

Exploring Mathematics Form 2

Answers

Chapter 1 Numbers

Activity 1.1

This statement is correct. The number 34.00 has four significant figures, since the end digits of decimal numbers are significant. The number 34 has two significant figures since it has only two non-zero digits. The number 3 400 has two significant figures, since the end zeroes of whole numbers are not significant. The number 0.034 has two significant figures, since the leading zeroes in decimal numbers are not significant.

Exercise 1.1

- (a) 5 (b) 7 000 (c) 3
- (a) 1 (b) 2 (c) 3 (d) 3
(e) 3 (f) 1 (g) 3 (h) 3
(i) 4 (j) 6
- The digits 1 and 9 are significant. The zero digits are not significant. This is because the zeroes at the end of a whole number are not significant.
- The answer is 0.000108. The number has three significant figures.
- 1 472 801
(a) 7 (b) 400 000
(c) (i) Disagree. The number has seven significant figures.
(ii) Jabari has not included the zero digit as a significant figure.
- Digits in **bold** are significant.
(a) **10** = 2 s.f. (b) **508** = 3 s.f.
(c) **45 000** = 2 s.f. (d) **200 780** = 5 s.f.
(e) **1 709 300** = 5 s.f. (f) **9.7** = 2 s.f.
(g) **0.01** = 1 s.f. (h) **0.05408** = 4 s.f.
(i) **9.670** = 4 s.f. (j) **0.0400** = 3 s.f.

Activity 1.2

- 4 200 000 2. 4 234 000
- The second number is more accurate.
- Cash registers are programmed to round off automatically to the nearest hundredth. It is useful to use rounding off when shopping, to decide approximately how much money is needed.

Rounding off helps to simplify numbers when explaining something to somebody that involves a large number, such as the distance between the earth and the moon.

- Answers will vary but here are some possible ideas: TB is contagious and spreads through the air. People living with HIV are at much greater risk of becoming sick with TB. A total of 1.77 million people died from TB in 2007. (World Health Organisation) In the 1940s, scientists discovered the first of several drugs now used to treat TB.

Exercise 1.2

- (a) 346 000 (b) 350 000
(c) 300 000 (d) 346 100
(e) 346 090
- (a) 0.057 (b) 0.06 (c) 0.0574
- (a) 14 600 (b) 580 000
(c) 96 420 (d) 1 740 000
(e) 7 300 (f) 1
(g) 0.07 (h) 0.8422
- The answer is 11 881. Correct to three significant figures, this is 11 900.
- The answer is 0.0524. Correct to one significant figure, this is 0.05.
- (a) Monday: 210 Tuesday: 180
Wednesday: 190 Thursday: 72
Friday: 45
(b) 697 students
- 80 000
-

Child	Mass on first day of treatment	Mass rounded off to two significant figures	Mass rounded off to three significant figures
A	12.89 kg	13 kg	12.9 kg
B	13.00 kg	13 kg	13.0 kg
C	25.69 kg	26 kg	25.7 kg
D	16.943 kg	17 kg	16.9 kg
E	17.005 kg	17 kg	17.0 kg

9. (a) There are many possible answers, but some correct answers are:
68 401; 68 432; 68 449; 68 429; 68 418
(b) 68 370; 68 371; 68 372; 68 373; 68 374

Exercise 1.3

- (a) $1\ 500 \div 100 = 15$
(b) $43\ 700 + 900 + 1\ 400 = 46\ 000$
(c) $100 \times 400 = 40\ 000$
(d) $28\ 100 - 14\ 400 = 13\ 700$
- (a) $23\ 100 + 15\ 500 = 38\ 600$
(b) $1.03 \times 2.6 = 2.678 \approx 2.68$
(c) $4\ 860 \div 100 = 48.6$
(d) $5\ 830 - 2\ 960 = 2\ 870$
- (a) $123.8 - 56.1 = 67.7$
(b) $3\ 563.7 + 690.0 = 4\ 253.7$
(c) $3.5 \times 2 = 7$ (d) $2.8 \div 1.2 = 2.3$
- $14\ 800 + 13\ 100 = 27\ 900$ spectators
- $4.35\text{ cm} \times 4.35\text{ cm} = 18.9225\text{ cm}^2 \approx 18.92\text{ cm}^2$
- $3\ 750 \div 60 = 62.5 \approx 60$ boxes of oranges
- $P4.25 \times 20 = P85 \approx P90$
- Round off the number of people to 80.
He should buy approximately 160 cups, 80 plates, 160 serviettes and 240 plastic spoons.
- (a) $3\ 570 + 12\ 700 = 16\ 300$
(b) $3\ 570 + 12\ 710 = 16\ 280$
(c) $3\ 569.0 + 12\ 709.0 = 16\ 278.0$
(d) 16 277.968
(e) (c) was the most accurate.
(f) (a) was the least accurate.
- (a) $P134.85 \approx P130$
 $P200 - P130 = P70$
She will get approximately P70 change.
(b) Our estimation would suggest that this is not the correct change. The correct change is in fact P65.15.

Activity 1.3

This is a practical activity to be completed in the classroom.

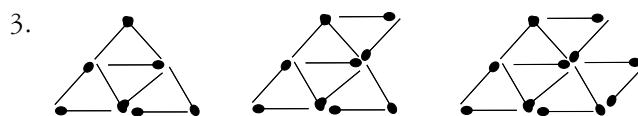
Exercise 1.4

1. Consider setting up a table as follows:

End of day	Money in the piggy bank	Money added to the piggy bank	Money in the piggy bank at the end of the day
1	P1	P1	P2
2	P2	P2	P4
3	P4	P4	P8
4	P8	P8	P16
5	P16	P16	P32

- (a) There will be P32 in the piggy bank at the end of the fifth day.
(b) Looking at the pattern in the 4th column of the table, it appears that there is a sequence developing. i.e. 2; 4; 8; 16; 32 ... are all powers of 2: $2^1, 2^2, 2^3, 2^4, 2^5$
It follows, therefore, that on the eighth day, the total money in the piggy bank will be $2^8 = P256$
(c) Since $2^{10} = 1\ 024$ and $2^9 = 512$, it appears that by the end of the 10th day, there will be more than P1 000 in the piggy bank.

2. If each of the three free-standing matchsticks are placed at each vertex of the triangle shown, in a vertical position, then brought together to form one vertex at the top, a triangular pyramid is formed. Four triangular faces will therefore result.



3.
4. The rabbit and the snail are together covering the distance at 9 km per hour (i.e. on adding their speeds). So, they will cover the distance of 81 km in 9 hours. This means in 9 hours they will meet, and the rabbit will have travelled $9 \times 8 = 72$ km. The snail will have covered 9 km.
Another option is to use trial and error in a table.

Time elapsed	Distance of rabbit from Rabbittown	Distance of snail from Snailville	Total distance covered
1	8 km	1 km	9 km
2	16 km	2 km	18 km
3	24 km	3 km	27 km
4	32 km	4 km	36 km
5	40 km	5 km	45 km
6	48 km	6 km	54 km
7	56 km	7 km	63 km
8	64 km	8 km	72 km
9	72 km	9 km	81 km
10	80 km	10 km	
11	88 km	11 km	

Using the table, we can see that the total distance of 81 km is covered after 9 hours. At that point, the rabbit and snail meet. The snail will have covered 9 km and the rabbit will have covered 72 km.

5. Since Dikeledi got 10 sweets, she must have won 10 more games than Pako. If Pako won 13 games, Dikeledi must have won 23 games. There were therefore 36 games played in total. Since they played one game each day, Pako was there for 36 days.

6. The triangle is the odd one out as it only has three line segments. The other shapes all have four line segments.
7. L, M, N
This is the sequence of consonants.
8. $7 \times 7 \times 7 \times 7 \times 7 = 16\,807$ measures of flour
9. 1991 and 2112

Extension activity

These are some possible answers:

- $1 + 2 + 3 + 4 - 5 + 6 + 7 - 8 + 9 = 100$
 $12 + 3 - 4 + 5 + 6 + 7 + 8 + 9 = 100$
 $123 - 4 - 5 - 6 - 7 + 8 - 9 = 100$
 $123 + 4 - 5 + 6 + 7 - 8 + 9 = 100$
 $123 + 45 - 67 + 8 - 9 = 100$
 $123 - 45 - 67 + 89 = 100$
 $12 - 3 - 4 + 5 - 6 + 7 + 8 + 9 = 100$
 $12 + 3 + 4 + 5 - 6 - 7 + 8 + 9 = 100$
 $1 + 23 - 4 + 5 + 6 + 7 + 8 - 9 = 100$
 $1 + 23 - 4 + 5 + 6 + 7 + 8 + 9 = 100$
 $1 + 2 + 3 - 4 + 5 + 6 + 7 + 8 + 9 = 100$

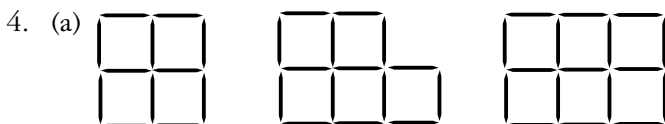
Exercise 1.5

1. (a) 10, 12, 14 (b) 35, 40, 45
 (c) -9, -11, -13 (d) $1\frac{1}{2}$, $1\frac{3}{4}$, 2
 (e) 0.0001, 0.00001, 0.000001
 (f) -500, -1 000, -1 500
 (g) 0, 10, 20 (h) $\frac{4}{8}$, $\frac{3}{8}$, $\frac{2}{8}$
 (i) 3.5, 4, 4.5 (j) 33%, 40%, 47%

2.

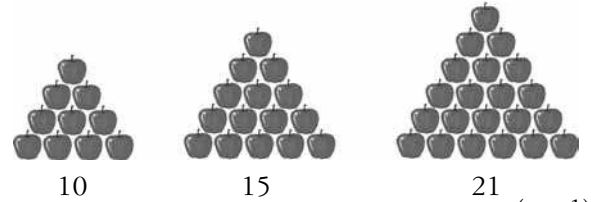
Term number	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈	T ₉	T ₁₀
Term	-2	0	2	4	6	8	10	12	14	16

3. (a) 37, 33, 29, 25, 21, 17, 13, 9, 5, 1
 (b) $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$, $\frac{7}{3}$, $\frac{8}{3}$, $\frac{9}{3}$, $\frac{10}{3}$
 (c) 5.5, 4.6, 3.7, 2.8, 1.9, 1, 0.1
 (d) 100%, 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20%
 (e) -28, -25, -22, -19, -16, -13, -10
 (f) 6, 3, 1.5, 0.75, 0.375, 0.1875
 (Rule: divide by 2)



- (b) The sequence 1, 2, 3, etc. relates to the number of squares in each picture pattern. Another possible sequence relates to the number of matchsticks in each arrangement: 4, 7, 10, 12, 15, 17, ...

5. (a)



- (b) The general rule for this sequence is $x_n = \frac{n(n+1)}{2}$
 In simpler terms, the rule could be stated as: add 1 to the term number, multiply the sum by the term number and divide the product by 2.

