several Estimations of the Population Parameters

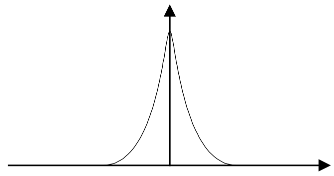
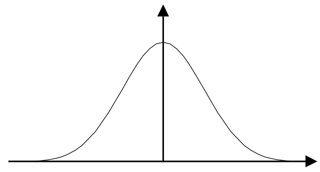
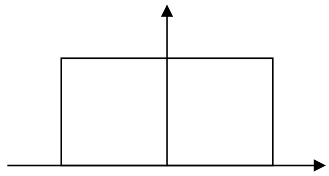
When the probability distribution is unknown, it is tempting to consider a non parametric approach. When the population is not normal, the average is not an estimator with minimal variance. We propose to choose a non parametric estimator with minimal variance.

Two important questions when dealing with control charts are:

- How often will there be false alarms where we look for an assignable cause but nothing has changed?
- How quickly will we detect certain kinds of systematic changes, such as mean shifts?

We will show that the ARL is better (or equal) for a non parametric control chart than a Shewhart chart whatever the distribution of the population.

Table 1. Minimal variance estimator.

Type	Estimator	Typical application
Impulsive	$\tilde{X} = X_{\left(\frac{n+1}{2}\right)}$ (n odd)	
Normal	$\bar{X} = \frac{1}{n}\sum_{n} x_i$	
Uniform	$\frac{X_{(n)} + X_{(1)}}{2}$	

The main disadvantage of parametric methods is that theirs performances decrease as experimental conditions get different from the statistical model used (normal law for example). Thus, the average is the optimal estimator in terms of variance when the population is normally distributed (its variance reaches the Frechet limits). In opposition, the median has good performances when the population is heavy tailed and the midrange estimator has a good behavior for distributions which are close to the uniform law. This phenomenon is very remarkable in the application of the Hodges-Lehmann control charts, which are more efficient than the $\bar{X}$ control chart for leptokurtic populations (see Pappanastos and Adams [13]).

The second element to take into consideration is the choice of the location and the scale parameters of the population. How justify the choice of the median rather than the average or another parameter? This choice can be justified by the cost of non quality defined by Taguchi.

3.2. Choice of the Location Parameter

When the distribution of the population is symmetrical, the choice of the location parameter of the population is simple since the average and the median are superposed on the symmetry axis of the distribution. In the opposite case, this choice is disputable since the median and the average do not correspond to the same geometrical characteristic of the law. The question is then to know whether it is preferable to choose the average rather than the median or any other parameter?

We have therefore chosen a criterion that allows one to determine an optimal parameter. The cost of the non quality seems to be a good criterion to justify the choice of the location parameter. So, we have established the parameter of location that minimizes the loss in the Taguchi sense, when it coincides with the target of the process.

First of all, let us remind that the average loss is defined by:

$$\bar{L} = K \cdot E\left[\left(X - \mu_0\right)^2\right], \tag{1}$$

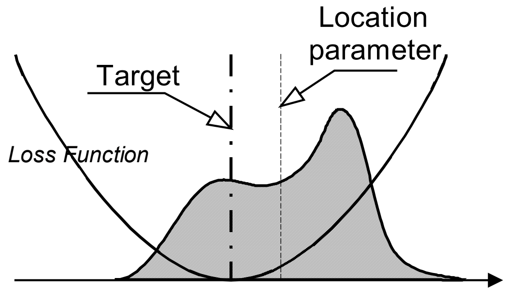


Figure 2. Choice of the location parameter.

- X is the value of a criterion,
- μ_0 is the target of the production,
- K is a constant that depends on the cost of an out-of-specifications measure.

The parameter of location is supposed to be placed on the target: $\mu = \mu_0$,

$$\begin{aligned}\bar{L} &= K \cdot E\left[\left(X - \mu\right)^2\right] \\ &= K \cdot \left(E\left[X^2\right] - 2\mu \cdot E\left[X\right] + \mu^2\right).\end{aligned}$$

The extremum of the function is obtained by deriving with respect to μ:

$$\frac{\partial \bar{L}}{\partial \mu} = K\left(2 \cdot E\left(X\right) - 2\mu\right) = 0 \iff \mu = E(X).$$

The parameter of location of the estimator is therefore the average because it minimizes the loss when it is equal to the target. This demonstration is independent from the distribution of the population. Concerning the dispersion parameter, we choose the standard deviation because it is the most common parameter of dispersion.

