



Monitoring the Coefficient of Variation Using Control Charts with Run Rules

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Abstract: Monitoring the coefficient of variation (CV) is a successful approach to Statistical Process Control when the process mean and standard deviation are not constant. In recent years the CV has been investigated by many researchers as the monitored statistic for several control charts. Viewed under this perspective, this paper presents a new efficient method to monitor the CV by means of Run Rules (RR) type charts. Tables are provided to show the statistical run length properties of Shewhart- γ , RR_{2,3}- γ , RR_{3,4}- γ and RR_{4,5}- γ control charts for several combinations of in control CV values γ_0 , sample size n and shift size τ . Indeed, comparative studies have been performed to find the best control chart for each combination. An example illustrates the use of these charts on real data gathered from a metal sintering process.

Keywords: Average run length, coefficient of variation, Markov chain, run rules.

1. Introduction

Quality is one of the most important consumer decision factors. Furthermore, Quality control has become one of the most important strategies to increase the productivity of industrial and services organizations. One of the fundamental principles of the SPC (Statistical Process Control) is that a normally distributed process cannot be claimed to be *in-control* until it has a constant mean and variance. This implies that a shift in the mean and / or the standard-deviation makes the process *out-of-control*. However, control charting techniques were recently extended to various service sectors such as health, education, finance and various societal applications where the mean and the standard-deviation may not be constant all the time but the process may nevertheless be declared in-control if their ratio remains stable around a constant value. In this case, it is natural to explore the use of the coefficient of variation (CV, in short) γ which is a normalized measure of dispersion of a probability distribution that is defined as the ratio of the standard deviation σ to the mean μ . The CV has several applications in applied statistics and is receiving increased attention in quality control. Generally, it is widely used to compare data sets having different units or widely different means. Several published works have investigated, in the case of normality, the distribution of the sample coefficient of variation and its related inferential properties, see Hendricks and Robey [12], Iglewicz and Myers [14], Iglewicz *et al.* [15], Mahmoudvand and Hassani [21], McKay [22], Reh and Scheffler [28], Tian [33], Vangel [34], Verrill and Johnson [35] and Warren [36].

For instance, the CV is commonly used in renewal theory, queuing theory, and reliability theory. In the field of finance (see Sharpe [31]), it is interpreted as a measure of the risk faced by investors, by relating the volatility of the return on an asset to the expected value of the return. It is also adopted in chemical and biological assay quality control to validate results, see Reed *et al.* [27]. It is worth noting that there are many opportunities for SPC monitoring of the CV also in the fields of materials engineering and manufacturing. Often some quality characteristics related to the physical properties of products constituted by metal alloys or composite materials have a standard-deviation which is proportional to their population mean. Usually, these properties are related to the way atoms of a metal diffuse into another. Tool cutting life and several properties of sintered materials are typical examples from this setting.

As pioneers in this field, Kang *et al.* [16] developed a control chart for monitoring the CV using rational subgroups. In this chart, the mean may change from one sample to another, making useless to plot the (\bar{X}, R) or (\bar{X}, S) control charts. A common clinical chemistry-control problem in which repeated measurements of some characteristic (the amount of a chemical in a patient blood) is used to show that the CV is a potentially attractive tool in quality improvement, where neither the process mean nor the variance are constant. In Kang *et al.* [16]'s paper, the CV is monitored through a Shewhart-Type chart. This makes this chart sensitive to large shifts in the CV but not very sensitive to small to moderate shifts.

Hong *et al.* [13], proposed an EWMA-CV (Exponentially Weighted Moving Average) control chart in order to improve the Shewhart's type chart proposed by Kang *et al.* [16] and detect small shifts more efficiently. The *ARL* (Average Run Length) was used in this paper to investigate the performance of the EWMA-CV chart v.s. the Shewhart-CV control chart proposed by Kang *et al.* [16]. The results clearly showed that the EWMA-CV chart gives smaller out-of-control *ARL*'s than the Shewhart-CV chart proposed by Kang *et al.* [16].

Castagliola *et al.* [7] suggested a new method to monitor the CV by means of two one-sided EWMA charts (denoted EWMA-CV²) of the CV squared. The performance investigation with this chart requires an extensive computational analysis based on a Markov chain approach (while the Hong *et al.* [13] approach is only based on simulation). A numerical analysis demonstrated that the control chart proposed by Castagliola *et al.* [7] almost always yielded smaller *ARL* values than the control chart proposed by Hong *et al.* [13], even if the difference between these values is often rather small.

Very recently, Calzada and Scariano [5] suggested a synthetic control chart (denoted as SynCV) for monitoring the coefficient of variation. The results showed (see Table 8 in Calzada and Scariano [5]) that the out-of-control *ARL*s obtained for the SynCV chart are obviously smaller than the ones for the CV chart of Kang *et al.* [16] but the former are generally larger than the out-of-control *ARL*'s obtained for the EWMA-CV² chart of Castagliola *et al.* [7] as long as the increasing shift in the CV is not too large. Contrary to Castagliola *et al.* [7] in which the authors investigated both the CV increasing and decreasing cases, Calzada and Scariano [5] only provided results concerning the CV increasing case.

Although EWMA and synthetic type control charts efficiently detect small or moderate shifts in the process mean, their "relatively complexity of use" (i.e. two chart parameters, (K, λ) for EWMA type charts and (K, L) for synthetic type charts) has not been widely accepted everywhere by the quality practitioners. In this way, it is natural to explore the use of a Shewhart chart with supplementary run rules as an intermediate solution (a single parameter K needs to be selected for run rules type charts). The most popular Run Rules were suggested by the Western Electric Corporation [37]. Several Run Rules have also been

suggested by Bissell [4], Page [25], Roberts [30] and Wheeler [38]. As Champ and Woodall [8] pointed out, these rules may be stated in the following form: an out-of-control signal is given if r of the last ℓ standardized sample means fall in the interval (a, b) , where $r \leq \ell$ and $a < b$. A good discussion of some of these rules is given by Nelson [23]. Using a Markov chain approach, Champ and Woodall [8] obtained the exact formula to evaluate the Run-Length distribution and applied it to compute the time until the signal of an alarm for the Shewhart control chart with supplementary Run Rules. Palm [26] used their method to construct tables of Run-Length percentiles, when one or more of 3 additional Run Rules are used with a Shewhart chart for monitoring the mean (\bar{X} chart). Lowry *et al.* [20] used the Markov chain approach to study the Western Electric Run Rules for S - and R -charts, and suggested alternative rules, also based on runs. The supplementary Run Rules need to be used cautiously. Traditionally, the performance of a control chart is evaluated by the Average Run Length (ARL). The standard Shewhart \bar{X} control chart has an in-control ARL equal to $ARL_0 = 1 / 0.0027 = 370.4$. However, Champ and Woodall [8] found that a Shewhart control chart with the Western Electric Rules results in an in-control ARL of 91.75, in contrast to 370.4 for the Shewhart control chart alone. This means that the simultaneously use of Run Rules may cause an increasing number of false alarms and so may increase the cost of quality. Some other works that have discussed the problem of ARL of Shewhart charts with Run Rules are that of Acosta-Mejia [2], Antzoulakos and Rakitzis [3], Champ and Woodall [9], Divoky and Taylor [10], Fu *et al.* [11], Khoo [17], Klein [18], Palm [26], Riaz *et al.* [29], Shmueli and Cohen [32], Yasui *et al.* [39] and Zhang and Castagliola [40].

The goal of this paper is to introduce and compare several run rules charts for monitoring the CV. It is important to note that, as for all run rules type control charts, the run rules charts for monitoring the CV introduced in this paper will not outperform more *advanced* strategies like the EWMA-CV² chart proposed by Castagliola *et al.* [7] or the SynCV chart proposed by Calzada and Scariano [5]. Consequently, this paper must be considered as a framework for quality practitioners who already made the choice to implement an *intermediate* type of control charts. The remainder of the paper is organized as follows: in Section 2, the distribution properties of the sample coefficient of variation $\hat{\gamma}$ are discussed; in Section 3, three run rules charts for monitoring the CV are introduced and their run length performances (ARL , $SDRL$) are defined; in Section 4, a numerical analysis is conducted in order to compare the three run rules charts for monitoring the CV and to propose simple guidelines; in Section 5, an illustrative example is provided to show the implementation of one run rules chart for monitoring the CV; in Section 6, concluding remarks with comments and recommendations for future research are given.