6. (a)

Number of rows	1	2	3	4	5
Number of bricks	42	84	126	168	210

- (b) $30 \times 42 = 1\,260$ bricks

Exercise 1.6

1. (a) 7.5%, 9%, 10.5%
 (b) $1\frac{2}{3}$, $\frac{10}{3}$, $\frac{8}{3}$
 (c) 1 301, 1 401, 1 501
 (d) -3, 8, 19
 (e) 759, 749, 739 (f) $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$
2. 7, 10, 13, 16, 19

3. (a) $T_n = -3n + 5$ (b) $T_{100} = -295$
 4. (a) -3, -1, 1 (b) $T_n = 2n - 13$
 (c) The 12th term (d) $T_{25} = 37$
 5. (a) $T_n = 3n + 1$ (b) $T_7 = 22$

Revision exercise

1. (a) 1 (b) 4 (c) 4 (d) 1
 (e) 4 (f) 2
2. (a) (i) 3 800 (ii) 3 780
 (b) (i) 15 000 (ii) 15 050
 (c) (i) 490 (ii) 490
 (d) (i) 160 000 (ii) 163 970
 (e) (i) 9 200 (ii) 9 210
 (f) (i) 1 800 000 (ii) 1 784 540
3. (a) (i) 0.785 (ii) 0.78 (b) (i) 8.65 (ii) 8.65
 (c) (i) 0.344 (ii) 0.34 (d) (i) 164 (ii) 163.98
4. (a) $330 \times 190 = 62\,700 \approx 63\,000$
 (b) $4\,600 \div 450 = 10.222... \approx 10$
 (c) $0.067 + 0.1 = 0.167 \approx 0.17$
 (d) $45 - 33 = 12$
5. The smallest tap fills $\frac{1}{20}$ of the barrel in 1 minute.
 The middle tap fills $\frac{1}{12}$ of the barrel in 1 minute.
 The largest tap fills $\frac{1}{5}$ of the barrel in 1 minute.
 Together they fill $\frac{1}{20} + \frac{1}{12} + \frac{1}{5} = \frac{1}{3}$ of the barrel in 1 minute. Therefore, the whole barrel is filled in 3 minutes.

6. (a) 73, 71, 69 (b) 1.5, 1.75, 2
(c) 41%, 48%, 55% (d) -42, -52, -62
(e) 8, 13, 21 (Fibonacci sequence)
(f) $\frac{16}{10}$, $\frac{21}{10}$, $\frac{26}{10}$
7. 2, 5, 8, 11, 14
8. (a) $T_n = 3n$ (b) $T_n = -2n + 1$

Chapter 2 Squares, square roots, cubes and cube roots

Exercise 2.1

1.

Side length of square (cm)	Area of square (cm ²)
(a) 6	$6 \times 6 = 6^2 = 36$
(b) 7	$7 \times 7 = 7^2 = 49$
(c) 8	$8 \times 8 = 8^2 = 64$
(d) 9	$9 \times 9 = 9^2 = 81$
(e) 10	$10 \times 10 = 10^2 = 100$
(f) 11	$11 \times 11 = 11^2 = 121$

2. (a) 3^2 (b) 9^2 (c) $\left(\frac{1}{2}\right)^2$ (d) $\left(\frac{1}{8}\right)^2$
 3. (a) 9 (b) 1 (c) 16 (d) 169
 (e) $\frac{1}{4}$ (f) $\frac{1}{9}$ (g) 225
 (h) 10 000 (i) $\frac{16}{25}$ (j) $\frac{49}{100}$ (k) 0.16
 (l) 0.0001 (m) $\frac{25}{81}$ (n) 0.04 (o) $\frac{16}{49}$
 4. (a) 25 (b) 100 (c) 81 (d) 400
 (e) $\frac{1}{64}$ (f) 0.49 (g) 1 225 (h) 0.09
 (i) $\frac{4}{25}$ (j) $\frac{81}{100}$
 5. (a) 14.44 (b) 144 400
 (c) 0.1444 (d) 0.001444

6. Surface area of floor is $11 \text{ m} \times 11 \text{ m} = 121 \text{ m}^2$
 $121 \text{ m}^2 \div 1 \text{ m}^2 = 121$
 There are 121 tiles needed.

7. (a) $4 + 9 = 13$ (b) $49 + 16 = 65$
 (c) $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ (d) $0.01 + 0.36 = 0.37$
 (e) $\frac{1}{64} + \frac{9}{64} = \frac{10}{64}$ (f) $\frac{25}{36} + \frac{1}{9} = \frac{29}{36}$
 8. (a) $n = 2$ (b) $n = 25$ (c) $n = 10$

Exercise 2.2

1. (a) 2 (b) 5 (c) 7 (d) 10
 (e) 6 (f) $\frac{1}{2}$ (g) $\frac{1}{8}$ (h) 0.1
 (i) $\frac{3}{5}$ (j) $\frac{2}{3}$
 2. 12 cm
 3. (a) 1 (b) 2 (c) 9 (d) 4
 (e) 14 (f) 15 (g) $\frac{1}{5}$
 (h) $\frac{4}{3}$ or $1\frac{1}{3}$ (i) $\frac{3}{2}$ (j) 0.3
 4. 5 m
 5. (a) $225 = 5 \times 45$
 $= 5 \times 5 \times 9$
 $= 5^2 \times 3^2$
 $\sqrt{225} = 5 \times 3 = 15$

$$\begin{aligned} \text{(b) } 256 &= 2 \times 128 \\ &= 2 \times 2 \times 64 \\ &= 2 \times 2 \times 2^6 \\ &= 2^2 \times 2^2 \times 2^2 \times 2^2 \\ \sqrt{256} &= 2 \times 2 \times 2 \times 2 = 16 \end{aligned}$$

$$\begin{aligned} \text{(c) } 36 &= 2 \times 18 \\ &= 2 \times 2 \times 9 \\ &= 2 \times 2 \times 3 \times 3 \\ &= 2^2 \times 3^2 \\ \sqrt{36} &= 2 \times 3 = 6 \end{aligned}$$

$$\begin{aligned} \text{(d) } 400 &= 2 \times 200 \\ &= 2 \times 2 \times 100 \\ &= 2 \times 2 \times 10 \times 10 \\ &= 2 \times 2 \times 2 \times 5 \times 2 \times 5 \\ &= 2^2 \times 2^2 \times 5^2 \\ \sqrt{400} &= 2 \times 2 \times 5 = 20 \end{aligned}$$

$$\begin{aligned} \text{(e) } 1\,764 &= 2 \times 882 \\ &= 2 \times 2 \times 441 \\ &= 2 \times 2 \times 3 \times 147 \\ &= 2 \times 2 \times 3 \times 3 \times 49 \\ &= 2 \times 2 \times 3 \times 3 \times 7 \times 7 \\ &= 2^2 \times 3^2 \times 7^2 \\ \sqrt{1\,764} &= 2 \times 3 \times 7 = 42 \end{aligned}$$

6. 9

7. (a) $n = 13$ (b) $n = \frac{1}{8}$ (c) $n = 100$
 (d) $n = \frac{1}{4}$ (e) $n = 16$ (f) $n = 3$

Activity 2.1

3	6	$\frac{1}{10}$
$\frac{1}{3}$	12	8
10	$\frac{1}{2}$	9

Exercise 2.3

1. (a) 8 (b) 1 (c) 27 (d) 125
 (e) 1 000 (f) 216 (g) 729 (h) 64
 (i) $\frac{1}{8}$ (j) $\frac{8}{27}$
 2. (a) 64 (b) 125 (c) $\frac{1}{64}$ (d) $\frac{1}{125}$
 (e) 512 (f) 0.001 (g) 0.027 (h) $\frac{8}{27}$
 (i) $\frac{125}{343}$ (j) $\frac{27}{125}$

3. (a) $n = 27$ (b) $n = 3$ (c) $n = 4$
 (d) $n = 125$ (e) $n = 2$ (f) $n = 64$
4. 216 cm^3

5. Consider the possible dimensions of the bigger cubes.
 $2 \times 2 \times 2 = 8$ smaller cubes in each bigger cube.
 $3 \times 3 \times 3 = 27$ smaller cubes in each bigger cube.
 $4^3 = 64$
 $5^3 = 125$
 $6^3 = 216$
 Since we are not allowed to have any smaller cubes left over, the only two numbers that divide exactly into 256, are 8 and 64.
 She could make 32 bigger cubes, each cube having 8 smaller cubes.
 She could make 4 bigger cubes, each cube having 64 smaller cubes.
6. $8 \times 8 \times 8 = 512$ fish

Exercise 2.4

1. (a) 2 (b) 10 (c) 4 (d) 3
 (e) 5 (f) $\frac{1}{4}$ (g) $\frac{1}{6}$ (h) $\frac{2}{3}$
 (i) 0.2 (j) 0.5

2.

Volume of cube (cm ³)	27	64	729	1 000	1 331
Side length of cube (cm)	3	4	9	10	11

3. $5 \text{ m} \times 5 \text{ m} \times 5 \text{ m}$
4. (a) $5 + 6 = 11$ (b) $8 - 4 = 4$
 (c) $2 \times 10 = 20$ (d) $9 \div 3 = 3$
 (e) $3 + 5 = 8$ (f) $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$
 (g) $\frac{2}{3} \div \frac{2}{3} = 1$
5. $13\,824 = 2^3 \times 2^3 \times 2^3 \times 3^3$
 $\sqrt[3]{13\,824} = 2 \times 2 \times 2 \times 3 = 24$
6. $\sqrt[3]{\left(\frac{27}{8}\right)} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2} = 1\frac{1}{2}$
7. (a) $1\,331 = 11 \times 121$
 $= 11 \times 11 \times 11$
 $\sqrt[3]{1\,331} = 11$
 (b) $2\,197 = 13 \times 169$
 $= 13 \times 13 \times 13$
 $\sqrt[3]{2\,197} = 13$
 (c) $2\,744 = 2 \times 1\,372$
 $= 2 \times 2 \times 686$
 $= 2 \times 2 \times 2 \times 343$
 $= 2 \times 2 \times 2 \times 7 \times 49$
 $= 2 \times 2 \times 2 \times 7 \times 7 \times 7$
 $\sqrt[3]{2\,744} = 2 \times 7 = 14$

Exercise 2.5

1. (a) 50 (b) 63 (c) 54 (d) 124
 (e) 418 (f) 0.875 (g) 0.258
 (h) 0.8 (2 d.p.) (i) 0.47 (2 d.p.)
 (j) 2.4 (2 d.p.) (k) 2.6 (2 d.p.)
 (l) 0.77 (2 d.p.) (m) 2.39 (2 d.p.)
 (n) 3.63 (2 d.p.) (o) 0.02 (2 d.p.)
2. (a) 1 089 (b) 27 225 (c) 151 321
 (d) 164 025 (e) 1 000 000
 (f) 0.02 (2 d.p.) (g) 0.49
 (h) 15 (2 d.p.) (i) 0 (2 d.p.)
 (j) 34.03 (2 d.p.) (k) 745.29
 (l) 53.78 (2 d.p.) (m) 0.26 (2 d.p.)
 (n) 61.15 (2 d.p.) (o) 0.25
3. (a) 11 (b) 15 (c) -20 (d) 25
 (e) 63 (f) -0.3 (g) 0.75 (h) 1.2
 (i) 50.5 (j) 0.96 (2 d.p.)
 (k) 0.93 (l) 1.09 (2 d.p.)
 (m) 1.84 (2 d.p.) (n) 1.73 (2 d.p.)
 (o) -0.25
4. (a) 1 728 (b) 3 375 (c) -12 167
 (d) 1 030 301 (e) 125 000 000
 (f) 0.216 (g) 0 (2 d.p.)
 (h) 190.11 (2 d.p.) (i) 0.09 (2 d.p.)
 (j) 0.027 (k) 386.74 (2 d.p.)
 (l) 4.34 (2 d.p.) (m) 12.07 (2 d.p.)
 (n) -0.512 (o) -47.44 (2 d.p.)
5. 662 m^2
6. $12.8 \text{ cm} \times 12.8 \text{ cm} = 163.84 \text{ cm}^2$
 $10.4 \text{ cm} \times 10.4 \text{ cm} = 108.16 \text{ cm}^2$
 $163.84 \text{ cm}^2 - 108.16 \text{ cm}^2 = 55.68 \text{ cm}^2$
 The area of the picture frame is 55.68 cm^2 .
7. 5.6 m
8. 12.8 m
9. $25 \times 25 \times 25 = 15\,625$ toes, fingers, eyes, ears and noses altogether.
10. 11.35 (2 d.p.)