3.3. The L-Statistics

Order statistics have played a preponderant role in the development of robust and non parametric methods. They are usually used in problems of extreme values rejection with estimators such as α-truncated averages. For more information we invite readers to refer to Tassi [17], Capéràa [3] or David [5].

Let us consider a sample $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ of n independent observations sampled at a time k. $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ are ordered observations of $X_{(k)}$ such as $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$.

The variable $X_{(i)}$ is called the i^{th} order statistic (i = 1, 2, ... , n).

The linear combination of $X_{(k)}$'s ordered statistics with a vector C of real coefficient defines the statistic T_n called an L-statistic :

$$T_n = \sum_{i=1}^{n} c_i \cdot X_{(i)}. \tag{2}$$

L-statistics can be used to estimate location or dispersion parameters of a distribution. So, they are called L-estimators.

The estimator that we use can be applied to a very large number of distributions, since it concerns distributions which depend on location and scale parameters only. For such distributions, Lloyd [11] shows that parameters can be estimated by applying the generalized least-squares theory to an ordered sample. These estimators, that are linear combinations of ordered observations, are without bias and have a minimum variance. Indeed, the variance of the location Lloyd's estimator never exceeds the variance of the sample average. Appendix A develops the principle of the Least squares estimation.

4. The L-Chart Calculation

Lloyd's L-estimator construction requires the calculation of the covariance matrix and of the vector of moments of standardized ordered statistics. In order to guaranty that coefficients of the variance matrix are representative of the distribution of the population, it is necessary to estimate them from data of a stationary process.

Whereas calculation of the L-estimator coefficients is not very delicate for relatively stable processes, it is not the same for processes that are particularly disturbed. In the case of a stable process, stationarity of the process can be assumed over a reference period used to compute statistics of interest. In the other case, since periods of stability are short, it is difficult to collect enough data to calculate the L-estimator. The idea consists therefore in exploiting the few collected data to make Ω matrix and α vector coefficients converge to reasonable values. So we will use a bootstrap method.

4.1. Estimation of Coefficients by a Bootstrap Method

Bootstrap methods (see Efron [8]) are part of intensive calculation methods. The bootstrap methods belong to resampling methods because it reuses the original sample of process measurements to synthesize many additional samples to replicate a statistic (here the Least Squares L-estimator). Although, we do not have any information regarding the distribution of X, we wish to estimate the parameter L-estimator. Jones and Woodall [10] have used simulation studies to evaluate the performance of different bootstrap control chart procedures to monitor the mean of the process. The authors discuss the different procedures in two different situations: normal distribution and exponential distribution. The procedure used in our approach is close to Bajgier's Bootstrap control chart [1].

The bootstrap algorithm is relatively simple. We will summarize it in three steps:

- First of all, we consider an independent sample $E = (X_1, X_2, ..., X_m)$.
- Then, N samples are collected, in the empirical population that constitutes E. The sample thus collected is noted E^*.
- The last step consists in studying the behavior of the "bootstraped" statistic T^*, constructed from E^*. The resampling operation is iterated a large number of times to give an approximation of T^* with the Monte-Carlo method.

This approach implicitly assumes that the process is stable when the control limits are computed. Other procedures as discussed by Jones and Woodall [10] , prevent the necessity for this assumption. Different simulations [10] show that the bootstrap control chart to monitor the mean is interesting in the case of an extremely skewed distribution. In case of normal distribution, bootstrap procedures do not perform substantially better than the standard Shewhart chart in terms of resulting in-control ARL.

4.1.1. Application to an Ordered Sample

Studied statistics are ordered statistics associated to the calculation of elements of Ω and α. Bootstrap samples are the same size as subgroups of control charts: $N = n$.

$$T^* = X_{(i)} \quad i = 1, 2, \ldots, n$$

for elements of the vector α.

$$T^* = \left(X_{(i)} - \bar{X}_{(i)}\right)\left(X_{(j)} - \bar{X}_{(j)}\right) \quad 1 \le i < j \le n$$

for elements of the Ω matrix.