2. Properties of the (Sample) Coefficient of Variation

Let X be a positive random variable and let $\mu = E(X) > 0$ and $\sigma = \sigma(X)$ be the mean and standard-deviation of X respectively. By definition, the *coefficient of variation* γ of the random variable X is defined as

$$\gamma = \frac{\sigma}{\mu}.$$

Now, let us assume that $\{X_1, \dots, X_n\}$ is a sample of n normal i.i.d. (μ, σ) random variables. Let \bar{X} and S be the sample mean and the sample standard-deviation of X_1, \dots, X_n , i.e.,

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i,$$

and

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2}.$$

The sample coefficient of variation $\hat{\gamma}$ is defined as

$$\hat{\gamma} = \frac{S}{\bar{X}}.$$

By definition, $\hat{\gamma}$ is defined on $(0, +\infty)$. The distributional properties of the sample coefficient of variation $\hat{\gamma}$ have been studied by Hendricks and Robey [12], Iglewicz and Myers [14], Iglewicz *et al.* [15], McKay [22], Reh and Scheffler [28], Vangel [34] and Warren [36]. Among these authors, Iglewicz *et al.* [15] noticed that $\sqrt{n}/\hat{\gamma}$ follows a noncentral t distribution with $n-1$ degrees of freedom and noncentrality parameter \sqrt{n}/γ . Based on this property, it is easy to derive the c.d.f. (cumulative distribution function) $F_{\hat{\gamma}}(x|n, \gamma)$ of $\hat{\gamma}$ as

$$F_{\hat{\gamma}}(x|n, \gamma) = 1 - F_t\left(\frac{\sqrt{n}}{x} \mid n-1, \frac{\sqrt{n}}{\gamma}\right), \quad (1)$$

where $F_t(\cdot)$ is the c.d.f. of the noncentral t distribution with $n-1$ degrees of freedom and noncentrality parameter \sqrt{n}/γ . Inverting $F_{\hat{\gamma}}(x|n, \gamma)$ gives the inverse c.d.f. $F_{\hat{\gamma}}^{-1}(\alpha|n, \gamma)$ of $\hat{\gamma}$ as

$$F_{\hat{\gamma}}^{-1}(\alpha|n, \gamma) = \frac{\sqrt{n}}{F_t^{-1}\left(1-\alpha \mid n-1, \frac{\sqrt{n}}{\gamma}\right)}, \quad (2)$$

where $F_t^{-1}(\cdot)$ is the inverse c.d.f. of the non-central t distribution.

3. SPC Monitoring Using Run Rules Control Charts

Let us suppose that we observe subgroups $\{X_{k,1}, X_{k,2}, \dots, X_{k,n}\}$ of size n , at time $k=1, 2, \dots$. We assume that there is independence within and between these subgroups and we also assume that each random variable $X_{k,j}$ follows a normal (μ_k, σ_k) distribution where parameters μ_k and σ_k are constrained by the relation $\gamma_k = \sigma_k / \mu_k = \gamma_0$ when the process is in-control. This implies that from one subgroup to another, the values of μ_k and σ_k may change, but the coefficient of variation $\gamma_k = \sigma_k / \mu_k$ must be equal to some predefined in-control value $\gamma_0 = \sigma_0 / \mu_0$, common to all the subgroups where μ_0 is the in-control mean and σ_0 is the in-control standard-deviation.

Kang *et al.* [16] were the first to investigate the opportunity to monitor the coefficient of variation through a Shewhart type chart: in their paper, an application from the medical field is discussed to show the motivations which can lead quality practitioners to monitor the coefficient of variation instead of other sample statistics. In the remainder of this paper the Shewhart chart proposed by Kang *et al.* [16] is denoted as "SH- γ ". The control limits LCL_{SH} and UCL_{SH} proposed by Kang *et al.* [16] are probability type control limits with an assumed type I error rate of $\alpha_0 = 0.0027$, i.e. an in-control $ARL_0 = 370.4$. That is, LCL_{SH} and UCL_{SH} are respectively equal to:

$$LCL_{SH} = F_{\hat{\gamma}}^{-1}\left(\frac{\alpha_0}{2} \mid n, \gamma_0\right), \quad UCL_{SH} = F_{\hat{\gamma}}^{-1}\left(1 - \frac{\alpha_0}{2} \mid n, \gamma_0\right),$$

where $F_{\hat{\gamma}}^{-1}(\cdot)$ is the inverse c.d.f. of $\hat{\gamma}$ defined in (2).

The goal of this paper is to apply and evaluate several Run Rules strategies in order to monitor the CV. It is important to note that, like in Klein [18], we only focus on *pure* Run Rules type charts which only require warning limits and no control limit and assume that an out-of-control condition must be signaled only if the selected Run Rules pattern occurs.

In the 2-out-of-3 Run Rules (from now on, denoted as $RR_{2,3-\gamma}$ chart), an out-of-control signal is obtained if two out of three successive values $\hat{\gamma}_k$ are plotted above an upper warning limit UWL or two out of three successive points are plotted below a lower warning limit LWL with

$$LWL = \mu_0(\hat{\gamma}) - K_{2,3} \times \sigma_0(\hat{\gamma}), \quad (3)$$

$$UWL = \mu_0(\hat{\gamma}) + K_{2,3} \times \sigma_0(\hat{\gamma}), \quad (4)$$

where $K_{2,3}$ is a chart parameter to be defined and where $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$ are the mean and standard-deviation of the sample coefficient of variation $\hat{\gamma}$ when the process is in-control, i.e. $\gamma_k = \gamma_0$. Since there is no closed form for $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$, Hong *et al.* [13] suggested to use the approximations proposed by Reh and Scheffler [28], i.e.

$$\mu_0(\hat{\gamma}) = \gamma_0 \left(1 + \frac{1}{n} \left(\gamma_0^2 - \frac{1}{4} \right) + \frac{1}{n^2} \left(3\gamma_0^4 - \frac{\gamma_0^2}{4} - \frac{7}{32} \right) + \frac{1}{n^3} \left(15\gamma_0^6 - \frac{3\gamma_0^4}{4} - \frac{7\gamma_0^2}{32} - \frac{19}{128} \right) \right), \quad (5)$$

$$\sigma_0(\hat{\gamma}) = \gamma_0 \sqrt{\frac{1}{n} \left(\gamma_0^2 + \frac{1}{2} \right) + \frac{1}{n^2} \left(8\gamma_0^4 + \gamma_0^2 + \frac{3}{8} \right) + \frac{1}{n^3} \left(69\gamma_0^6 + \frac{7\gamma_0^4}{2} + \frac{3\gamma_0^2}{4} + \frac{3}{16} \right)}. \quad (6)$$

In order to investigate the $RR_{2,3-\gamma}$ chart, it is necessary to define all the possible states depending on the position of the last two points plotted on the chart, as pictured in Figure 1.

- (1) 1st point above UWL and 2nd point between the warning limits,
- (2) 1st point above UWL and 2nd point below LWL ,
- (3) 1st point between the warning limits and 2nd point above UWL ,
- (4) both points between the warning limits,
- (5) 1st point between the warning limits and 2nd point below LWL ,
- (6) 1st point below LWL and 2nd point above UWL ,
- (7) 1st point below LWL and 2nd point between the warning limits.

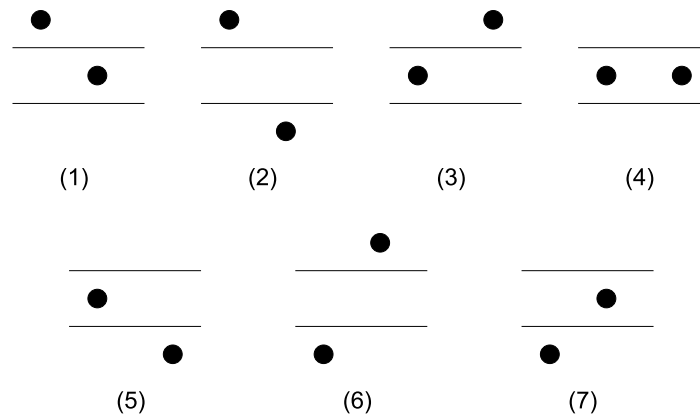


Figure 1. States of 2-out-of-3 Run Rules chart.