Activity 2.2

04/04/2016; 05/05/2025; 06/06/2036; 07/07/2049;
 08/08/2064; 09/09/2081

Revision exercise

1. (a) 16 (b) 49 (c) 144 (d) $\frac{1}{16}$
 (e) $\frac{81}{100}$ (f) 8 (g) 125 (h) 216
 (i) 0.001 (j) $\frac{27}{8}$
2. (a) 7 (b) 10 (c) 8 (d) 9
 (e) $\frac{1}{2}$ (f) $\frac{1}{4}$ (g) $\frac{3}{5}$ (h) 32
 (i) 28 (j) 51

$$3. \quad (a) 3 \qquad (b) 2 \qquad (c) 10 \qquad (d) \frac{1}{4}$$

$$(e) 0.2$$

$$(f) \begin{aligned} 1728 &= 2 \times 864 \\ &= 2 \times 2 \times 432 \\ &= 2 \times 2 \times 2 \times 216 \\ &= 2 \times 2 \times 2 \times 2 \times 108 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 36 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 12 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 6 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 3 \\ &= 2^3 \times 2^3 \times 3^3 \end{aligned}$$

$$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$$

$$(g) 3375 = 3 \times 1125$$

$$= 3 \times 3 \times 375$$

$$= 3 \times 3 \times 3 \times 125$$

$$= 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$= 3^3 \times 5^3$$

$$\sqrt[3]{3375} = 3 \times 5 = 15$$

$$4. \quad (a) 2209 \qquad (b) 0.2 \text{ (2 d.p.)}$$

$$(c) 12.17 \text{ (2 d.p.)} \qquad (d) 4096$$

$$(e) 1.42 \text{ (2 d.p.)} \qquad (f) 84$$

$$(g) 1.95 \qquad (h) 0.45 \text{ (2 d.p.)}$$

$$(i) 16 \qquad (j) 0.61 \text{ (2 d.p.)}$$

$$5. \quad \begin{aligned} \text{Side length of vegetable patch is 16 m. Perimeter is} \\ 16 \text{ m} \times 4 = 64 \text{ m.} \end{aligned}$$

$$\text{Fencing required is 64 m.}$$

$$6. \quad 2.5 \text{ m}$$

Chapter 3 Indices

Exercise 3.1

- (a) 4^3 (b) 2^5 (c) 10^2 (d) 9^4
 (e) 8^6 (f) 5^3 (g) 12^3 (h) 7^7
 (i) m^5
- (a) $3 \times 3 \times 3 \times 3$ (b) 8×8
 (c) $9 \times 9 \times 9$
 (d) $10 \times 10 \times 10 \times 10 \times 10$
 (e) $11 \times 11 \times 11 \times 11 \times 11 \times 11$
 (f) $12 \times 12 \times 12 \times 12$ (g) $137 \times 137 \times 137$
 (h) $4 \times 4 \times 4 \times 4 \times 4$
 (i) $7 \times 7 \times 7 \times 7 \times 7 \times 7$
 (j) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
- (a) t^3 (b) c^2d^2 (c) j^4y (d) kp^2q^2
 (e) u^4v^2 (f) $4yr^2$
- (a) $r \times r \times r \times r \times r$ (b) $e \times e \times e$
 (c) $t \times t \times m \times m$ (d) $h \times w \times w \times w \times w$
 (e) $5 \times f \times f \times f \times f \times f \times f \times y \times y \times y$
 (f) $u \times u \times u \times u \times n \times n \times n \times n \times b \times b \times b \times b \times b$
- (a) 10^6 (b) 6 (c) 10 and 6
 (d) $10 \times 10 \times 10 \times 10 \times 10 \times 10$
- $16 = 2^4$ and $16 = 4^2$
- Challenge:**
 (a) $8 \times 8 \times 8 \times 8 \times 8$ (b) $8^5 = 32\,768$

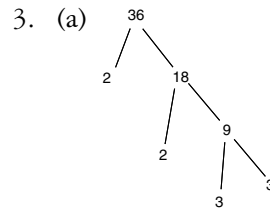
Exercise 3.2

- There may be different, correct answers to this question, since the students have not yet been shown how to use prime factors.
 (a) 10^2 (b) 9^2 (c) $4^2 \times 3$ (d) 5×2^3
 (e) 3×7^2 (f) $5^2 \times 7$ (g) $2^2 \times 10^2$
 (h) $2^2 \times 12^2$ (i) 5^4 (j) 10^3
- (a) $128 = 2^7$ (b) $256 = 2^8$
 (c) $1\,280 = 2^8 \times 5$ (d) $12\,800 = 2^9 \times 5^2$
 (e) $2^7 \times 5$
- $2 \times 3 \times 5^3$ or 6×5^3

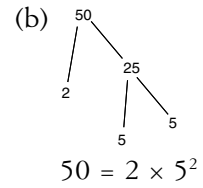
Exercise 3.3

- (a) no (b) yes (c) yes (d) no
 (e) yes (f) no (g) no (h) no
 (i) yes (j) yes
- (a) 1, 2, 3, 4, 6, 12
 (b) 1, 2, 4, 5, 10, 20
 (c) 1, 5, 7, 35
 (d) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
 (e) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
 (f) 1, 3, 5, 7, 15, 21, 35, 105

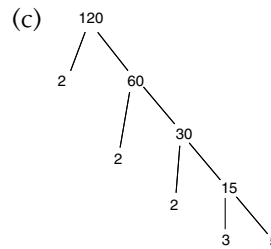
- 1, 2, 5, 10, 13, 26, 65, 130
- 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160
- 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
- 1, 2, 5, 10, 25, 50, 125, 250



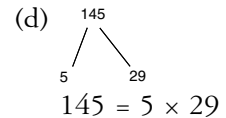
$$36 = 2^2 \times 3^2$$



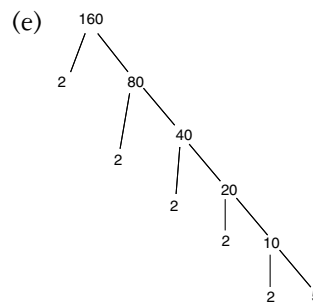
$$50 = 2 \times 5^2$$



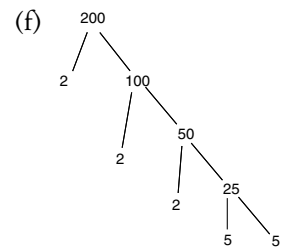
$$120 = 2^3 \times 3 \times 5$$



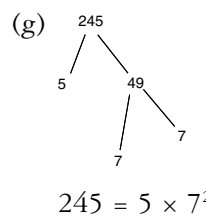
$$145 = 5 \times 29$$



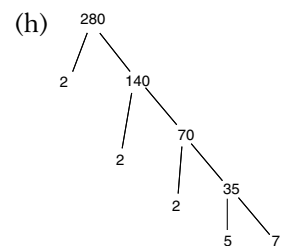
$$160 = 2^5 \times 5$$



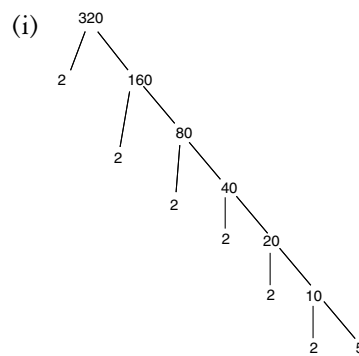
$$200 = 2^3 \times 5^2$$



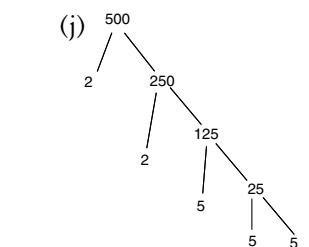
$$245 = 5 \times 7^2$$



$$280 = 2^3 \times 5 \times 7$$



$$320 = 2^6 \times 5$$



$$500 = 2^2 \times 5^3$$

4. (a)

5	145
29	29
	1

$$145 = 5 \times 29$$

(b)

2	260
2	130
5	65
13	13
	1

$$260 = 2^2 \times 5 \times 13$$

(i)

2	526
263	263
	1

$$526 = 2 \times 263$$

(j)

2	318
3	159
53	53
	1

$$318 = 2 \times 3 \times 53$$

(c)

2	108
2	54
3	27
3	9
3	3
	1

$$108 = 2^2 \times 3^3$$

(d)

3	99
3	33
11	11
	1

$$99 = 3^2 \times 11$$

(e)

2	270
3	135
3	45
3	15
5	5
	1

$$270 = 2 \times 3^3 \times 5$$

(f)

2	400
2	200
2	100
2	50
5	25
5	5
	1

$$400 = 2^4 \times 5^2$$

(g)

2	210
3	105
5	35
7	7
	1

$$210 = 2 \times 3 \times 5 \times 7$$

(h)

2	180
2	90
3	45
3	15
5	5
	1

$$180 = 2^2 \times 3^2 \times 5$$

Activity 3.1

Consider the first 60 prime numbers:

2 3 5 7 11 13 17 19 23 29 31 37 41
 43 47 53 59 61 67 71 73 79 83 89 97 101
 103 107 109 113 127 131 137 139 149 151 157 163 167
 173 179 181 191 193 197 199 211 223 227 229 233 239
 241 251 257 263 269 271 277 281

Students test any 10 prime numbers. They could set their work out as follows:

$$13 \times 13 - 1 = 168 \quad (168 \div 24 = 7)$$

$$29 \times 29 - 1 = 840 \quad (840 \div 24 = 35)$$

Activity 3.2

- (a) $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
 (b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$
 (c) $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$
 (d) $x \times x \times x \times x \times x \times x \times x = x^7$
 (e) $p \times p \times p \times p \times p \times p \times p \times p = p^8$
 (f) $r \times r \times r \times r \times r \times r \times r = r^7$

4. It appears as though we could add the indices of the numbers being multiplied. The sum is the index of the product (answer). This would be a much shorter method than using repeated multiplication.