E_* is the empirical moment calculated from B bootstrap samples. The moment of bootstrapped statistics is calculated by the relationship:

$$E_*\left[T^*\right] = \frac{1}{B}\sum_{i=1}^{B} T_i^* . \tag{3}$$

4.1.2. Example of Application

- We proceed to a random sampling with sample size $n = 5$ in order to construct the control chart. As the process is moderately stable, only $m = 30$ measures are collected (six SPC samples) in a relatively short period of time to minimize the effect of the process evolution. This sample is the reference used for the bootstrap method to construct the L -estimator.
- $B = 1000$ ordered sample are generated with sample size $n = 5$.
- It's possible to collect up $C_{m+n-1}^{n} = (m+n-1)!/\,n!(m-1)!$ different samples from the reference sample. $C_{m+n-1}^{n} = 278256$ is the number of combinations.
- Samples synthesized are then used to calculate the Ω matrix and the α vector coefficients.
- If the location parameter of the population is the average, then $\mu_p = (1/B)\sum_{i=1}^{B}(1/n)\sum_{j=1}^{n} X_{i,j}^*$, the average of the whole bootstrap samples.
- The scale parameter is the standard deviation of the $B*n$ individual measures.
- When computation is accomplished, coefficients are the following:

Table 2. L-estimator of position.

L-estimator of position					
	c_1	c_2	c_3	c_4	c_5
Bootstrap method	0.55	–0.030	–0.042	–0.027	0.55
Optimal coefficients	0.53	–0.045	–0.046	–0.010	0.57

The coefficients obtained by the bootstrap method are very close to the optimal coefficients. This result is confirmed by Jones and Woodall [10] in the case of a normal and exponential distribution for a bootstrap method to monitor the mean of the process.

We define the relative efficiency by the relation $ER = \frac{Var(\hat{\mu})}{Var(\overline{X})}$.

The relative efficiency of the L-estimator of position is *0.49* with the coefficients estimated by the bootstrap method and *0.47* for the optimal coefficients.

Table 3. L-estimator of dispersion.

L-estimator of dispersion					
	c_1	c_2	c_3	c_4	c_5
Bootstrap method	–0.49	0.034	–0.062	–0.0047	0.47
Optimal Coefficients	–0.47	0.026	0.0013	–0.023	0.46

The coefficients obtained by the bootstrap method give reliable estimations.

4.2. Control Limits

Most of control charts based on Shewhart concepts, like range charts (*R*), standard deviation charts (*S*), attributes charts (*p, np, c* and *u*) have theirs limits to ±3 standard deviation of the estimator used:

$$UCL = \mu_0 + 3\sigma_{L-estimator\ of\ location},$$
$$LCL = \mu_0 - 3\sigma_{L-estimator\ \text{of location}}. \tag{4}$$

With *UCL* : Upper Control Limit, *LCL* : Lower Control Limit. α risk is of course different from the *0.27%* known for $\overline{X}$ charts but remains in an acceptable order of magnitude. Another approach consists in using bootstrap method for control limits calculation close to *0.27%.*

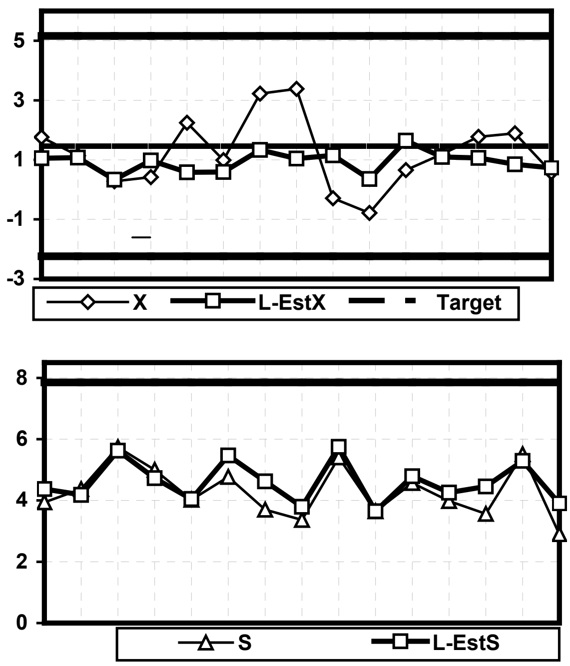


Figure 3. L-chart and Shewhart chart.