Thus a new point, i.e. the third point of the run, will lead to one of the above seven states or into the 8th state where two out of the three successive points are plotted out of the warning limits, i.e. an out-of-control status is signaled. The run length performances (ARL and $SDRL$) of the $RR_{2,3-\gamma}$ chart can be obtained by using the following Markov chain matrix $\mathbf{P}_{(8 \times 8)}$ corresponding to the eight previously defined states (where the 8th state corresponds to the absorbing state)

$$\mathbf{P} = \begin{pmatrix} \mathbf{Q} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{pmatrix} = \left(\begin{array}{cccccc|c} 0 & 0 & 0 & p_C & p_L & 0 & 0 & p_U \\ 0 & 0 & 0 & 0 & 0 & 0 & p_C & p_L + p_U \\ p_C & p_L & 0 & 0 & 0 & 0 & 0 & p_U \\ 0 & 0 & p_U & p_C & p_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_U & p_C & p_L \\ p_C & 0 & 0 & 0 & 0 & 0 & 0 & p_L + p_U \\ 0 & 0 & p_U & p_C & 0 & 0 & 0 & p_L \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right),$$

where $\mathbf{0} = (0, 0, \dots, 0)^T$, $\mathbf{Q}_{(7 \times 7)}$ is matrix of transient probabilities, the vector $\mathbf{r}_{(7 \times 1)}$ satisfies $\mathbf{r} = \mathbf{1} - \mathbf{Q}\mathbf{1}$ (i.e. row probabilities must sum to 1), with $\mathbf{1} = (1, 1, 1, 1, 1, 1, 1)^T$. The corresponding vector $\mathbf{q}_{(7 \times 1)}$ of initial probabilities associated with the transient states is equal to $\mathbf{q} = (0, 0, 0, 1, 0, 0, 0)^T$ (i.e. the initial state is the fourth one). The probabilities $p_L = P(\hat{\gamma} \leq LWL)$, $p_U = P(\hat{\gamma} \geq UWL)$ and $p_C = P(LWL \leq \hat{\gamma} \leq UWL)$ are equal to

$$\begin{aligned} p_L &= F_{\hat{\gamma}}(LWL | n, \gamma_1), \\ p_U &= 1 - F_{\hat{\gamma}}(UWL | n, \gamma_1), \\ p_C &= 1 - p_L - p_U, \end{aligned}$$

where $F_{\hat{\gamma}}(\cdot | n, \gamma_1)$ is the c.d.f. of $\hat{\gamma}$ as defined in (1) and where $\gamma_1 = \tau\gamma_0$ is an out-of-control value for the CV. Values of $\tau \in (0, 1)$ correspond to a decrease of the nominal coefficient of variation, while values of $\tau > 1$ correspond to an increase of the nominal coefficient of variation.

The number of steps L until the process reaches the absorbing state is a *Discrete PHase-type* (or DPH) random variable of parameters (\mathbf{Q}, \mathbf{q}) , (see Neuts [24] or Latouche and Ramaswami [19]). Consequently, the mean (ARL) and the standard-deviation ($SDRL$) of the run length L of the $RR_{2,3-\gamma}$ chart are equal to

$$\begin{aligned} ARL &= v_1, \\ SDRL &= \sqrt{\mu_2}, \end{aligned}$$

with

$$\begin{aligned} v_1 &= \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{1}, \\ v_2 &= 2\mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-2} \mathbf{Q} \mathbf{1}, \\ \mu_2 &= v_2 - v_1^2 + v_1. \end{aligned}$$

The $RR_{2,3-\gamma}$ chart may easily be extended to “longer” Run Rules like, for instance, the 3-out-of-4 and the 4-out-of-5 Run Rules (denoted as the $RR_{3,4-\gamma}$ and $RR_{4,5-\gamma}$ charts in the following). An example of application of longer Run Rules can be found in Capobianco *et al.* [6] where a $RR_{3,4-\bar{X}}$ chart is implemented in order to monitor the mean of a quality characteristic in a piston manufacturing line and is demonstrated to outperform the

the average RL will be denoted ARL_0 and the standard deviation of the RL will be denoted $SDRL_0$. On the contrary, when a process is out-of-control, the average RL will be denoted ARL_1 and the standard deviation of the RL is $SDRL_1$. A control chart is considered better than its competitors if it has the smaller ARL_1 value for a specific mean shift τ when ARL_0 is the same for all the charts.

In practice, it is interesting for the quality practitioner to find out the optimal chart parameter K^* (i.e. in our case K_{23}^* , K_{34}^* or K_{45}^*) such that

$$ARL(\gamma_0, \gamma_0, K^*, n) = ARL_0,$$

i.e. the optimal chart parameter K^* such that when the process is functioning at the nominal coefficient of variation $\gamma = \gamma_0$, then $ARL = ARL_0$, where ARL_0 is some predefined "in-control" ARL value.

Table 1 provides out-of-control ($ARL_1, SDRL_1$) of the $SH - \gamma$ chart for different combinations of $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case). The value for the in-control ARL is set at $ARL_0 = 370.4$. For the same combinations of n , γ_0 and τ , Tables 2, 3 and 4 also provide out-of-control ($ARL_1, SDRL_1$) of the $RR_{2,3} - \gamma$, $RR_{3,4} - \gamma$ and $RR_{4,5} - \gamma$ charts, respectively, as well as the optimal chart parameters K_{23}^* , K_{34}^* and K_{45}^* . For instance, concerning the increasing case, if $n = 10$, $\gamma_0 = 0.15$ and $\tau = 1.1$ (i.e. $\gamma_1 = 0.165$) then $ARL_1 = 123.1$ and $SDRL_1 = 122.6$ for the $SH - \gamma$ chart (in Table 1) while, for the $RR_{2,3} - \gamma$, $RR_{3,4} - \gamma$ and $RR_{4,5} - \gamma$ charts, the corresponding values of ARL_1 and $SDRL_1$ in Tables 2, 3 and 4 are (74, 72.2), (81.1, 78.4) and (89.2, 85.6), respectively. Concerning the decreasing case, if $n = 15$, $\gamma_0 = 0.2$ and $\tau = 0.8$ (i.e. $\gamma_1 = 0.16$) then $ARL_1 = 74$ and $SDRL_1 = 73.5$ for the $SH - \gamma$ chart (in Table 1) while, for the $RR_{2,3} - \gamma$, $RR_{3,4} - \gamma$ and $RR_{4,5} - \gamma$ charts, the corresponding values of ARL_1 and $SDRL_1$ in Tables 2, 3 and 4 are (45.9, 44.1), (20.9, 18.4) and (15.3, 12.2), respectively.

In particular, as it can be noticed in Table 2 for the $RR_{2,3} - \gamma$ chart, the performance of this two-sided scheme reveals the problem of ARL -biased performance (see for instance Acosta-Mejia [1]), especially for small sample sizes. This fact seriously affects the simultaneous monitoring of increasing or decreasing shifts in CV, since it is more difficult to detect certain (decreasing) shifts in the process parameter. One way to overcome this problem is to suggest a one-sided (downward) version of the $RR_{2,3} - \gamma$ chart with a single lower warning limit $LWL = \mu_0(\hat{\gamma}) - K_{2,3} \times \sigma_0(\hat{\gamma})$. The run length performances (ARL and $SDRL$) of the downward $RR_{2,3} - \gamma$ chart can be obtained the same way as for the two-sided case by using the following matrix of transient probabilities $\mathbf{Q}_{(3 \times 3)}$ (corresponding to states 4, 5 and 7 in Figure 1, re-labeled 1, 2 and 3)

$$\mathbf{Q} = \begin{pmatrix} 1 - p_L & p_L & 0 \\ 0 & 0 & 1 - p_L \\ 1 - p_L & 0 & 0 \end{pmatrix},$$

with the vector $\mathbf{q}_{(3 \times 1)} = (1, 0, 0)^T$ of initial probabilities associated with the transient states. For example, in Table 2, for the (two-sided) $RR_{2,3} - \gamma$ chart and for $n = 5$, $\gamma_0 = 0.05$ and $\tau = 0.9$ we have $K_{2,3}^* = 1.934$, $ARL_1 = 1179.5$ and $SDRL_1 = 1177.5$. We clearly have an ARL -biased situation since $ARL_1 > ARL_0 = 370.4$. If we use a downward one-sided $RR_{2,3} - \gamma$ chart, the chart parameter $K_{2,3}^* = 1.604$ and $ARL_1 = 182.2$ and $SDRL_1 = 180.4$. Using such a downward one-sided $RR_{2,3} - \gamma$ chart allows to overcome the ARL -biased situation and improve the performance of the $RR_{2,3} - \gamma$ chart.