Exercise 3.4

- (a) $5 \times 5 \times 5 \times 5 \times 5 = 5^5$
 (b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8$
 (c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$
 (d) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^7$
 (e) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{10}$
 (f) $a \times a \times a \times a \times a \times a \times a \times a = a^8$
- (a) 2^{11} (b) 5^6 (c) 9^{10} (d) 8^{12}
 (e) 10^9 (f) y^{14}
- (a) $2^2 + 2^3 = 4 + 8 = 12$. But $12 \neq 2^5$
 (b) $3^2 + 3^4 = 9 + 81 = 90$. But $90 \neq 3^6$
 (c) $10^4 + 10^2 = 10\,000 + 100 = 10\,100$.
 But $10\,100 \neq 10^6$

We can conclude that the law of multiplying powers does not apply for addition of powers with the same base.

Activity 3.3

3. (a) $\frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2^3$
 (b) $\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3^3$
 (c) $\frac{7 \times 7}{7} = 7$
 (d) $\frac{8 \times 8 \times 8 \times 8}{8 \times 8} = 8^2$
 (e) $\frac{y \times y \times y \times y \times y \times y \times y}{y \times y \times y} = y^4$
 (f) $\frac{x \times x \times x \times x \times x}{x \times x \times x} = x^2$
4. It appears as though we could subtract the indices of the numbers being divided. The difference is the index of the quotient (answer). This would be a much shorter method than using repeated multiplication and cancelling.

Exercise 3.5

1. (a) $\frac{5 \times 5 \times 5}{5} = 5^2$
 (b) $\frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6 \times 6} = 6$
 (c) $\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} = 3^4$
 (d) $\frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4} = 4^3$
 (e) $\frac{t \times t \times t \times t \times t \times t \times t \times t}{t \times t \times t} = t^4$
 (f) $\frac{k \times k \times k \times k \times k \times k \times k}{k \times k \times k \times k \times k} = k$
2. (a) 9^3 (b) 4^3 (c) 8^6
 (d) 12^4 (e) 10^6 (f) g^{11}
3. (a) $6^5 - 6^2 = 7\,776 - 36 = 7\,740$, but $7\,740 \neq 6^3$
 (b) $3^7 - 3^2 = 2\,187 - 9 = 2\,178$, but $2\,178 \neq 3^5$
 (c) $2^4 - 2^3 = 16 - 8 = 8$, but $8 \neq 2^1$

Activity 3.4

3. (a) $5^3 \times 5^3 \times 5^3$
 $= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5)$
 $= 5^9$
 (b) $7^4 \times 7^4 \times 7^4$
 $= (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7)$
 $= 7^{12}$
 (c) $10^2 \times 10^2 \times 10^2 \times 10^2$
 $= (10 \times 10) \times (10 \times 10) \times (10 \times 10) \times (10 \times 10)$
 $= 10^8$
 (d) $e^5 \times e^5 \times e^5$
 $= (e \times e \times e \times e \times e) \times (e \times e \times e \times e \times e) \times (e \times e \times e \times e \times e)$
 $= e^{15}$
 (e) $x^2y \times x^2y \times x^2y$
 $= (x \times x \times y) \times (x \times x \times y) \times (x \times x \times y)$
 $= x^6y^3$

(f) $5h^3r^2 \times 5h^3r^2 \times 5h^3r^2 \times 5h^3r^2$
 $= (5 \times h \times h \times h \times r \times r) \times$
 $(5 \times h \times h \times h \times r \times r) \times$
 $(5 \times h \times h \times h \times r \times r) \times$
 $(5 \times h \times h \times h \times r \times r)$
 $= 5^4h^{12}r^8$

4. Multiply the exponent of the number or letter within the brackets by the exponent outside the brackets.

Exercise 3.6

1. (a) 5^8 (b) 3^{12} (c) 2^{16}
 (d) f^8 (e) b^8c^{12} (f) $3^3g^{12}h^6$
2. (a) 3^{10} (b) 7^{21} (c) v^{30}
 (d) 4^4r^{32} (e) 2^4i^{24} (f) $5^3w^{27}x^{15}$

Activity 3.5

2. (a) $\frac{\ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell}{\ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell} = 1$
 but $\frac{c^{10}}{c^{10}} = c^{10-10} = c^0$
 (b) $\frac{\ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell}{\ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell} = 1$
 but $\frac{c^8}{c^8} = c^{8-8} = c^0$
 (c) $\frac{\ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell}{\ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell \times \ell} = 1$
 but $\frac{c^9}{c^9} = c^{9-9} = c^0$

Activity 3.6

2. (a) $\frac{\ell \times \ell \times \ell}{\ell \times \ell \times \ell \times c \times c} = \frac{1}{c^2}$
 But $\frac{c^3}{c^5} = c^{3-5} = c^{-2}$
 (b) $\frac{\ell \times \ell \times \ell \times \ell}{\ell \times \ell \times \ell \times \ell \times c} = \frac{1}{c}$
 But $\frac{c^4}{c^5} = c^{4-5} = c^{-1}$
 (c) $\frac{\ell \times \ell}{\ell \times \ell \times c \times c \times c \times c \times c \times c} = \frac{1}{c^6} \cdot \frac{1}{c^6}$
 But $\frac{c^2}{c^8} = c^{2-8} = c^{-6}$

Activity 3.7

3. $(y^3)^{\frac{1}{3}}$
 $= y^{3 \times \frac{1}{3}}$
 $= y^1$
 $= y$
 When we raise a power to a power, we multiply the exponents. So, if one power is the inverse of the other power, the product will be 1. In other words, we will simply be left with the base of the original problem.

Exercise 3.7

1. (a) 1 (b) 1 (c) 1 (d) 1
 (e) 1 (f) 2 (g) 9 (h) 4
 (i) 12 (j) b

2. (a) $\frac{1}{6^2}$ (b) $\frac{1}{8^5}$ (c) $\frac{1}{p^8}$
 (d) $\frac{7^5}{9}$ (e) $\frac{2}{5^2}$ (f) 2^6
 (g) $\frac{1}{12^3}$ (h) $\frac{1}{7} + 1 = 1\frac{1}{7}$
 (i) $8k^8$ (j) $\frac{a^7}{b^3}$

8. $2^{4x} = 2^{16}$
 $\therefore 4x = 16$
 $\therefore x = 4$

Extension activity

A	a^7	H	a^{12}	O	$5a^8$	V	$\frac{5}{a^6}$
B	$\frac{1}{a^7}$	I	$5a$	P	a^2	W	1
C	a^{20}	J	$\frac{2}{4^2}$ or $\frac{1}{2^3}$	Q	4	X	$2a^9$
D	a^1 or a	K	a^{27}	R	a^{14}	Y	a^9
E	a^4	L	$\frac{1}{5^2}$	S	$\frac{5}{a^5}$	Z	$(-3)^5 a^5$
F	$\frac{1}{a^3}$	M	$2^3 a^9$	T	$\frac{1}{a^{15}}$		
G	$3^3 a^{15}$	N	$2a$	U	$24a^{14}$		

1. BOXING 2. HOCKEY
 3. TENNIS 4. SOCCER

Exercise 3.8

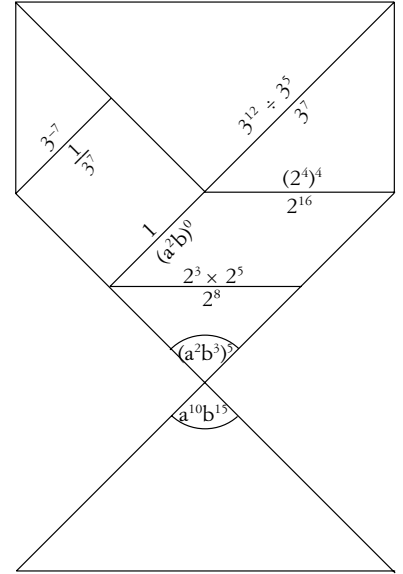
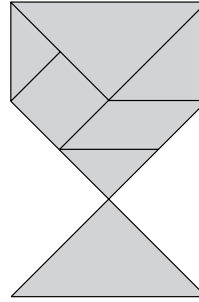
1. (a) Population at time 0 = $50 \times 2^0 = 50$
 Population at time 1 = $50 \times 2^1 = 100$
 Population at time 2 = $50 \times 2^2 = 200$
 Population at time 3 = $50 \times 2^3 = 400$
 Population at time 8 = $50 \times 2^8 = 12\,800$
 (b) $t = 50 \times 2^t$
2. (a) $2^1, 2^2, 2^3, 2^4$
 (b) $2^5 = 32$ orange beads
3. (a) $a^8 b^6$ (b) 1 (c) $\frac{6a^2 b}{5}$ (d) a^{19}
 (e) $12a^4$ (f) $\frac{4a}{3b^2 c^6}$
4. 5^4 fleas 5. $a = 10$ 6. 2×3^4 trees
- 7.

Index	Base 2	Base 3	Base 4	Base 5
1	2	3	4	5
2	4	9	16	25
3	8	27	64	125
4	16	81	256	625
5	32	243	1 024	3 125
6	64	729	4 096	15 625
7	128	2 187	16 384	78 125
8	256	6 561	65 536	390 625
9	512	19 683	262 144	1 953 125
10	1 024	59 049	1 048 576	9 765 625

- (a) 2, 4, 6 or 8 (b) 1, 3, 7 or 9
 (c) Base 4: Only the digits 4 and 6 are seen in the units places.
 Base 5: Only the digit 5 is seen in the units places.

Activity 3.8

This is a wine glass.