4.3. ARL for L-Chart and Shewhart Chart

The particular context of L-chart use makes a great difficulty for the analytical determination of the ARL. Under the assumption that the distribution is a mixture of different individual distributions, each case is a particular case. To study the ARL, we have used the Monte Carlo method for various situations (200 million sampling by simulation).

Let us consider a stratified situation based on six elementary normal distributions:

$$N_1(0,1),\ N_2(5.5,1),\ N_3(-3.2,1),\ N_4(5,1),\ N_5(5,1),\ N_6(-3.5,1).$$

We have $\mu = 1.4667$ and $\sigma = 3.9965$.

This example is a simulation of injection molding with six cavities. We suppose that the L-chart consists of a sample of three measures. The L-estimator of position is determined by the linear equation: $T = 0.52X(1) - 0.07X(2) + 0.55X(3)$

The equivalent Shewhart chart consists of a sample of three measures. The $\bar{X}$ estimator of position is determined by the linear equation:

$$\bar{X} = 1/3\,(X(1) + X(2) + X(3)).$$

For $k = 0$, the ARL is quite different to the standard 370 due to a hard non-normality of the distribution. In the same way, the ARL are not symmetrical.

Ours different simulations show that when the distribution is close to a normal law, the L chart and Shewhart ARL are very close, and that when non-normality is pronounced, the L chart improves the effectiveness of detection.

Table 4. ARL L-chart versus Shewhart chart.

Shift $k.\sigma$	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.8	2.2	2.8
Shewhart	26555	40827	2787.8	101.9	15.35	8.00	5.45	3.38	1.79	1.29	1.04
L-chart	2673	2161	97.7	16.2	8.59	6.83	4.83	3.11	1.31	1.12	1.01

Shift $k.\sigma$	–0.2	–0.4	–0.6	–0.8	–1	–1.2	–1.4	–1.8	–2.2	–2.8
Shewhart	2241	129.2	30.01	14.8	9.26	5.32	3.26	1.96	1.27	1.02
L-chart	146	35.6	18.6	11.0	8.30	5.80	2.81	1.34	1.15	1.00

5. Study of an Injection Press

The plastic industry uses an important number of technologies for processing plastics: thermoforming technique, transfer molding, injection molding, or spray up. Among these techniques, we have been particularly interested in injection presses which are the most classical processes. These processes allow one to manufacture full forms by plastic injection in a mold. For profitability evident reasons, molds are usually composed of several cavities: the multi-cavity injection presses.

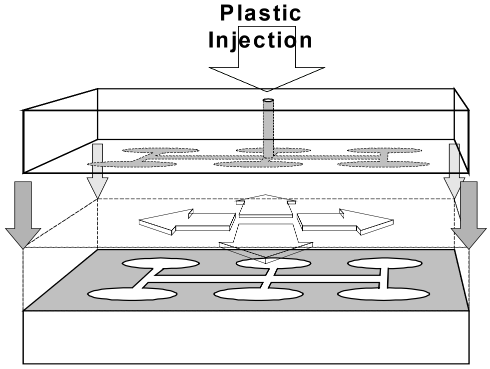


Figure 4. Plastic injection principle.

	Average	Std
Cavity N°1	30.224	0.0101
Cavity N°2	30.169	0.0090
Cavity N°3	30.134	0.0081
Cavity N°4	30.167	0.0069

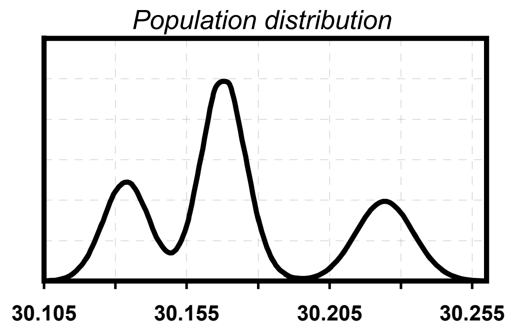


Figure 5. Population distribution.

The best sampling procedure consists in taking one part by cavity. The interest to use L-chart is to be able to decrease sampling size without too much deteriorating the efficiency of the control chart. To validate this procedure, we have set up an L-chart in the EUDICA Company, which produces parts for pharmaceutical industry.

In order to determine the characteristic to study, we processed data coming from an anterior capability analysis. It comes out of this study that the length of the piston has a stratified distribution. Thirty data were independently collected from each cavity for this capability study. The influence of cavities is significant and the graphic shows a sharp stratification of the population (Figure 5).