Table 1. $(ARL_1, SDRL_1)$ of the SH- γ chart for $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case). The in-control ARL is $ARL_0 = 370.4$.

| $n = 5$ | | | | |
|----------|-------------------|------------------|-------------------|------------------|
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | (51.5,51.0) | (51.8,51.3) | (52.4,51.9) | (53.1,52.6) |
| 0.6 | (102.3,101.8) | (102.9,102.4) | (103.8,103.3) | (105.1,104.6) |
| 0.7 | (184.7,184.2) | (185.5,185.0) | (186.9,186.4) | (188.7,188.2) |
| 0.8 | (308.4,307.9) | (309.4,308.9) | (310.9,310.4) | (313.0,312.5) |
| 0.9 | (445.7,445.2) | (446.1,445.6) | (446.6,446.1) | (447.3,446.8) |
| 1.1 | (159.9,159.4) | (160.6,160.1) | (162.0,161.5) | (164.0,163.4) |
| 1.2 | (64.7,64.2) | (65.3,64.8) | (66.4,65.9) | (68.1,67.6) |
| 1.5 | (10.6,10.1) | (10.8,10.3) | (11.1,10.6) | (11.6,11.1) |
| 2.0 | (2.9,2.3) | (2.9,2.4) | (3.1,2.5) | (3.2,2.7) |
| 2.5 | (1.7,1.1) | (1.8,1.2) | (1.8,1.2) | (1.9,1.3) |
| $n = 7$ | | | | |
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | (18.4,17.9) | (18.6,18.1) | (18.9,18.4) | (19.3,18.8) |
| 0.6 | (45.7,45.2) | (46.1,45.6) | (46.8,46.3) | (47.7,47.2) |
| 0.7 | (102.8,102.3) | (103.5,103.0) | (104.8,104.3) | (106.5,106.0) |
| 0.8 | (212.1,211.6) | (213.2,212.7) | (215.0,214.5) | (217.5,217.0) |
| 0.9 | (384.2,383.7) | (385.0,384.5) | (386.2,385.7) | (387.8,387.3) |
| 1.1 | (141.2,140.7) | (142.2,141.7) | (143.8,143.3) | (146.1,145.6) |
| 1.2 | (50.3,49.8) | (50.9,50.4) | (52.0,51.5) | (53.7,53.2) |
| 1.5 | (7.2,6.7) | (7.4,6.8) | (7.6,7.1) | (8.0,7.5) |
| 2.0 | (2.1,1.5) | (2.1,1.5) | (2.2,1.6) | (2.3,1.7) |
| 2.5 | (1.3,0.7) | (1.4,0.7) | (1.4,0.8) | (1.5,0.8) |
| $n = 10$ | | | | |
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | (6.2,5.7) | (6.2,5.7) | (6.4,5.8) | (6.5,6.0) |
| 0.6 | (17.9,17.4) | (18.1,17.6) | (18.5,18.0) | (19.0,18.5) |
| 0.7 | (50.1,49.6) | (50.7,50.2) | (51.5,51.0) | (52.8,52.3) |
| 0.8 | (131.7,131.2) | (132.7,132.2) | (134.5,134.0) | (136.8,136.3) |
| 0.9 | (311.6,311.1) | (312.8,312.3) | (314.6,314.1) | (317.1,316.6) |
| 1.1 | (120.3,119.8) | (121.3,120.8) | (123.1,122.6) | (125.7,125.2) |
| 1.2 | (37.1,36.6) | (37.7,37.2) | (38.7,38.2) | (40.2,39.7) |
| 1.5 | (4.8,4.2) | (4.9,4.4) | (5.1,4.6) | (5.4,4.9) |
| 2.0 | (1.5,0.9) | (1.6,0.9) | (1.6,1.0) | (1.7,1.1) |
| 2.5 | (1.1,0.4) | (1.1,0.4) | (1.2,0.4) | (1.2,0.5) |
| $n = 15$ | | | | |
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | (2.2,1.6) | (2.2,1.6) | (2.2,1.7) | (2.3,1.7) |
| 0.6 | (6.2,5.6) | (6.3,5.7) | (6.4,5.9) | (6.6,6.1) |
| 0.7 | (20.5,20.0) | (20.8,20.2) | (21.2,20.7) | (21.9,21.4) |
| 0.8 | (70.1,69.6) | (70.9,70.4) | (72.2,71.7) | (74.0,73.5) |
| 0.9 | (229.9,229.4) | (231.3,230.8) | (233.6,233.1) | (236.7,236.2) |
| 1.1 | (95.8,95.3) | (96.9,96.4) | (98.8,98.3) | (101.3,100.8) |
| 1.2 | (25.0,24.5) | (25.5,25.0) | (26.3,25.8) | (27.5,27.0) |
| 1.5 | (3.0,2.5) | (3.1,2.5) | (3.2,2.7) | (3.4,2.9) |
| 2.0 | (1.2,0.5) | (1.2,0.5) | (1.2,0.5) | (1.3,0.6) |
| 2.5 | (1.0,0.2) | (1.0,0.2) | (1.0,0.2) | (1.1,0.2) |

Table 2. Optimal chart parameter $K_{2,3}^*$ and corresponding $(ARL_1, SDR L_1)$ of the $RR_{2,3}-\gamma$ chart, for $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case). The in-control ARL is $ARL_0 = 370.4$.