Exercise 3.9

1. (a) 5 000 (b) 78 000 (c) 0.002
 (d) 0.00045 (e) 287 000 (f) 0.31
 (g) 990 000 000 (h) 24 500
 (i) 0.0000059 (j) 8 640
2. (a) $a = 3$ (b) $a = 4$ (c) $a = 5$ (d) $a = 6$
 (e) $a = -2$ (f) $a = 1$ (g) $a = -3$
 (h) $a = 3$ (i) $a = 4$
3. (a) (ii) (b) (iii) (c) (i)
 (d) (i) (e) (ii)
4. (a) 3×10^3 (b) 1.4×10^4
 (c) 5×10^4 (d) 1.5×10^5
 (e) 1×10^6 (f) 6.75×10^2
 (g) 1.28×10^4 (h) 5×10^{-2}
 (i) 6×10^{-3} (j) 7×10^{-4}
 (k) 6.43×10^{-3} (l) 1.23×10
 (m) 1.508×10^2 (n) 1.308785×10^4
5. (a) Dinosaur: 1×10^4 kg (b) Car: 1×10^3 kg
 (c) Asian elephant: 5 000 kg
 (d) Blue whale: 1.7×10^5 kg
 (e) Building: 7 300 000 kg

Extension activity

330 200 000 000 000 000 000 000 000 kg

Exercise 3.10

1. (a) 2×10^{-9} g/ml (b) 1.2×10^{-5} g/ml
 (c) 0.000005 g/ml
2. 300 000 000 m/s

3. (a) 7×10^{-6} g (b) $0.0007 = 7 \times 10^{-4}$ g
 (c) $0.0021 = 2.1 \times 10^{-3}$ g
4. $1\ 020\ 000 = 1.02 \times 10^6$ Smarties
5. (a) 5.9×10^6 tons (b) 2.565 tons
 (c) $2\ 565\ \text{kg} = 2.565 \times 10^3$ kg

Revision exercise

1. (a) $2 \times 2 \times 2 \times 2$
 (b) $6 \times 6 \times 6$ (c) $5 \times 5 \times 5 \times 5 \times 5$
 (d) $2 \times 2 \times 2 \times 2 \times 2 \times 2$ (e) 3×3
 (f) $4 \times 4 \times 4 \times p \times p \times p \times p \times p \times p$
2. (a) 3^4 (b) 2^5 (c) 10^3 (d) 8^4
 (e) y^5 (f) $5^3 c^3$
3. (a) 6^2 (b) 2×5^2 (c) 13×5
 (d) 2×6^2 (e) $2 \times 3^2 \times a$
4. (a) $2^2 \times 5$ (b) $2 \times 3 \times 5$
 (c) $2^2 \times 5^2$ (d) $2^3 \times 3 \times 5$ (e) 2×5^3
5. (a) The law of multiplying powers
 (b) The rule of zero indices
 (c) The law of dividing powers
 (d) The rule of fractional indices
 (e) The rule of negative indices
 (f) The law of raising a power to a power
6. (a) 7^{12} (b) 3 (c) $\frac{1}{b^{11}}$
 (d) $3^7 u^{35}$ (e) 4^4
7. (a) 4×10^3 (b) 1.25×10^6
 (c) 4.98×10^2 (d) 8×10^{-2}
 (e) 6.5×10^{-4}
8. (a) 9.79×10^2 m (b) 8.9×10^3 m
 (c) 2.72×10^0 m (d) 8.18×10^2 m
 (e) 6×10^0 m
- From tallest to shortest:
 8.9×10^3 m, 9.79×10^2 m, 8.18×10^2 m,
 6×10^0 m, 2.72×10^0 m

Chapter 4 Money

Exercise 4.1

- (a) (i) P7 767.50 (ii) P11 595.00 (iii) P12 117.00
 (b) Rent: P9 600.00 (c) Shoes and beads
 (d) Overheads
 (e) Since her rent is most costly, perhaps she could consider moving premises.
 She could look at reducing her telephone bill. Perhaps staff are using the telephone for private reasons.
 She could investigate getting her shoes from a different source, thereby reducing the cost of the shoes.

2. (a)

Cost statement for Sunny Farm		
October 2010		
Type of cost	Calculation	Total cost
1. Labour cost		
(a) Farm labourers	– Cost per person 200 hours × P15.50/h = P3 100.00 – Cost: 5 × P3 100 = P15 500.00	
(b) Receptionist	– Cost: 160 hours × P15.45 = P2 472.00	
(c) Accountant	– Cost: 40 hours × P70.70 = P2 828.00	
(d) Packers	– Cost per person 200 × P14.70 = P2 940.00 – Cost: 5 × P2 940.00 = P14 700.00	
(e) Cleaners	– Cost per person P12.70 × 180 = P2 286.00 – Cost: 3 × P2 286.00 = P6 858.00	
	Total labour cost	P42 358.00
2. Materials cost		
(a) Chicken feed	100 kg × P35.00/kg = P3 500.00	
(b) Incubator costs	P1 570.00	
(c) Nesting material	150 kg × P7.50/kg = P1 125.00	
(d) Newborn chicks	80 × P8.70 = P696.00	
(e) Vaccines	P1 780.00	
(f) Packaging material	P1 460.00	
	Total materials cost	P10 131.00

3. Overheads		
(a) Water	P1 760.00	
(b) Electricity	P2 790.00	
(c) Rent	P13 540.00	
(d) Telephone	P1 890.00	
(e) Repairs	P5 790.00	
(f) Advertising	P1 630.00	
	Total overheads cost	P27 400.00
	Subtotal:	P79 889.00
	V.A.T at 10%:	P7 988.90
	Total:	P87 877.90

(b) Overheads and labour costs are particularly high.

(c) P32 122.10

3.

Cost statement for Kitso's soap business		
Any month		
Type of cost	Calculation	Total cost
1. Labour cost		
(a) Part-time soap maker	– Cost: 50 × P20.60 = P1 030.00	
	Total labour cost	P1 030.00
2. Materials cost		
(a) Base	1 000 × P4.30 = P4 300.00	
(b) Fragrance	1 000 × P1.20 = P1 200	
(c) Packaging and labelling material	1 000 × P2.50 = P2 500.00	
(d) Colour	1 000 × P0.40 = P400.00	
	Total materials cost	P8 400.00
3. Overheads		
(a) Water and electricity	P120.00	
(b) Rent	P300.00	
(d) Telephone	P45.00	
(e) Insurance	P50.00	
	Total overheads cost	P515.00
	Subtotal	P9 945.00
	V.A.T at 10%	P994.50
	Total	P10 939.50

Exercise 4.2

- (a) P3.00 (b) P2.60 (c) P16.20
 (d) P19.50 (e) P2.30 (f) P15.90
 (g) P16.20 (h) P1.10
 (i) P28.80 (j) P13.60
- (a) (i) P3.00 (ii) P3.30 (iii) P3.00
 (iv) P4.90 (v) P4.10
 (b) P18.30

3. (a) P11.60 (b) P53.90 (c) P40.10
(d) P10.80 (e) P139.00
4. $(3 \times P16.20) + (2 \times P4.10) = P56.80$

Exercise 4.3

1. (a) P30.00 (b) P103.40 (c) P83.30
(d) P173.60 (e) P99.40 (f) P226.00
2. (a) P111.00 (b) P581.20 (c) P388.20
(d) P171.20 (e) P211.60 (f) P169.80
3. Cost by surface mail: P127.50. Cost by airmail:
P179.00
She will save P51.50 if she mails them by
surface mail.

Activity 4.1

1. (a) Lesego Kwena (b) Lesego Kwena
(c) The bank official (d) 8
(e) 145.60
(f) One hundred and forty-five pula and sixty
thebe
(g) To keep track of your deposit.
(h) No

2. (a) (i) Mrs R. Madome
(ii) Pula
(iii) Day, month, year
(iv) Fifty-seven pula
(v) Notes 50: Qty: 1 Amount: 50
Coins 5: Qty: 1 Amount: 5
Coins 2: Qty: 1 Amount: 2
- (b) (i) The branch outlet of the bank where the
account is held
(ii) Mrs R Madome's
(iii) Yes
(iv) 124.70
(v) On the cheque

Activity 4.2

Account name: Student's name
Account currency: Pula
Acc no: The account number
Date: The date
Amount withdrawal: P250.00
Branch: Branch of the bank at which the account is held
Amount in words: Two hundred and fifty pula
Customer signature: Student to sign.

Exercise 4.4

1.

BARCLAYS		Barclays Bank of Botswana Limited	29-14-67
SEROWE BRANCH		Letsatsi Date	05/04/2009
Duela Pay	<i>Payless Stores</i>	kana Molaelwa or Order	
<i>One hundred and fifty seven</i>		P	157.80
<i>Pula eighty thebe</i>		MOKIBELO J PRESTIGE	<i>Mokibelo</i>
Please do not write below this line or fold this voucher			
⑈0000 1 7 4 1 2 9 1 4 6 7 1 0 0 0 0 0 0 1 5 3 2 3 7 5 1 1 0 1			

2. (a) (i) Woolworths (ii) Mr P. Masole (iii) P384.30
(iv) Three hundred and eighty-four pula and
thirty thebe
- (b) (i) Water Utilities Corporation
(ii) Mrs A. Molatole (iii) P846.50
(iv) Eight hundred and forty-six pula and fifty
thebe
- (c) (i) Movie Magic (ii) M. Sesana (iii) P114.40
(iv) One hundred and fourteen pula and forty
thebe

Exercise 4.5

1.



We will deliver

MONEY ORDER APPLICATION FORM
 (*FOR CUSTOMER TO COMPLETE IN BLOCK LETTERS*)

M011

OFFICE OF PAYMENT: _____

*Names of recipient ① Mr Richard Mudongo

*Postal address of recipient ② 25 Office Towers, Main Street, Gaborone

*Message Ref no: TV1087MAL③

*Optional services are available at additional charges.(Tick the box if you want the service.)

The Post office to advise the recipient by phone that the money order is waiting for collection..
 Recipient Cell/Tel _____

* The Post office to provide you with a copy of certified payment. **④**
 Your fax _____ (Placed in your box if no fax)

*Amount to be paid in words (Pula) One hundred⑤ and twenty five pula and fifty thebe

Amount to be paid in figures	
Pula	thebe
⑥ 125	50

*Name/s of sender ⑦ Mr Maolosi

*Postal address of sender ⑧ 13 Sturgeon Street, Gaborone

*Senders Tel/Cell ⑨ 045 074 1445 Time the form was handled over 10:00am

(ISSUING OFFICE USE ONLY)

ISSUING OFFICE _____

Order Number					

Charges	
P	t
Commission	
Fax	
Optional	
Total	

ISSUING OFFICE STAMP _____

Date of issue _____

Place adhesive Number here _____

Commission
 Fax
 Optional
 Total

Issuing officer's signature _____ Checked by _____

Mailed/Faxed by _____ Date _____ Time _____

NOTE: (FOR USE BY PAYING OFFICE ONLY)

(Individual)

Name of recipient _____ Signature of recipient _____

Identity number _____

COMPANY STAMP _____

(Company)

Names of representative collecting _____

Signature of Representative _____

Identity Number _____

Names of paying officer _____ Signature of paying officer _____

PAYING OFFICE STAMP _____

Recipient phoned: date _____ Time _____

Recipient advised: date _____ Time _____

Sender faxed: date _____ Time _____

Faxed Office Copy

- You are guaranteed the money, since the money order has already been paid for.
- No cash is sent so the money cannot be stolen.
- P38.40
 $P350 + P38.40 = P388.40$
- P34.50

Activity 4.3

- Mrs K. Moswela
- China
- Mr R. Lee
- US\$560.00
- P3 808.00
- Barclays Bank
- To ensure that the writing is legible.