5.1. L-estimator Coefficients

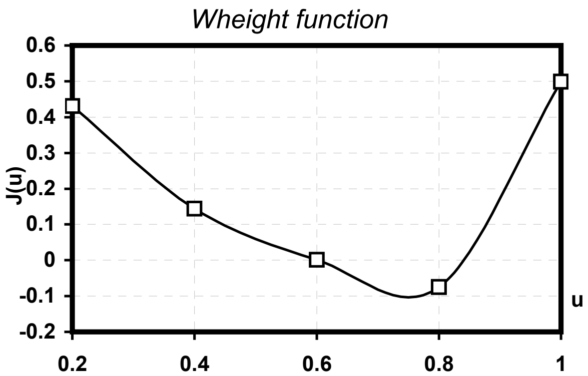


Figure 6. L-estimator coefficients.

Since the distribution of the population has light tails, the weight function of the L-estimator of position is *U* shaped and tends to the midrange estimator.

The relative efficiency of the L-estimator is especially significant when the distribution of the population is highly non-normal.

$$Var(\hat{\mu}) = \sigma^2 \cdot \frac{\alpha^t \cdot \Omega \cdot \alpha}{\det\left(A^t \cdot \Omega^{-1} \cdot A\right)} = 1.31.10^{-4}, \quad Var\left(\bar{X}\right) = 2.44.10^{-4},$$

and so the relative efficiency is : $ER\left(\bar{X}, \hat{\mu}\right) = 1.31/\ 2.44 = 0.54.$

These values are obtained by approximating the population standard deviation by the empirical population standard deviation.

5.2. Process Control with L-Chart

In order to construct the L-chart, the target has been chosen to be the middle of specifications. Samples are randomly collected by subgroups of size *n* = *5*. Control charts limits are respectively calculated by:

Table 4. Control limits.

	Shewhart	***L-Chart***
Formula	$\mu_0 \pm 3\sigma/\sqrt{n}$	$\mu_0 \pm 3\sigma_{L-estimator}$
UCL	30.197	30.184
LCL	30.103	30.116

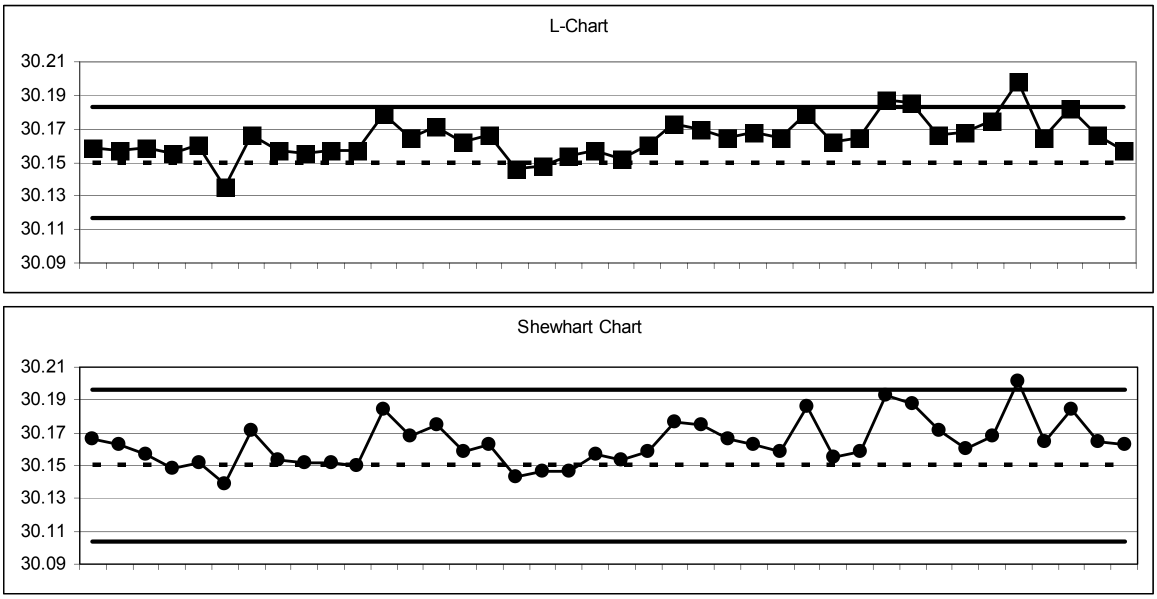


Figure 7. L-chart and Shewhart chart.