| $n = 5$ | | | | |
|----------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{2,3}^* = 1.934$ | $\gamma_0 = 0.10$ $K_{2,3}^* = 1.936$ | $\gamma_0 = 0.15$ $K_{2,3}^* = 1.942$ | $\gamma_0 = 0.20$ $K_{2,3}^* = 1.949$ |
| 0.5 | (39.7, 38.0) | (43.8, 42.1) | (52.2, 50.5) | (68.7, 67.0) |
| 0.6 | (118.9, 117.1) | (132.6, 130.7) | (161.0, 159.1) | (217.7, 215.8) |
| 0.7 | (329.4, 327.5) | (369.5, 367.6) | (453.2, 451.3) | (621.3, 619.4) |
| 0.8 | (825.4, 823.5) | (925.9, 924.0) | (1134.4, 1132.4) | (1547.1, 1545.2) |
| 0.9 | (1179.5, 1177.5) | (1251.4, 1249.5) | (1376.9, 1375.0) | (1558.5, 1556.6) |
| 1.1 | (101.6, 99.8) | (101.5, 99.7) | (101.5, 99.7) | (101.7, 99.9) |
| 1.2 | (39.1, 37.4) | (39.3, 37.6) | (39.6, 37.8) | (40.0, 38.3) |
| 1.5 | (8.3, 6.8) | (8.4, 6.8) | (8.5, 7.0) | (8.8, 7.2) |
| 2.0 | (3.4, 1.9) | (3.4, 1.9) | (3.5, 2.0) | (3.6, 2.1) |
| 2.5 | (2.6, 1.0) | (2.6, 1.0) | (2.6, 1.0) | (2.7, 1.1) |
| $n = 7$ | | | | |
| τ | $\gamma_0 = 0.05$ $K_{2,3}^* = 1.930$ | $\gamma_0 = 0.10$ $K_{2,3}^* = 1.932$ | $\gamma_0 = 0.15$ $K_{2,3}^* = 1.935$ | $\gamma_0 = 0.20$ $K_{2,3}^* = 1.940$ |
| 0.5 | (8.2, 6.7) | (8.6, 7.1) | (9.3, 7.8) | (10.5, 9.0) |
| 0.6 | (24.1, 22.5) | (25.6, 23.9) | (28.4, 26.7) | (33.2, 31.5) |
| 0.7 | (76.8, 75.0) | (82.3, 80.5) | (92.7, 90.9) | (110.9, 109.0) |
| 0.8 | (245.0, 243.1) | (263.5, 261.6) | (299.1, 297.2) | (361.2, 359.3) |
| 0.9 | (651.1, 649.2) | (690.6, 688.7) | (763.1, 761.2) | (880.1, 878.1) |
| 1.1 | (88.4, 86.7) | (88.3, 86.5) | (88.2, 86.4) | (88.2, 86.4) |
| 1.2 | (30.3, 28.6) | (30.4, 28.8) | (30.7, 29.0) | (31.1, 29.4) |
| 1.5 | (6.0, 4.5) | (6.1, 4.6) | (6.2, 4.7) | (6.4, 4.9) |
| 2.0 | (2.7, 1.2) | (2.8, 1.2) | (2.8, 1.2) | (2.9, 1.3) |
| 2.5 | (2.2, 0.5) | (2.2, 0.6) | (2.3, 0.6) | (2.3, 0.7) |
| $n = 10$ | | | | |
| τ | $\gamma_0 = 0.05$ $K_{2,3}^* = 1.930$ | $\gamma_0 = 0.10$ $K_{2,3}^* = 1.931$ | $\gamma_0 = 0.15$ $K_{2,3}^* = 1.933$ | $\gamma_0 = 0.20$ $K_{2,3}^* = 1.936$ |
| 0.5 | (3.4, 1.8) | (3.4, 1.9) | (3.6, 2.0) | (3.7, 2.2) |
| 0.6 | (7.9, 6.3) | (8.2, 6.6) | (8.7, 7.1) | (9.5, 7.9) |
| 0.7 | (25.0, 23.4) | (26.2, 24.6) | (28.4, 26.7) | (31.9, 30.2) |
| 0.8 | (95.0, 93.2) | (100.1, 98.3) | (109.6, 107.7) | (125.0, 123.2) |
| 0.9 | (368.8, 366.9) | (387.6, 385.7) | (421.8, 419.9) | (476.6, 474.7) |
| 1.1 | (73.9, 72.2) | (73.9, 72.2) | (74.0, 72.2) | (74.1, 72.3) |
| 1.2 | (22.4, 20.7) | (22.5, 20.9) | (22.8, 21.2) | (23.3, 21.6) |
| 1.5 | (4.4, 2.9) | (4.4, 2.9) | (4.5, 3.0) | (4.6, 3.1) |
| 2.0 | (2.3, 0.7) | (2.3, 0.7) | (2.4, 0.7) | (2.4, 0.8) |
| 2.5 | (2.1, 0.3) | (2.1, 0.3) | (2.1, 0.3) | (2.1, 0.4) |
| $n = 15$ | | | | |
| τ | $\gamma_0 = 0.05$ $K_{2,3}^* = 1.929$ | $\gamma_0 = 0.10$ $K_{2,3}^* = 1.930$ | $\gamma_0 = 0.15$ $K_{2,3}^* = 1.931$ | $\gamma_0 = 0.20$ $K_{2,3}^* = 1.933$ |
| 0.5 | (2.2, 0.5) | (2.2, 0.5) | (2.2, 0.6) | (2.3, 0.6) |
| 0.6 | (3.5, 2.0) | (3.6, 2.1) | (3.7, 2.2) | (3.8, 2.3) |
| 0.7 | (9.2, 7.7) | (9.5, 8.0) | (10.0, 8.5) | (10.8, 9.2) |
| 0.8 | (37.4, 35.7) | (38.9, 37.2) | (41.6, 39.9) | (45.9, 44.1) |
| 0.9 | (200.6, 198.7) | (208.9, 207.1) | (223.8, 221.9) | (247.0, 245.1) |
| 1.1 | (57.4, 55.6) | (57.6, 55.8) | (57.8, 56.1) | (58.3, 56.5) |
| 1.2 | (15.3, 13.7) | (15.4, 13.8) | (15.7, 14.1) | (16.1, 14.5) |
| 1.5 | (3.2, 1.7) | (3.2, 1.7) | (3.3, 1.8) | (3.4, 1.9) |
| 2.0 | (2.1, 0.3) | (2.1, 0.3) | (2.1, 0.4) | (2.1, 0.4) |
| 2.5 | (2.0, 0.1) | (2.0, 0.1) | (2.0, 0.1) | (2.0, 0.1) |

Table 3. Optimal chart parameter $K_{3,4}^*$ and corresponding (ARL_1, SDR_1) of the $RR_{3,4-\gamma}$ chart, for $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case). The in-control ARL is $ARL_0 = 370.4$.

| $n = 5$ | | | | |
|---------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{3,4}^* = 1.392$ | $\gamma_0 = 0.10$ $K_{3,4}^* = 1.390$ | $\gamma_0 = 0.15$ $K_{3,4}^* = 1.387$ | $\gamma_0 = 0.20$ $K_{3,4}^* = 1.381$ |
| 0.5 | (8.3, 6.0) | (8.4, 6.1) | (8.7, 6.3) | (9.0, 6.6) |
| 0.6 | (19.9, 17.4) | (20.3, 17.8) | (21.0, 18.5) | (22.1, 19.6) |
| 0.7 | (53.7, 51.1) | (55.0, 52.3) | (57.2, 54.5) | (60.4, 57.7) |
| 0.8 | (151.0, 148.2) | (154.7, 151.9) | (161.0, 158.2) | (170.4, 167.6) |
| 0.9 | (388.7, 385.8) | (396.3, 393.4) | (409.2, 406.3) | (427.9, 424.9) |
| 1.1 | (120.9, 118.1) | (120.9, 118.1) | (120.8, 118.0) | (120.9, 118.1) |
| 1.2 | (46.0, 43.4) | (46.2, 43.6) | (46.5, 43.9) | (47.0, 44.4) |
| 1.5 | (10.0, 7.6) | (10.1, 7.7) | (10.3, 7.9) | (10.5, 8.1) |
| 2.0 | (4.5, 2.1) | (4.5, 2.2) | (4.6, 2.2) | (4.7, 2.3) |
| 2.5 | (3.6, 1.0) | (3.6, 1.1) | (3.6, 1.1) | (3.7, 1.2) |

| $n = 7$ | | | | |
|---------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{3,4}^* = 1.393$ | $\gamma_0 = 0.10$ $K_{3,4}^* = 1.392$ | $\gamma_0 = 0.15$ $K_{3,4}^* = 1.389$ | $\gamma_0 = 0.20$ $K_{3,4}^* = 1.386$ |
| 0.5 | (4.5, 2.1) | (4.5, 2.1) | (4.6, 2.2) | (4.7, 2.3) |
| 0.6 | (8.8, 6.5) | (9.0, 6.6) | (9.2, 6.9) | (9.5, 7.2) |
| 0.7 | (23.5, 21.0) | (24.0, 21.5) | (24.8, 22.3) | (26.1, 23.5) |
| 0.8 | (76.3, 73.6) | (78.1, 75.3) | (81.0, 78.3) | (85.3, 82.6) |
| 0.9 | (265.5, 262.6) | (270.7, 267.8) | (279.6, 276.7) | (292.4, 289.5) |
| 1.1 | (101.0, 98.2) | (101.1, 98.4) | (101.5, 98.7) | (102.0, 99.2) |
| 1.2 | (33.8, 31.2) | (34.0, 31.4) | (34.5, 31.9) | (35.1, 32.5) |
| 1.5 | (7.1, 4.8) | (7.2, 4.9) | (7.4, 5.0) | (7.6, 5.2) |
| 2.0 | (3.7, 1.2) | (3.7, 1.2) | (3.8, 1.3) | (3.9, 1.4) |
| 2.5 | (3.2, 0.5) | (3.2, 0.6) | (3.2, 0.6) | (3.3, 0.7) |