Activity 4.4

FUND TRANSFER SLIP		Standard Chartered	
From Account		To Account	
Account Name	Mrs Dube	Account Name	Baby Care
Account Currency	Pula	Account Currency	Pula
Account Number	7309965143289	Account Number	7282216098723
For Bank Use Only		Branch	
		Date	02062010
		Transfer Amount	
		Currency	P
		Amount	1780.00
		Amount in Words	One thousand seven hundred and eighty pula
Standard Chartered Bank Botswana Ltd.		Processed By	Checked By
		MBDube Customer Signature	

FUND TRANSFER SLIP		Standard Chartered	
From Account		To Account	
Account Name	Mr Sesana	Account Name	Ms. M. Sesana
Account Currency	Pula	Account Currency	Pula
Account Number	7308765132400	Account Number	7198346721997
For Bank Use Only		Branch	
		Date	05022010
		Transfer Amount	
		Currency	P
		Amount	4560.00
		Amount in Words	Four thousand five hundred and sixty pula
Standard Chartered Bank Botswana Ltd.		Processed By	Checked By
		Sesana Customer Signature	

Exercise 4.6

- | Type of loan | Explanation | Advantages | Disadvantages |
|----------------------------|--|---|--|
| 1. Variable home loan | The interest rate moves up or down in accordance with movements in global interest rates. | Repayments fall when interest rates fall. | Repayments rise when interest rates rise |
| 2. Fixed rate home loan | The interest rate is fixed for a certain period of the home loan period. | You are always assured of how much you will pay each month. | If interest rates fall, you do not benefit in any way. |
| 3. Interest-only home loan | You repay only the interest on the principal for 1 – 5 years. Thereafter, you must start making principal and interest repayments. | Initially, you pay much less which is good if you are short of cash. | There will be a sudden increase in repayments at the end of the interest-only period. |
| 4. Introductory home loan | An initial low interest rate is charged in order to attract borrowers. This rate usually only lasts for around 12 months and then reverts to the standard interest rate. | Payments in the low-interest period can reduce the principal quickly. | There will be a sudden increase in repayments when the low interest rate period is over. |
| 5. No deposit home loan | No deposit is required. | Perfect for borrowers that do not have enough money for a deposit. | Higher interest rate than traditional loans. |

- A secured loan is a loan in which the borrower promises some asset, such as a car or property, as security for the loan. Example: A mortgage loan where the house is the asset. An unsecured loan is not secured against any assets. Example: A personal loan.
- This depends on whether or not Mrs Molefe can afford the 10% deposit. If she can pay the deposit, she will pay P1 350.00 in interest per year. For the second option, she will pay P1 650.00 interest per year. So option 1 is best if she can afford the deposit.
- This will all depend on whether or not Kwena is sure that she can make her repayments.
Option 1: A higher interest rate, but this is a better option if she thinks she might not be able to make a payment, as she then won't lose her car.
Option 2: A lower interest rate, but she must be sure that she can maintain her payments as her car may be repossessed if she does not pay in time.

Activity 4.5

- Farm insurance
- Life insurance
- Vehicle insurance
- Business insurance
- Landlord insurance

Exercise 4.7

- P10 000.00
- P3 250.00
- P6 750.00
- P15 000.00
- P1 905.00
- Plan A: P450.00 per month
Plan B: P270.00 per month. (Remember to first subtract the deposit before calculating the interest.)

Exercise 4.8

- (a) P25.00 (b) P60.00 (c) P320.00
(d) P1 125.00 (e) P34 200.00
- P450.00 3. P840.00
- $P480.00 + P8\ 000 = P8\ 480$
- P42 500.00 6. P5 400.00
- (a) P4 500
(b) Her monthly interest repayments will be P375. Taking all her other expenses into account, this might be a financial strain each month.

Exercise 4.9

- P1 680.00 2. P3 380.00
- P175.00
- (a) P2 160.00 (b) P210.00
- (a) P2 199.00 (b) P4 448.00
(c) P2 249.00

Activity 4.6

- Microwave: P181.00
Fridge: P390.00
Sleeper couch: P840.00
Television: P291.00
Vacuum cleaner: P241.00
- HP is perfect for people who do not have enough money to buy the item up front. The big disadvantage is the much larger overall total cost using the HP option.

Exercise 4.10

- (a) P86.87 (b) P108.13 (c) P92.49
(d) P113.00 (e) P116.94
- (a) P117.35 (b) P694.19
(c) P120.35 (d) P671.69
(e) P121.90 (f) P124.95
(g) P659.13 (h) P655.38
(i) P135.88 (j) P649.00
- (a) P694.75 (b) P8 177.25
(c) P98 127.00
- $P102.88 - P90.24 = P12.64$
- $P89.37 + P86.99 + P112.44 + P98.74 = P387.54$
- P1 938.44

Activity 4.7

- (a) P0 (b) P0 (c) P750.00
(d) $P1\ 500.00 + P250.00 = P1\ 750.00$
(e) $P5\ 250.00 + P1\ 875.00 = P7\ 125.00$

Exercise 4.11

- (a) P0 (b) P0 (c) P250.00
(d) $P1\ 500.00 + P2\ 050.00 = P3\ 550.00$
(e) $P5\ 250.00 + P4\ 230.00 = P9\ 480.00$
(f) $P5\ 250.00 + P980.63 = P6\ 230.63$
(g) $P10\ 875.00 + P8\ 825.00 = P19\ 700.00$
(h) $P10\ 875.00 + P27\ 750.00 = P38\ 625.00$
- (a) $P10\ 875.00 + P450.00 = P11\ 325.00$
(b) P110 475.00
(c) Yes, he will be able to save P706.25 per month.

- (a) P5 250.00 + P3 093.75 = P8 343.75
(b) P695.31 (c) P8 179.69
- Mrs Moswela: Annual tax is P1 625.00
Mr Koloji: Annual tax is P1 475.00
Mrs Moswela: Net monthly income: P4 947.92
Mr Koloji: Net monthly income: P4 835.42
False. Mrs Moswela takes home more money per month.
- (a) P1 875.00 (b) P3 375.00
- Botswana residents pay no tax on the first P30 000 of earnings, whereas a non-resident is charged 5% tax on all money earned up to P60 000.
- (a) P3 500.00 (b) P175.00
(c) P3 325.00

Exercise 4.12

Answers are all to 2 decimal places.

- | Item | Price excl. sales tax | Price incl. sales tax |
|--|-----------------------|-----------------------|
| 1. Frothy cappuccino | P6.50 | P7.02 ≈ P7.00 |
| 2. Choc-nut delight | P18.35 | P19.818 ≈ P19.80 |
| 3. Creamy chicken casserole | P39.90 | P43.092 ≈ P43.10 |
| 4. Tropical salad with apple and cashews | P17.80 | P19.224 ≈ P19.20 |
| 5. Rich caramel surprise | P20.70 | P22.356 ≈ P22.40 |

- (a) P210.60
(b) P198.00 with VAT, so P12.60 less
- Tax is P170.10, so total bill is P2 060.10.
- (a) P411.80 (b) P176.50
- (a) P359.10
(b) P39.00 – P34.10 = P4.90 more
- (a) P5 716.48 ≈ P5 716.50
(b) P5 716.48 ≈ P5 716.50
(c) Answers are the same.
- Challenge:** P13 750.00

Exercise 4.13

- (a) P2 050.00 (b) P1 615.00
(c) P8 400.00 (d) P15 525.00
- (a) P1 585.00 (b) P1 350.00
(c) P18 785.00
- P20 295.00
- (a) P4 860.00 (b) P3 375.00
(c) P56 835.00
- (a) P200 200.00 (b) P4 104.00

Exercise 4.14

- P70.00 2. P240.00
- (a) P70.00 (b) P140.00 (c) P90.00
(d) P110.00 (e) P190.00 (f) P60.00
(g) P160.00 (h) P260.00
- (a) P190.00 (b) P6 700.00
(c) P69 000.00
- To keep track of ownership and to find stolen cars.
- 4 500 kg
- Cost of car: P43 250.00; VAT (10%): P4 325.00; registration: P90.00. Total needed: P47 665.00. Her uncle is incorrect. She needs at least P47 665.00 to buy the vehicle.

Activity 4.8

The advantage is that if you sell a lot of merchandise, you will get more money. This motivates sales people to work harder. The disadvantage is that if you don't sell, no matter how hard you work, you will get no pay.

Exercise 4.15

- (a) Thapelo: P961.20 (b) Atlang: P166.00
(c) Julia: P797.20
- P37 800.00
- Job 1: P348.00 for the week. Job 2: P255.00 plus commission.
If Joe wants a secure income, then job 1 is better. If he thinks the television business is a good one and he is sure that he can fix many televisions, then job 2 might be a better option, as he could earn good commission.
- P20 880.00 5. P6 200.00
-

Hungry Jacks – Commission record			
Employee	Commission rate (% of the value of the meals sold)	Total value of the meals sold for the week	Money made in commission
(a) Ngwakwana	5%	P1 250	P62.50
(b) John	7.5%	P980	P73.50
(c) Boitshepo	6%	P1 000	P60.00
(d) Lesego	8%	P850	P68.00
(e) Kitso	6.5%	P1 300	P84.50

Kitso is the star employee of the month.

- Challenge:**
P13 250.00

Revision exercise

1. Water, electricity, rent, telephone, advertising, etc.
2. P267.20
3. Payee, date of cheque, amount in figures, amount in words, signature.
4. A cheque draws money from your own cheque account. If you have no money in your cheque account, the cheque will “bounce” and not be payable. A money order is purchased from a post office or other institution. It is paid for in cash, so the recipient is assured of getting the money.
5. Sending foreign currency abroad.
6. P250.00
7. P8 500.00
8. P1 350.00
9. P4 477.00
10. (a) P5 812.50
- (b) P484.38
11. P492.75
12. P8 720.00
13. P46 820.00
14. P869.00

Chapter 5 Matrices

Exercise 5.1

1. (a) 3 rows; 3 columns (b) 2 rows; 2 columns
(c) 3 rows; 2 columns (d) 3 rows; 5 columns

2. (a) $\begin{pmatrix} 10 & 12 & 8 \\ 15 & 10 & 12 \\ 14 & 8 & 12 \end{pmatrix}$

- (b) 3rd row; 1st column
(c) 2nd row; 3rd column

3. (a) $\begin{pmatrix} 50 & 80 & 100 & 150 \\ 120 & 180 & 200 & 150 \\ 300 & 200 & 150 & 400 \\ 250 & 250 & 300 & 350 \\ 280 & 250 & 100 & 350 \end{pmatrix}$

- (b) Day 4 (c) Day 3
(d) 2nd row; 3rd column

4. (a) 2nd row (b) 1st column
(c) First Sunday: 65 Second Sunday: 88
Difference: 23 people

5. (a) $\begin{pmatrix} 5 & -1 & 5 \\ -2 & 0 & 4 \\ 2 & 4 & -1 \end{pmatrix}$

- (b) 2nd April (c) Lift 1
(d) You could be trapped in the lift, as the lift may stop working. It is always best to use the stairs as an exit point when leaving a building that is on fire.

Activity 5.1

1 and 2 – Answers will vary.