Figure 7 presents the evolution of the process according to the L-chart and the $\bar{X}$ chart. This graphic shows that the L-chart detects an out-of-control state before the $\bar{X}$ chart. The L-chart also provides a minimum variance estimation which allows one to extract trends of the process.

6. Conclusion

In order to define the research framework, we focused our attention on two points:

- First, we expected to use an optimal estimator in terms of variance.
- Second, we wanted to propose a non parametric approach to provide this new control chart to a large application area.

So, our work took interest in order statistics which represent a preferential tool for non-parametric methods. The Least Squares L-estimator proposed by Lloyd was particularly adapted to our problem because of its properties:

- It is based on a non parametric model of scale and location.
- It provides a non biased estimation of the location and scale parameters.
- It provides a minimal variance estimation compared to estimators constructed from a linear combination observations.
- The estimated parameters can be defined by the user.

To satisfy these objectives, we have proposed a new control chart (the L-Chart) based on the L-statistics, which is particularly suitable for non-normal processes.

6.1. The Choice of the Estimated Parameters

The choice of the location parameter can be discussed when the population is not symmetrical since the central position of the distribution has no longer meaning. We have therefore proposed to define this parameter according to the loss criterion defined by Taguchi. With this criterion, we have shown that when the location parameter was the population average, the loss caused by non quality could be minimum if the loss function was symmetrical.

6.2. The L-Chart Construction

The calculation of Lloyd L-estimator coefficients requires the knowledge of the covariance matrix of order statistics. To reduce the reference period, we have proposed to use a bootstrap method. This procedure is particularly convenient because it provides coefficients very close to theirs asymptotical values with a reduced reference sample.

6.3. The L-Charts Performances

Our work brought some interesting characteristics of the L-chart and particularly that L-charts are less disturbed than $\bar{X}$ charts. As a consequence, trends of the process and out-of-control points are detected with much more accuracy and precise adjustments are provided.

6.4. Industrial Application

The study of multi-cavity injection presses has shown that strong non-normality of criteria was an industrial reality. In these cases of non-normality, we have shown that the L-chart could offer better performances than conventional $\bar{X}$ charts.

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Appendix: The Least Squares Estimation

We shortly remind construction of the Least Squares L-estimator. We define respectively μ and σ as the location and the scale parameters of the distribution function. These parameters, that we want to estimate by the method of the least squares, are not necessarily the average and the standard deviation of the population. Let us note $(x_{(1)}, x_{(2)}, \ldots, x_{(n)})$ the ordered observations of the $X_{(k)}$ vector. We introduce the standardized variables $U_{(r)}$ of rank *r:* $U_{(r)} = \left(x_{(r)} - \mu\right)/\sigma$, then we construct moments of order 1 and 2 of the variable *U*:

$$E\left[U_{(r)}\right] = \alpha_r \quad Var\left[U_{(r)}\right] = \omega_{rr}, \quad Cov\left[U_{(r)}, U_{(s)}\right] = \omega_{rs}, \text{ with } \theta = \begin{pmatrix} \mu \\ \sigma \end{pmatrix}, \quad A = \begin{bmatrix} 1 & \alpha_1 \\ \vdots & \vdots \\ 1 & \alpha_n \end{bmatrix},$$

Ω is a ($n \times n$) matrix of ω_{rs} elements, such that : $Cov\left(X\right)=\sigma^2 \cdot \Omega$.

Since the model is an ordinary multiple linear model written as $E[X]=A.\theta$, the θ vector is estimated with the Generalized Least Squares theory:

$$\theta=(A^t.\Omega^{-1}.A)^{-1}.A^t.\Omega^{-1}.X \quad . \qquad \text{(A1)}$$

The estimator variance is given by:

$$Var\left[\hat{\mu}\right]=\sigma^2 \cdot \frac{\alpha^t \cdot \Omega \cdot \alpha}{\det\left(A^t \cdot \Omega^{-1} \cdot A\right)}, \quad Var\left[\hat{\sigma}\right]=\sigma^2 \cdot \frac{e^t \cdot \Omega \cdot e}{\det\left(A^t \cdot \Omega^{-1} \cdot A\right)}.$$

These relations allow one to easily compare performances of the average to the location L-estimator ones.

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