| $n = 10$ | | | | |
|----------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{3,4}^* = 1.393$ | $\gamma_0 = 0.10$ $K_{3,4}^* = 1.392$ | $\gamma_0 = 0.15$ $K_{3,4}^* = 1.391$ | $\gamma_0 = 0.20$ $K_{3,4}^* = 1.388$ |
| 0.5 | (3.3, 0.7) | (3.3, 0.7) | (3.3, 0.8) | (3.4, 0.8) |
| 0.6 | (5.0, 2.6) | (5.0, 2.7) | (5.1, 2.7) | (5.2, 2.9) |
| 0.7 | (11.4, 9.0) | (11.6, 9.2) | (11.9, 9.5) | (12.4, 10.0) |
| 0.8 | (38.8, 36.2) | (39.7, 37.1) | (41.1, 38.5) | (43.1, 40.5) |
| 0.9 | (174.6, 171.7) | (178.0, 175.1) | (183.7, 180.9) | (192.1, 189.2) |
| 1.1 | (80.1, 77.4) | (80.5, 77.7) | (81.1, 78.4) | (82.0, 79.2) |
| 1.2 | (23.7, 21.2) | (24.0, 21.5) | (24.4, 21.9) | (25.1, 22.5) |
| 1.5 | (5.3, 2.9) | (5.3, 3.0) | (5.4, 3.1) | (5.6, 3.2) |
| 2.0 | (3.3, 0.6) | (3.3, 0.7) | (3.3, 0.7) | (3.4, 0.8) |
| 2.5 | (3.1, 0.2) | (3.1, 0.3) | (3.1, 0.3) | (3.1, 0.3) |

Table 3. (Continued).

| $n = 15$ | | | | |
|----------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{3,4}^* = 1.393$ | $\gamma_0 = 0.10$ $K_{3,4}^* = 1.393$ | $\gamma_0 = 0.15$ $K_{3,4}^* = 1.391$ | $\gamma_0 = 0.20$ $K_{3,4}^* = 1.390$ |
| 0.5 | (3.0,0.2) | (3.0,0.2) | (3.0,0.2) | (3.0,0.2) |
| 0.6 | (3.5,1.0) | (3.5,1.0) | (3.5,1.0) | (3.6,1.1) |
| 0.7 | (6.1,3.7) | (6.1,3.8) | (6.3,3.9) | (6.5,4.1) |
| 0.8 | (19.0,16.6) | (19.4,16.9) | (20.0,17.5) | (20.9,18.4) |
| 0.9 | (105.5,102.8) | (107.6,104.8) | (111.0,108.3) | (116.0,113.2) |
| 1.1 | (58.6,55.9) | (59.1,56.4) | (59.9,57.2) | (61.0,58.3) |
| 1.2 | (15.7,13.2) | (15.9,13.5) | (16.2,13.8) | (16.7,14.3) |
| 1.5 | (4.0,1.6) | (4.1,1.7) | (4.1,1.7) | (4.2,1.8) |
| 2.0 | (3.1,0.3) | (3.1,0.3) | (3.1,0.3) | (3.1,0.4) |
| 2.5 | (3.0,0.1) | (3.0,0.1) | (3.0,0.1) | (3.0,0.1) |

Table 4. Optimal chart parameter $K_{4,5}^*$ and corresponding (ARL_1, SDR_1) of the $RR_{4,5-\gamma}$ chart, for $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case). The in-control ARL is $ARL_0 = 370.4$.

| $n = 5$ | | | | |
|---------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{4,5}^* = 1.051$ | $\gamma_0 = 0.10$ $K_{4,5}^* = 1.049$ | $\gamma_0 = 0.15$ $K_{4,5}^* = 1.046$ | $\gamma_0 = 0.20$ $K_{4,5}^* = 1.041$ |
| 0.5 | (6.2,3.0) | (6.2,3.0) | (6.3,3.1) | (6.3,3.1) |
| 0.6 | (11.8,8.6) | (11.8,8.7) | (12.0,8.8) | (12.2,9.0) |
| 0.7 | (28.4,25.1) | (28.6,25.3) | (29.0,25.7) | (29.5,26.2) |
| 0.8 | (80.0,76.4) | (80.6,77.1) | (81.6,78.1) | (83.0,79.4) |
| 0.9 | (236.5,232.7) | (237.5,233.8) | (239.3,235.5) | (241.6,237.8) |
| 1.1 | (144.5,140.8) | (145.3,141.6) | (146.6,142.9) | (148.4,144.7) |
| 1.2 | (54.4,50.9) | (54.9,51.4) | (55.8,52.3) | (57.0,53.5) |
| 1.5 | (11.8,8.7) | (12.0,8.8) | (12.2,9.1) | (12.6,9.4) |
| 2.0 | (5.6,2.4) | (5.7,2.4) | (5.7,2.5) | (5.9,2.7) |
| 2.5 | (4.6,1.1) | (4.6,1.2) | (4.7,1.2) | (4.7,1.3) |

| $n = 7$ | | | | |
|---------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{4,5}^* = 1.049$ | $\gamma_0 = 0.10$ $K_{4,5}^* = 1.048$ | $\gamma_0 = 0.15$ $K_{4,5}^* = 1.045$ | $\gamma_0 = 0.20$ $K_{4,5}^* = 1.041$ |
| 0.5 | (4.6,1.1) | (4.6,1.2) | (4.6,1.2) | (4.6,1.2) |
| 0.6 | (6.9,3.8) | (7.0,3.8) | (7.1,3.9) | (7.2,4.0) |
| 0.7 | (15.2,12.0) | (15.3,12.1) | (15.6,12.4) | (15.9,12.7) |
| 0.8 | (45.9,42.5) | (46.3,42.9) | (47.1,43.7) | (48.2,44.8) |
| 0.9 | (172.5,168.8) | (173.7,170.0) | (175.7,172.0) | (178.4,174.7) |
| 1.1 | (114.8,111.1) | (115.6,112.0) | (117.1,113.4) | (119.1,115.5) |
| 1.2 | (37.8,34.4) | (38.3,34.9) | (39.0,35.7) | (40.1,36.7) |
| 1.5 | (8.4,5.2) | (8.5,5.3) | (8.6,5.5) | (8.9,5.8) |
| 2.0 | (4.7,1.3) | (4.7,1.3) | (4.8,1.4) | (4.9,1.5) |
| 2.5 | (4.2,0.5) | (4.2,0.6) | (4.2,0.6) | (4.3,0.7) |

Table 4. (Continued).

| $n = 10$ | | | | |
|----------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{4,5}^* = 1.047$ | $\gamma_0 = 0.10$ $K_{4,5}^* = 1.046$ | $\gamma_0 = 0.15$ $K_{4,5}^* = 1.045$ | $\gamma_0 = 0.20$ $K_{4,5}^* = 1.042$ |
| 0.5 | (4.1,0.4) | (4.1,0.4) | (4.1,0.4) | (4.1,0.4) |
| 0.6 | (5.0,1.6) | (5.0,1.7) | (5.0,1.7) | (5.1,1.8) |
| 0.7 | (8.9,5.8) | (9.0,5.9) | (9.1,6.0) | (9.3,6.2) |
| 0.8 | (26.1,22.8) | (26.4,23.1) | (26.9,23.6) | (27.6,24.3) |
| 0.9 | (119.1,115.5) | (120.3,116.6) | (122.1,118.5) | (124.6,121.0) |
| 1.1 | (86.9,83.3) | (87.7,84.2) | (89.2,85.6) | (91.2,87.7) |
| 1.2 | (25.6,22.3) | (26.0,22.7) | (26.6,23.3) | (27.5,24.2) |
| 1.5 | (6.2,3.1) | (6.3,3.1) | (6.4,3.3) | (6.6,3.4) |
| 2.0 | (4.2,0.6) | (4.3,0.7) | (4.3,0.7) | (4.3,0.8) |
| 2.5 | (4.0,0.2) | (4.1,0.2) | (4.1,0.3) | (4.1,0.3) |