3. A matrix is less bulky than a table, showing only the essential numbers, rather than the headings and the lines drawn in to differentiate between each row and column. Matrices can be added, subtracted, multiplied and divided, whereas information in a table would need to be transferred out of the table before any operations could be performed on the data. A table, however, is perhaps easier to use if the data is to be analysed as is. This is because a table has headings that explain the data in words.

Exercise 5.2

1. (a) 1×4 (b) 3×1 (c) 3×3 (d) 3×5
(e) 2×3 (f) 4×4

2. (c) and (f)

3. (a) 3×4 (b) 2×2 (c) 4 (d) -3
(e) 3rd row; 4th column

4. (a) Any matrix with 3 rows and 2 columns.
(b) Any matrix with 2 rows and 2 columns.
 $\begin{pmatrix} 8 & 16 & 24 \\ 0 & 0 & 0 \\ 4 & 8 & 12 \end{pmatrix}$

5. False. The order is 2×4 .

6. (a) 4×5 (b) 13×10 (c) 6×7 (d) 8×1

7. (a) $a_{11} = 1; a_{12} = 4; a_{13} = 5; a_{21} = 8; a_{22} = 2; a_{23} = 9$
(b) $b_{11}; b_{22}; b_{33}; b_{44}; b_{55}; b_{66}; b_{77}; b_{88}$

Activity 5.2

1. A: 2×2 B: 1×3 C: 2×2 D: 4×1

2. No. There are not enough rows in A to match with the number of rows in D.
3. No. There are not enough rows in C to match with the number of rows in D.
4. Yes. There are the same number of rows and columns in both A and C.

Exercise 5.3

1. (a) $\begin{pmatrix} 7 & 4 \\ 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 11 \\ 1 \\ -1 \\ -1 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & 5 & 2 & -1 \\ 2 & 3 & 1 & 8 \\ -2 & 9 & -5 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 12 & -10 & 13 \\ 14 & 15 & -7 \\ 1 & -1 & 11 \end{pmatrix}$

(f) $(0 \ 13 \ 8)$

2. (a) $\begin{pmatrix} 11 & 5 & 1 \\ 0 & 1 & 3 \end{pmatrix}$; Order is 2×3 .

(b) $\begin{pmatrix} 2 & -1 \\ 4 & 5 \\ 8 & 4 \end{pmatrix}$; Order is 3×2 .

(c) Matrices A and B and matrices C and D do not have the same number of rows and columns.

3. (a) Answers will vary, but here is one correct answer:

$$A = \begin{pmatrix} 3 & 5 & 7 \\ 5 & 3 & 7 \end{pmatrix}$$

$$B = \begin{pmatrix} -4 & -2 & -6 \\ -6 & -4 & -2 \end{pmatrix}$$

(b) Answers will vary, but using the matrices from

(a): $A + B = \begin{pmatrix} -1 & 3 & 1 \\ -1 & -1 & 5 \end{pmatrix}$

4. (a) $X + Y = \begin{pmatrix} 15 & 3 & -6 \\ 10 & 3 & 11 \end{pmatrix}$

(b) Addition is not possible.

(c) $E + F = \begin{pmatrix} 1 & 2 & -2 \\ 2 & -2 & -6 \\ 9 & 8 & 2 \end{pmatrix}$

(d) $B + C = \begin{pmatrix} 12 & 11 \\ -4 & -1 \\ 29 & -18 \end{pmatrix}$

(e) Addition is not possible.

5. $6 + x = 2$, so $x = -4$ $w + 0 = 8$, so $w = 8$
 $2 + y = -3$, so $y = -5$ $-3 + 7 = z$, so $z = 4$

Exercise 5.4

1. (a) $\begin{pmatrix} -2 & -2 \\ -8 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} -5 & 2 & 2 \\ 1 & -4 & -2 \end{pmatrix}$
 (c) Matrices F and H do not have the same number of rows and columns.

2. (a) $\begin{pmatrix} 2 & -1 \\ -2 & -1 \\ 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -1 & -2 \\ -5 & -2 & -7 \\ -7 & 0 & 6 \end{pmatrix}$

- (c) $(-2 \ 5 \ 2 \ -9)$ (d) $\begin{pmatrix} 1 & -2 & -5 \\ 0 & 2 & -2 \\ 3 & 5 & -4 \end{pmatrix}$

- (e) $\begin{pmatrix} -2 \\ -1 \\ -1 \\ 8 \end{pmatrix}$ (f) $\begin{pmatrix} 2 & 4 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ -1 & 0 & -3 & 1 \\ 7 & 1 & -2 & -4 \end{pmatrix}$

3. (a) $N = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$ (b) $O = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{pmatrix}$

- (c) $O - N = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix}$

4. (a) $\begin{pmatrix} -7 & -1 & 2 \\ -6 & 1 & -1 \\ 6 & -2 & 8 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 1 & -5 \\ 4 & -3 & -1 \\ -6 & 8 & -2 \end{pmatrix}$

- (c) $\begin{pmatrix} -3 & 0 & -3 \\ -2 & -2 & -2 \\ 0 & 6 & 6 \end{pmatrix}$

5. (a) $x - 1 = -3$, so $x = -2$
 $2 - 4 = y$, so $y = -2$

- (b) $x - 0 = -3$, so $x = -3$
 $3 - y = -1$, so $y = 4$

- (c) $2x - 4 = 2$ $5 - (y + 1) = 0$
 $\therefore 2x = 6$ $\therefore 5 - y - 1 = 0$
 $\therefore x = 3$ $\therefore 4 - y = 0$
 $\therefore y = 4$

Exercise 5.5

1. (a) $\begin{pmatrix} 5 & 0 \\ -10 & 15 \end{pmatrix}$ (b) $(-6 \ 0 \ 2 \ -12)$

- (c) $\begin{pmatrix} -4 & 12 & 12 \\ 8 & 20 & 12 \\ -12 & 16 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2\frac{1}{2} \\ -1 \\ 1\frac{1}{2} \end{pmatrix}$

- (e) $\begin{pmatrix} 72 & -24 & 36 & 48 \\ -48 & 12 & 0 & 60 \end{pmatrix}$ (f) $\begin{pmatrix} 15p & -6p \\ 12p & 9p \end{pmatrix}$

2. (a) $\begin{pmatrix} 8 & 4 & 6 \\ -2 & 0 & -4 \end{pmatrix}$ (b) $\begin{pmatrix} -8 & 0 \\ 16 & 20 \end{pmatrix}$

- (c) $\begin{pmatrix} -5 & -2 \\ -3 & -1 \end{pmatrix}$

- (d) $\begin{pmatrix} 15 & 6 \\ 9 & 3 \end{pmatrix} + \begin{pmatrix} -4 & 0 \\ 8 & 10 \end{pmatrix} = \begin{pmatrix} 11 & 6 \\ 17 & 13 \end{pmatrix}$

- (e) $\begin{pmatrix} -4 & -2 & -3 \\ 1 & 0 & 2 \end{pmatrix}$

- (f) $\begin{pmatrix} 20 & 8 \\ 12 & 4 \end{pmatrix} - \begin{pmatrix} -6 & 0 \\ 12 & 15 \end{pmatrix} = \begin{pmatrix} 26 & 8 \\ 0 & -11 \end{pmatrix}$

3. (a) $\begin{pmatrix} 1 & 2 & 3 \\ 5 & 10 & 8 \\ 3 & 2 & 4 \end{pmatrix}$

- (b) Bucket in the shower to catch excess water.

- (c) 9 ℓ (d) Wednesday

- (e) $\begin{pmatrix} 5 & 10 & 15 \\ 25 & 50 & 40 \\ 15 & 10 & 20 \end{pmatrix}$ (f) 190 ℓ

4. $\begin{pmatrix} -10 & -6 & -4 & -2 \\ 8 & -10 & 4 & 0 \\ 0 & 2 & -6 & -12 \\ -6 & -4 & 2 & 0 \end{pmatrix}$

5. $Y = \begin{pmatrix} 3 & 4 & 7 \\ 10 & 12 & 9 \\ 2 & 1 & 5 \end{pmatrix}$

6. $8x = -16$, so $x = -2$

- $8y = -8$, so $y = -1$

- $8z = 96$, so $z = 12$

7. (a) $\begin{pmatrix} 23.25 & 15.50 & 12.40 \\ 31.00 & 18.60 & 15.50 \\ 40.30 & 38.75 & 31.00 \end{pmatrix}$

- (b) $\begin{pmatrix} 232.50 & 155.00 & 124.00 \\ 310.00 & 186.00 & 155.00 \\ 403.00 & 387.50 & 310.00 \end{pmatrix}$

- (c) P2 263.00

Exercise 5.6

1. (a) 11 (b) -5 (c) 70

- (d) Not possible

2. $x = -2$

3. (a) $(12 \ 25 \ 32)$ (b) $\begin{pmatrix} 5.00 \\ 3.50 \\ 6.00 \end{pmatrix}$
 (c) P339.50

4. (a) 24 (b) 5

- (c) The number of columns of matrix W is not equal to the number of rows of matrix Y.

Activity 5.3

1. $(16 \ 22)$ 2. $(-14 \ -6)$

3. $(-26 \ -26)$ 4. $(8 \ 32)$

Exercise 5.7

1. (a) $\begin{pmatrix} 2 & -8 \\ -1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 6 & 32 \\ 8 & 6 \end{pmatrix}$ (c) $\begin{pmatrix} -14 & -8 \\ -1 & 0 \end{pmatrix}$

- (d) $\begin{pmatrix} 5 & -4 \\ -2 & 8 \end{pmatrix}$ (e) $\begin{pmatrix} 17 & -7 \\ -26 & -53 \end{pmatrix}$

- (f) $\begin{pmatrix} -14 & -20 \\ -56 & -60 \end{pmatrix}$

2. (a) $\begin{pmatrix} 9 & 1 \\ -3 & -4 \end{pmatrix}$ (b) $\begin{pmatrix} 16 & 2 \\ -1 & -2 \end{pmatrix}$

- (c) $\begin{pmatrix} 9 & -2 \\ 19 & 8 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 \\ -6 & -5 \end{pmatrix}$

- (e) $\begin{pmatrix} 9 & 2 \\ 0 & 1 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & -6 \\ 18 & 13 \end{pmatrix}$

- (g) $\begin{pmatrix} -1 & 4 \\ -8 & 7 \end{pmatrix}$ (h) $\begin{pmatrix} 2\frac{1}{2} & 2\frac{1}{2} \\ -1\frac{1}{2} & 3\frac{1}{2} \end{pmatrix}$

3. (a) $m - 10 = -7$, so $m = 3$
 $n = 18$
 (b) $0 + 2m = 6$, so $m = 3$
 $1 + 3 = n$, so $n = 4$
4. (a) (i) $\begin{pmatrix} 8 & -8 \\ 31 & -20 \end{pmatrix}$ (ii) $\begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}$ (iii) $\begin{pmatrix} 5 & -4 \\ 2 & 0 \end{pmatrix}$
 (b) The original matrix stays the same.
 (c) Answers will vary, but learners will see that multiplying by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ will leave them with the same matrix that they started with.