| $n = 15$ | | | | |
|----------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| τ | $\gamma_0 = 0.05$ $K_{4,5}^* = 1.046$ | $\gamma_0 = 0.10$ $K_{4,5}^* = 1.045$ | $\gamma_0 = 0.15$ $K_{4,5}^* = 1.044$ | $\gamma_0 = 0.20$ $K_{4,5}^* = 1.042$ |
| 0.5 | (4.0,0.1) | (4.0,0.1) | (4.0,0.1) | (4.0,0.1) |
| 0.6 | (4.2,0.6) | (4.2,0.6) | (4.2,0.6) | (4.2,0.6) |
| 0.7 | (5.8,2.6) | (5.8,2.6) | (5.9,2.7) | (6.0,2.8) |
| 0.8 | (14.4,11.3) | (14.6,11.5) | (14.9,11.7) | (15.3,12.2) |
| 0.9 | (75.3,71.8) | (76.2,72.6) | (77.6,74.1) | (79.6,76.1) |
| 1.1 | (60.9,57.4) | (61.6,58.2) | (62.9,59.4) | (64.7,61.2) |
| 1.2 | (16.6,13.4) | (16.8,13.6) | (17.3,14.1) | (17.9,14.7) |
| 1.5 | (4.9,1.6) | (5.0,1.7) | (5.0,1.7) | (5.1,1.8) |
| 2.0 | (4.1,0.2) | (4.1,0.3) | (4.1,0.3) | (4.1,0.3) |
| 2.5 | (4.0,0.1) | (4.0,0.1) | (4.0,0.1) | (4.0,0.1) |

In Tables 2, 3 and 4, the couples (ARL_1, SDR_1) in bold characters are the ones for which ARL_1 is smaller than for the SH- γ chart. As it can be immediately noticed, the cases where the SH- γ chart outperforms the RR $_{2,3-\gamma}$, RR $_{3,4-\gamma}$ and RR $_{4,5-\gamma}$ charts are confined to large values (i.e. increasing case) of τ , say $\tau \geq 1.5$. When $\tau \in [0.5, 1.5)$, the RR $_{2,3-\gamma}$, RR $_{3,4-\gamma}$ and RR $_{4,5-\gamma}$ charts always outperform the SH- γ chart. In order to have an overall evaluation of these charts, Table 5 summarizes which chart has the smaller ARL_1 values for different combinations of $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case). Some conclusions can be drawn from Table 5:

- when $\tau < 1$ and n is small (i.e. $n = \{5, 7\}$) the RR $_{4,5-\gamma}$ chart tends, in general, to outperform the other charts,
- when $\tau < 1$ and n is larger (i.e. $n = \{10, 15\}$) the RR $_{2,3-\gamma}$ or RR $_{3,4-\gamma}$ chart tends to be more efficient,
- when $\tau \in [1, 1.5)$, the RR $_{2,3-\gamma}$ chart outperforms the other charts.
- when $\tau \geq 1.5$ the best choice is the SH- γ chart.

Table 5. Best control charts for $n = \{5, 7, 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.15, 0.2\}$, $\tau = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ (i.e. decreasing case) and $\tau = \{1.1, 1.2, 1.5, 2, 2.5\}$ (i.e. increasing case).

| $n = 5$ | | | | |
|---------|------------------------------|------------------------------|------------------------------|------------------------------|
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.6 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.7 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.8 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.9 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 1.1 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.2 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.5 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 2.0 | SH- γ | SH- γ | SH- γ | SH- γ |
| 2.5 | SH- γ | SH- γ | SH- γ | SH- γ |

| $n = 7$ | | | | |
|---------|------------------------------|------------------------------|------------------------------|------------------------------|
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | RR _{3,4} - γ | RR _{3,4} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.6 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.7 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.8 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.9 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 1.1 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.2 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.5 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 2.0 | SH- γ | SH- γ | SH- γ | SH- γ |
| 2.5 | SH- γ | SH- γ | SH- γ | SH- γ |

| $n = 10$ | | | | |
|----------|------------------------------|------------------------------|------------------------------|------------------------------|
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | RR _{3,4} - γ | RR _{3,4} - γ | RR _{3,4} - γ | RR _{3,4} - γ |
| 0.6 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.7 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.8 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.9 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 1.1 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.2 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.5 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 2.0 | SH- γ | SH- γ | SH- γ | SH- γ |
| 2.5 | SH- γ | SH- γ | SH- γ | SH- γ |

| $n = 15$ | | | | |
|----------|------------------------------|------------------------------|------------------------------|------------------------------|
| τ | $\gamma_0 = 0.05$ | $\gamma_0 = 0.1$ | $\gamma_0 = 0.15$ | $\gamma_0 = 0.2$ |
| 0.5 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 0.6 | RR _{3,4} - γ | RR _{3,4} - γ | RR _{3,4} - γ | RR _{3,4} - γ |
| 0.7 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.8 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 0.9 | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ | RR _{4,5} - γ |
| 1.1 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.2 | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ | RR _{2,3} - γ |
| 1.5 | SH- γ | SH- γ | SH- γ | SH- γ |
| 2.0 | SH- γ | SH- γ | SH- γ | SH- γ |
| 2.5 | SH- γ | SH- γ | SH- γ | SH- γ |

As we already stressed in the Introduction section, the use of supplementary run rules is clearly an intermediate solution between pure Shewhart type charts (easy and well accepted in industry) and more advanced charts like EWMA (or CUSUM) and synthetic type charts (viewed as more complex and not so spread in industry). The consequence is that the results in terms of out-of-control ARL_1 values are globally not in favor of the $RR-\gamma$ charts. Concerning the EWMA-CV² chart, a comparison between Tables 2–4 and Tables 1 and 2 in Castagliola *et al.* [7] clearly demonstrates that the EWMA-CV² chart outperforms the $RR-\gamma$ charts for any shift $\tau \in [0.5, 2.5]$. Concerning the SynCV chart, the results are more contrasted. In the case of an increasing shift, based on Tables 5 and 6 in Calzada and Scariano [5], the SynCV chart also outperforms the $RR-\gamma$ charts for any shift $\tau \geq 1$. But, in the case of a decreasing shift $\tau \in [0.5, 1)$ (not evaluated in Calzada and Scariano [5]), the computations we conducted demonstrate that the $RR-\gamma$ charts outperform the SynCV chart. For instance, if we consider the case $n = 5$, $\gamma_0 = 0.05$ and $\tau = 0.5$, ARL_1 for the SynCV chart is 74.7 (the control limits are $LCL = 0.01222$ and $UCL = 0.08183$) while, for the $RR_{2,3}-\gamma$ chart $ARL_1 = 39.7$ (for the same conditions, $ARL_1 = 8.3$ for the $RR_{3,4}-\gamma$ chart and $ARL_1 = 6.2$ for the $RR_{4,5}-\gamma$ chart, i.e. even better).

Table 6. Phase I and phase II datasets from a sintering process.

| Phase I | | | | Phase II | | | |
|---------|-------------|-------|------------------|----------|-------------|--------|------------------|
| k | \bar{X}_k | S_k | $\hat{\gamma}_k$ | k | \bar{X}_k | S_k | $\hat{\gamma}_k$ |
| 1 | 664.2 | 268.9 | 0.405 | 1 | 906.4 | 476.0 | 0.525 |
| 2 | 705.6 | 308.6 | 0.437 | 2 | 805.1 | 493.9 | 0.614 |
| 3 | 1051.5 | 539.9 | 0.513 | 3 | 1584.7 | 1050.8 | 0.663 |
| 4 | 1047.3 | 359.0 | 0.343 | 4 | 663.4 | 304.8 | 0.459 |
| 5 | 618.2 | 136.3 | 0.221 | 5 | 1012.1 | 367.4 | 0.363 |
| 6 | 781.4 | 446.4 | 0.571 | 6 | 863.2 | 350.4 | 0.406 |
| 7 | 797.8 | 342.5 | 0.429 | 7 | 1068.3 | 150.8 | 0.141 |
| 8 | 678.9 | 275.4 | 0.406 | 8 | 697.1 | 253.2 | 0.363 |
| 9 | 848.3 | 320.5 | 0.378 | 9 | 1024.6 | 120.9 | 0.118 |
| 10 | 1015.3 | 453.7 | 0.447 | 10 | 355.3 | 235.2 | 0.662 |
| 11 | 777.4 | 276.4 | 0.356 | 11 | 485.6 | 106.5 | 0.219 |
| 12 | 813.9 | 170.7 | 0.210 | 12 | 1224.3 | 915.4 | 0.748 |
| 13 | 716.9 | 397.4 | 0.554 | 13 | 1365.0 | 1051.6 | 0.770 |
| 14 | 937.6 | 421.2 | 0.449 | 14 | 704.0 | 449.7 | 0.639 |
| 15 | 915.1 | 331.9 | 0.363 | 15 | 1187.2 | 1105.9 | 0.932 |
| 16 | 873.2 | 285.0 | 0.326 | 16 | 1130.0 | 680.6 | 0.602 |
| 17 | 984.3 | 573.7 | 0.583 | 17 | 824.7 | 393.5 | 0.477 |
| 18 | 819.3 | 156.2 | 0.191 | 18 | 921.2 | 391.6 | 0.425 |
| 19 | 839.0 | 244.0 | 0.291 | 19 | 870.3 | 730.0 | 0.839 |
| 20 | 585.8 | 322.3 | 0.550 | 20 | 1561.0 | 1652.2 | 1.058 |