Revision exercise

1. (a) 2×4 (b) 2×2 (c) 3×3 (d) 1×4
2. (b) and (c)
3. (a) $\begin{pmatrix} 8 & 2 & 2 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 \\ 1 & -3 \end{pmatrix}$
 (c) $\begin{pmatrix} 1 & 0 & 1 \\ 8 & 4 & 3 \\ 0 & -1 & 8 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -2 \\ -1 & -4 \\ -8 & 0 \end{pmatrix}$
4. (a) $\begin{pmatrix} 25 & 20 \\ 1.25 & 2.50 \end{pmatrix}$ (b) $\begin{pmatrix} 300 & 240 \\ 15 & 30 \end{pmatrix}$

Clown Around Circus will cost P315 and Whizz Bang Circus will cost P270. It will be cheaper to go to Whizz Bang Circus.

5. (a) 12 (b) Not possible
 (c) Not possible (d) $\begin{pmatrix} 17 & -6 \\ -3 & 2 \end{pmatrix}$
 (e) $\begin{pmatrix} 17 & 6 \\ 3 & 2 \end{pmatrix}$ (f) $\begin{pmatrix} 30 & 12 \\ -6 & 0 \end{pmatrix}$
 (g) $\begin{pmatrix} 1\frac{1}{2} & -1 \\ \frac{1}{2} & 1 \end{pmatrix}$ (h) Not possible
 (i) Not possible (j) $\begin{pmatrix} 34 & -12 \\ -6 & 4 \end{pmatrix}$

Chapter 6 Angle properties

- 75°
- (a) $c = 48^\circ$ (b) $a = 30^\circ$
(c) $c = 30^\circ$ (d) 51°
(e) $b = 50^\circ; c = 50^\circ$ (f) $b = 63^\circ;$
 $a = 49^\circ; c = 68^\circ$
- (a) $q = 79^\circ$ (b) $j = 66^\circ$ (c) $u = 53^\circ$
(d) $g = 36^\circ$ (e) $o = 45^\circ$
- (a) $3 = 87^\circ; 2 = 93^\circ; 1 = 59^\circ$
(b) $3 = 92^\circ; 2 = 88^\circ; 1 = 63^\circ$
- (a) $x + 2x + x + 20^\circ = 180^\circ$
(b) $4x = 160^\circ$
 $x = 40^\circ$
(c) $c = 80^\circ; d = 40^\circ; e = 60^\circ$
- Challenge:**
 $2x + 3x + 4x = 180^\circ$
 $\therefore 9x = 180^\circ$
 $\therefore x = 20^\circ$
 $e = 40^\circ; f = 60^\circ; g = 80^\circ$

Exercise 6.2

- 54°
- PQ = PR
- (a) $b = c = 62^\circ$ (b) $c = 35^\circ; b = 110^\circ$
(c) $b = c = 48^\circ; a = 84^\circ$ (d) $b = c = 65^\circ$
- E = 45°
- If 80° is the vertex angle, then the base angles will be 50° each. If 80° is one of the base angles, then the other base angle will be 80° and the vertex angle will be 20° .

Activity 6.1

Let the triangle be triangle ABC. $AB = AC$. We know that angle B = angle C since the triangle is isosceles. If angle B is greater than or equal to 90° , then angle C is greater than or equal to 90° . Let us assume that angles B and C are equal to 90° . If $B = C = 90^\circ$, then angle $B + \text{angle } C = 180^\circ$. But the sum of the three interior angles of a triangle is 180° so that leaves 0° for angle A. Angle A cannot be 0° or else there will be no vertex at A. Therefore, angles B and C cannot be equal to 90° . It follows that if angle B and angle C are greater than 90° , then their sum will be greater than 180° . We cannot have this since the interior angles of any triangle always add up to 180° and no more. Therefore, angles B and C must be acute (less than 90°).

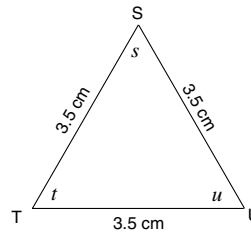
Activity 6.2

Triangle MNO is an equilateral triangle which means that angle M = angle N = angle O. Let each angle be equal to x . Since the sum of the interior angles of a triangle is 180° , $x + x + x = 180^\circ$. So, $3x = 180^\circ$ and $x = 60^\circ$. Therefore, every angle of an equilateral triangle is equal to 60° .

Exercise 6.3

- (a) $g = h = i = 60^\circ$

2.



Angle $t = 60^\circ$

- (a) $a = b = c = 60^\circ$
(b) $x = 60^\circ$ therefore $a = b = c = 60^\circ$
(c) $a = b = c = 60^\circ$ (d) $x = y = z = 60^\circ$
- Yes DEF is an equilateral triangle.
Reasons:
DE = EF because DE and EF are radii of the circle. Therefore, base angles must be equal as this triangle is an isosceles triangle. But vertex angle E is 60° , so angles D and F must also be 60° . We can therefore say that the triangle is an equilateral triangle.
- (a) and (b) Shape JKMN drawn with line KL drawn too.
(c) Since KMNL is a rectangle, opposite sides are equal in length. Therefore, $KL = 5$ units in length. But we have already been given that $JK = JL = 5$ units. Therefore, triangle JKL is an equilateral triangle. Therefore, $j = k = l = 60^\circ$.
(d) An equilateral triangle

Activity 6.3

Investigation:

It does not matter where the house is built. The sum of the three driveway lengths will always be the same. This only works for an equilateral triangle, not other triangles. Pick any point P inside an equilateral triangle. The sum of the perpendicular distances from any point P inside the triangle to the three sides of the triangle is always the same.

Activity 6.4

Proving that $r = m + n$:

$m + n + o = 180^\circ$ (interior angles of a triangle)

But $o + r = 180^\circ$ (adjacent angles on a straight line)

$$o + r = m + n + o$$

$$\begin{aligned} \therefore r &= m + n + o - o \\ &= m + n \end{aligned}$$

Proving that $p = n + o$:

$m + n + o = 180^\circ$ (interior angles of a triangle)

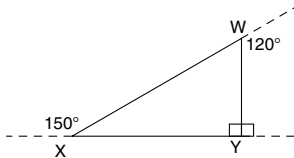
But $p + m = 180^\circ$ (adjacent angles on a straight line)

$$p + m = m + n + o$$

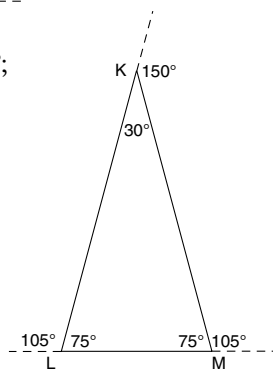
$$\begin{aligned} \therefore p &= m + n + o - m \\ &= n + o \end{aligned}$$

Exercise 6.4

- (a) $x = 113^\circ$ (b) $x = 55^\circ$
(c) $x = 130^\circ$ (d) $x = 60^\circ$
- (a) Exterior angle at N = 120°
(b) $l = 50^\circ$; $m = 70^\circ$ and $n = 60^\circ$
- (a) Exterior angle at Y = 90° and at W = 120°
(b)



- Exterior angle at K = 150° ;
exterior angle at L = 105° ;
exterior angle at M = 105°



- 120°

Activity 6.5

Investigation:

A complete revolution is 360° . Students will see that as they walk around the triangle, they are turning their bodies at each exterior angle. When they end, they are left facing where they started. They have walked a complete revolution. This helps to understand that the sum of the exterior angles of a triangle is 360° .

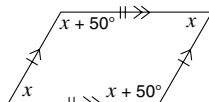
Exercise 6.5

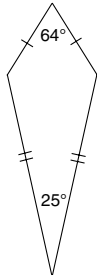
- $x = 60^\circ$; $y = 120^\circ$ (b) $y = 95^\circ$; $x = 37^\circ$
- $x = 57^\circ$; $y = 123^\circ$
- $y = 77^\circ$; $x = 26^\circ$; $z = 103^\circ$
- $x = 91^\circ$; $y = 55^\circ$; $z = 34^\circ$
- $4x + 10^\circ = 180^\circ$
 $\therefore 4x = 170^\circ$
 $\therefore x = 42.5^\circ$, so angle $a = 52.5^\circ$, $b = 42.5^\circ$ and $c = 85^\circ$

- $x = 60^\circ$; $y = z = 30^\circ$
- $x = y = 77^\circ$; $z = 154^\circ$
- $x = 115^\circ$; $y = 125^\circ$; $z = 120^\circ$
- $x = y = 45^\circ$; $z = 135^\circ$

Exercise 6.6

- (a) All angles are equal to 90° .
(b) Opposite angles are equal. Diagonals bisect at 90° .
(c) One pair of opposite angles are equal. Diagonals intersect at 90° .
(d) All angles are equal to 90° . Diagonals bisect at 90° .
(e) No angles are necessarily equal. For a right-angled trapezium, one angle is equal to 90° . For an isosceles trapezium, base angles are equal.
(f) Opposite angles are equal.
- (a) 68° (b) 60° (c) 132° (d) 70°
(e) 141° (f) 45°
- (a) $a = b = c = d = 90^\circ$
(b) $q = 43^\circ$; $p = r = 137^\circ$
(c) $k = m = 90^\circ$
(d) $x = 120^\circ$; $y = 40^\circ$; $w = 80^\circ$
(e) $e = g = 110^\circ$; $f = h = 70^\circ$
(f) $v = 45^\circ$ (alternate angles); $w = 135^\circ$; $t = 90^\circ$
(g) $d = b = 96^\circ$; $a = c = 84^\circ$
(h) $o = 70^\circ$ (corresponding angles); $m = 128^\circ$
- (a) $k = 75^\circ$; $i = 102^\circ$
(b) $x = 80^\circ$; $u = 105^\circ$; $v = 95^\circ$
- 78°

- 

- 

- (a) $x = 30^\circ$ and $2x = 60^\circ$
(b) $a = 74^\circ$; $c = 44^\circ$; $b = 118^\circ$
- Yes, if we know one angle, then we know two angles, since opposite angles are equal. Once we know two angles, we can subtract their sum from 360° to find the sum of the two remaining angles. Since the two remaining angles are equal, we can divide their sum by 2 to find each individual angle.

10. The two smaller angles are equal to 83° . The two larger angles are equal to 97° .

Revision exercise

- (a) $x = 73^\circ$ (b) $x = 50^\circ$ (c) $x = y = 67.5^\circ$
(d) $x = 30^\circ; y = 121^\circ$ (e) $x = y = z = 60^\circ$
(f) $x = 88^\circ; y = 115^\circ; z = 153^\circ$
- (a) $x = 40^\circ; y = 130^\circ$
(b) $x = 65^\circ; y = z = 115^\circ$
(c) $x = 96^\circ$ (d) $x = 120^\circ; y = 65^\circ$
(e) $x = 63^\circ; y = 104^\circ$
(f) $x = 49^\circ; y = 90^\circ; z = 131^\circ$