5. An Illustrative Example

The proposed example considers actual data from a sintering process (an operation of powder metallurgy whereby compressed metal powder is heated to a temperature that allows bonding of the individual particles) kindly provided by an Italian company that manufactures sintered mechanical parts. The process manufactures parts which are required to guarantee a pressure test drop time T_{pd} from 2 bar to 1.5 bar larger than 30 sec as a quality characteristic related to the pore shrinkage. Using molten copper to fill pores during the sintering process allows the drop time to be significantly extended. In fact, the larger the quantity Q_C of

molten copper absorbed within the sintered compact during cooling, the larger is the expected pressure drop time T_{pd} . A preliminary regression study relating T_{pd} to the quantity Q_C of molten copper has demonstrated the presence of a constant proportionality $\sigma_{pd} = \gamma_{pd} \times \mu_{pd}$ between the standard deviation of the pressure drop time and its mean. To perform SPC by means of control charts the quality practitioner decided to monitor the coefficient of variation $\gamma_{pd} = \sigma_{pd} / \mu_{pd}$ in order to detect changes in the process variability. Given the nominal quantity of copper Q_C , a Phase I dataset of $m=20$ sample data, each having sample size $n=5$, have been collected; they are listed in Table 6 (left side). The analysis of the Phase I data resulted in an estimate $\hat{\gamma}_0=0.417$ based on a root-mean-square computation and proved that the sintering process is perfectly in-control.

To design the charts, the assumed increase in the coefficient of variation was set equal to $\tau=1.25$. In fact, accordingly to the process engineer experience, a shift of 25% in the coefficient of variation should be interpreted as a signal that something is going wrong in the production of the parts. The parameters of the $RR_{2,3-\gamma}$, $RR_{3,4-\gamma}$ and $RR_{4,5-\gamma}$ charts which are optimal for detecting a shift from $\gamma_0=0.417$ to $\gamma_1=\gamma_0 \times 1.25=0.521$ (i.e. increase of 25%) when $n=5$ are found by the optimizing algorithm to be $K_{2,3}^*=2.017$, $K_{3,4}^*=1.325$ and $K_{4,5}^*=0.989$, respectively. The corresponding couples $(ARL_1, SDRL_1)$ are $(32.8, 31.1)$, $(36.7, 34.1)$ and $(47.4, 44.0)$, respectively (to be compared with $(58.8, 58.3)$ for the $SH-\gamma$ chart). The most efficient chart in this case seems to be the $RR_{2,3-\gamma}$ chart. Using (5) and (6) we have $\mu_0(\hat{\gamma})=0.4074$, $\sigma_0(\hat{\gamma})=0.1733$ and the warning limits of the $RR_{2,3-\gamma}$ chart are

$$LWL = 0.4074 - 2.017 \times 0.1733 = 0.0579$$

$$UWL = 0.4074 + 2.017 \times 0.1733 = 0.7569$$

A second set of data collected during Phase II of the chart implementation are presented in Table 6 (right side). These data consist of 20 new samples taken from the process after the occurrence of a special cause increasing process variability. The $\hat{\gamma}_k$, the warning limits $LWL=0.0579$ and $UWL=0.7569$ of the $RR_{2,3-\gamma}$ chart and the “ 3σ ” upper control limit $UCL=0.4074+3 \times 0.1733=0.9273$ are plotted in Figure 2 (remark: the “ 3σ ” lower control limit $LCL=0.4074+3 \times 0.1733=-0.1125$ is not used). As it can be noticed, the $RR_{2,3-\gamma}$ chart detects an out-of-control situation (in bold in Table 6), at the 15th sample, confirming the occurrence of a special cause as it was expected by the engineers.

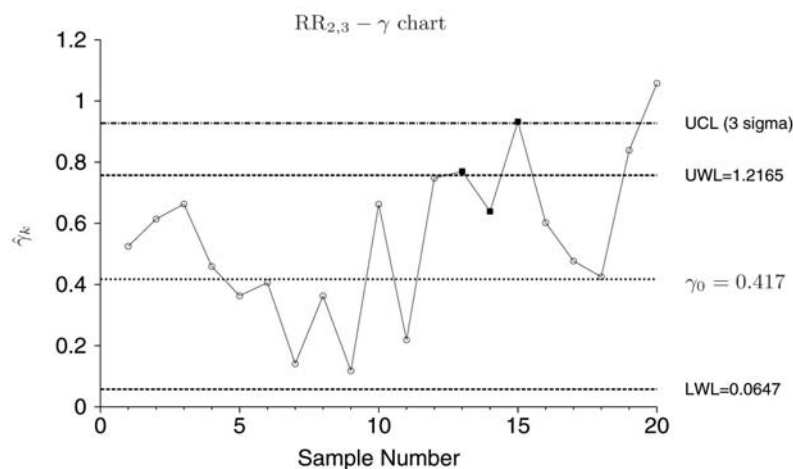


Figure 2. $RR_{2,3-\gamma}$ chart applied to the sintering process (Phase II).

6. Conclusions

Monitoring the coefficient of variation CV by means of a control chart is receiving growing attention in the context of SPC. There are many situations in which the sample mean and standard deviation vary naturally in a proportional manner when the process is in-control, in which case \bar{X} and S control charts cannot be implemented. In this paper several run rules type control charts are proposed to monitor the CV. An ARL computation based on a Markov Chain approach has been performed and optimal chart parameters K^* (i.e. in our case $K_{2,3}^*$, $K_{3,4}^*$ and $K_{4,5}^*$) have been proposed in order to detect specific shift size conditions. A comparison with the $SH-\gamma$ chart highlighted the following recommendations: when $\tau < 1$ and n is small (i.e. $n = \{5, 7\}$) the $RR_{4,5}-\gamma$ chart tends, in general, to outperform the other charts, when $\tau < 1$ and n is larger (i.e. $n = \{10, 15\}$) the $RR_{2,3}-\gamma$ or $RR_{3,4}-\gamma$ chart tends to be more efficient, when $\tau \in [1, 1.5)$, the $RR_{2,3}-\gamma$ chart outperforms the other charts and, finally, when $\tau \geq 1.5$ the best choice is the $SH-\gamma$ chart. The implementation of the proposed charts on a real set of data gathered from an industrial field showed how to implement the CV monitoring.

Future research should firstly be focused on extending the study to other strategies like the synthetic chart, adaptive type control charts (VSI and VSS) and the CUSUM chart. Since the value of the in-control CV γ_0 is rarely perfectly known, it is usually estimated from an in-control historical data set (Phase I). It is well known that when in-control parameters are estimated, the performance of control charts differs from the known parameters case due to the variability of the estimators used during the Phase I. A second research direction will be focused on evaluating the statistical performances of CV type charts when the in-control CV γ_0 is estimated, in order to provide guidelines about the number of Phase I samples to be used in practice and to suggest new specific chart parameters to be used in the estimated parameter case allowing similar in-control ARL 's as for the known parameter case. Finally, a systematic study of the relation between a shift in the process mean and/or dispersion and the shift of the coefficient of variation would provide a deeper understanding about how the charts monitoring CV are expected to react to out of control conditions originated by different special causes.

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